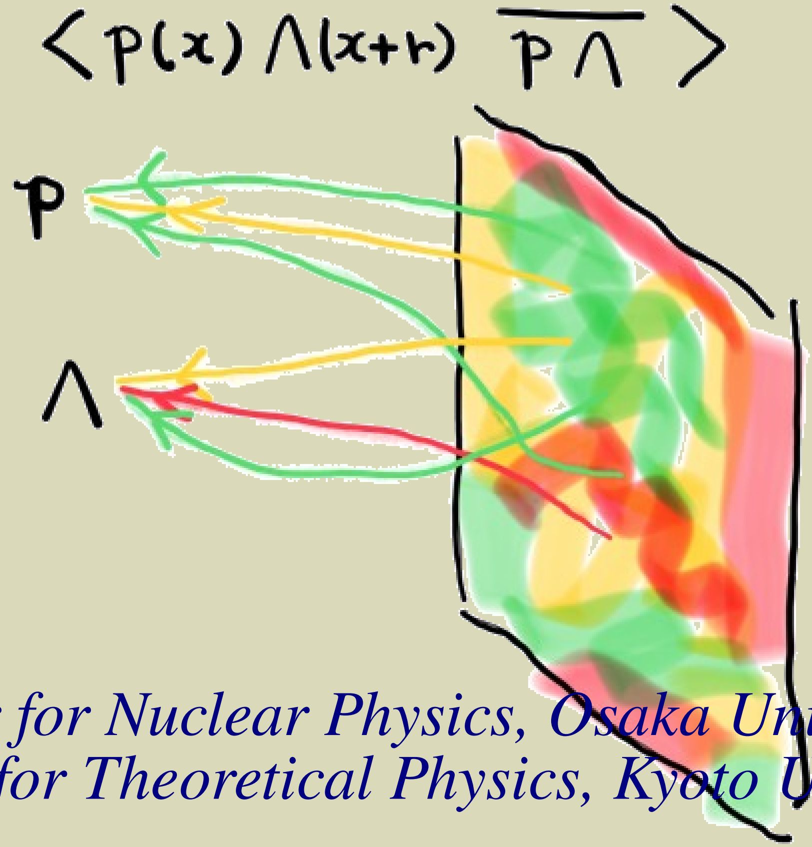
Lambda-Nucleon and Sigma-Nucleon potentials from space-time correlation function on the lattice

H. Nemura[1],



[1]*Reseach Center for Nuclear Physics, Osaka University, Japan*
Yukawa Institute for Theoretical Physics, Kyoto University, Japan

arXiv:1510.00903 [hep-lat]
arXiv:1810.04046 [hep-lat]

Outline

- Introduction
  - HAL QCD method for baryon-baryon interaction
- Preliminary results of LN-SN potentials at $(m_\pi, m_K) \approx (145, 525)$MeV
  - Single channel analysis for LN ==> central and tensor potentials
    - Phase shifts at low energy region below the SN threshold
  - LN-SN(I=1/2), central and tensor potentials
- Effective block algorithm for various baryon-baryon channels, CPC**207**,91(2016)[1510.00903]
  - New application of the algorithm
- Summary

Plan of research

QCD

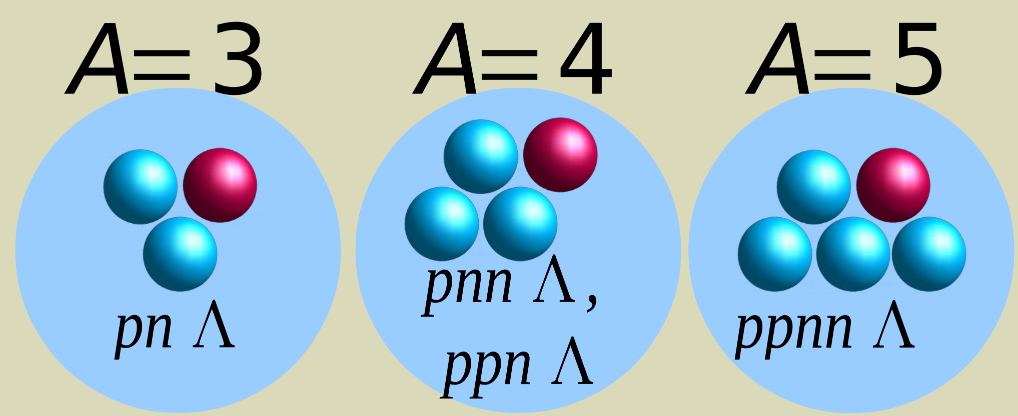
J-PARC,
JLab, GSI, MAMI, ...
YN scattering,
hypernuclei

Baryon interaction

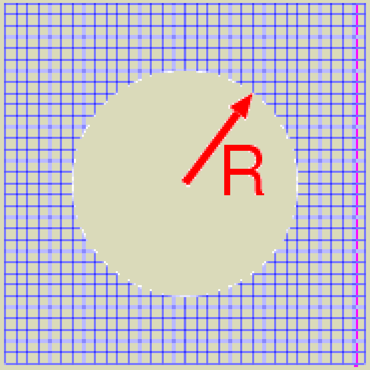
Structure and reaction of
(hyper)nuclei

Equation of State (EoS)
of nuclear matter

Neutron star and
supernova



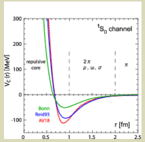
Multi-hadron on lattice



i) basic procedure:

asymptotic region

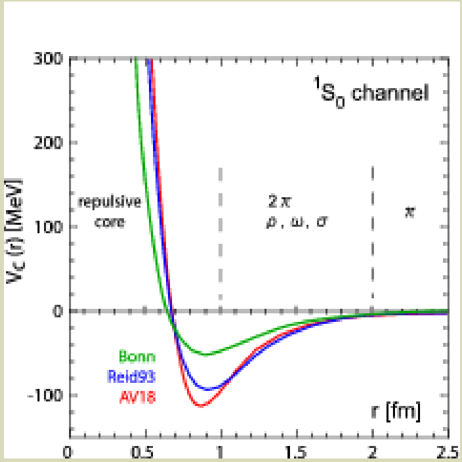
--> phase shift



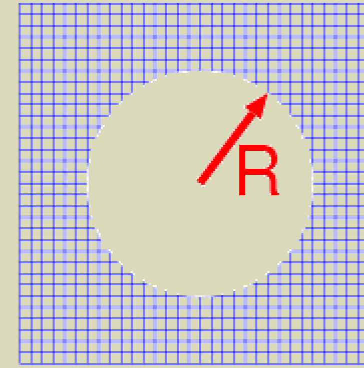
ii) HAL's procedure:

interacting region

--> potential



Multi-hadron on lattice



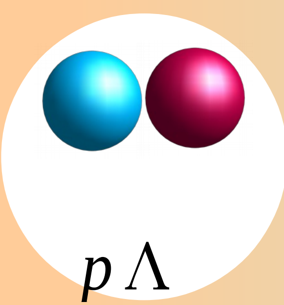
Lattice QCD simulation

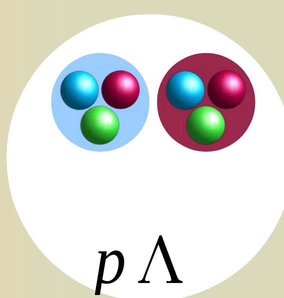
$$L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^\mu (i\,\partial_\mu - g\,t^a A^a_\mu) q - m\bar{q}q$$

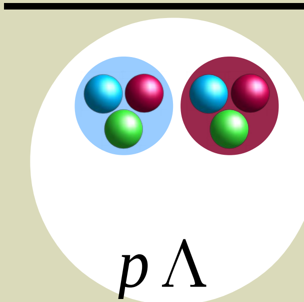
$$\langle O(\bar{q}, q, U)\rangle = \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U)$$

$$= \int dU \det D(U)\, e^{-S_U(U)} O(D^{-1}(U))$$

$$= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} O(D^{-1}(U_i))$$



$\rightarrow \langle$ 

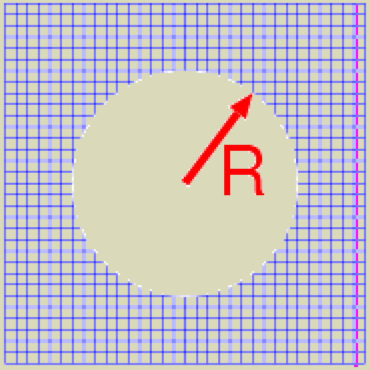
$(t)\;\overline{}$ 

$(t_0)\rangle$

Multi-hadron on lattice

i) basic procedure:

asymptotic region
(or temporal correlation)
--> scattering energy
--> phase shift



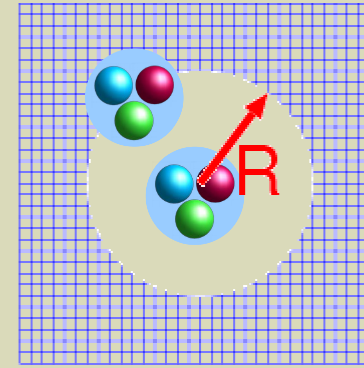
$$E = \frac{k^2}{2\mu}$$

$$k \cot \delta_0(k) = \frac{2}{\sqrt{\pi} L} Z_{00}(1; (kL/(2\pi))^2) = \frac{1}{a_0} + O(k^2)$$

$$Z_{00}(1; q^2) = \frac{1}{\sqrt{4\pi}} \sum_{\vec{n} \in \mathbf{Z}^3} \frac{1}{(n^2 - q^2)^s} \qquad \Re s > \frac{3}{2}$$

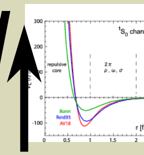
Luscher, NPB354, 531 (1991).
Aoki, et al., PRD71, 094504 (2005).

Multi-hadron on lattice



Lattice QCD simulation

$$L = -\frac{1}{4} G^a_{\mu\nu} G^{a\mu\nu} + \bar{q}\gamma^\mu (i\,\partial_\mu - g\, t^a A^a_\mu) q - m \bar{q} q$$



$$\langle O(\bar{q}, q, U) \rangle = \int dU\, d\bar{q}\, dq\, e^{-S(\bar{q}, q, U)} O(\bar{q}, q, U)$$

$$= \int dU \det D(U)\, e^{-S_U(U)} O(D^{-1}(U))$$

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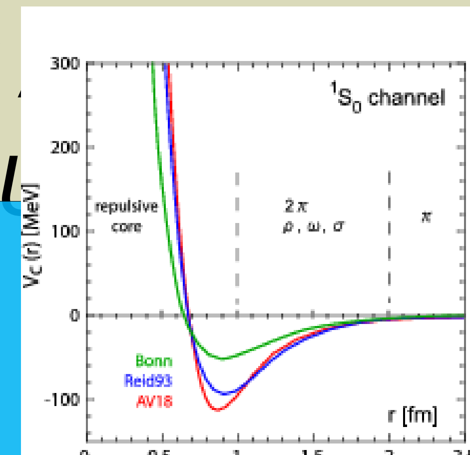
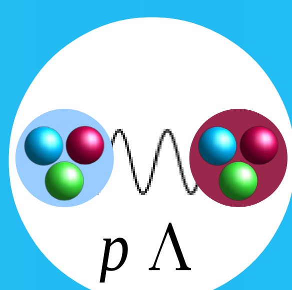
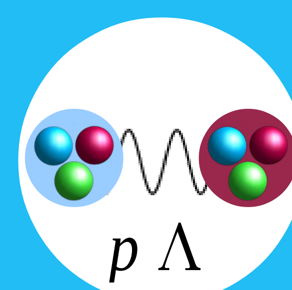


$$F^{(JM)}_{\alpha\beta}(\vec{r}, t - t_0)$$

$$\rightarrow \langle$$



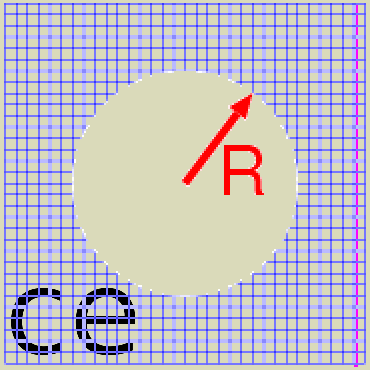
$$(\vec{r}, t)\ \overline{}$$



$$(t_0) \rangle$$

Calculate the scattering state

Multi-hadron on lattice



ii) HAL's procedure:

make better use of the lattice output ! (wave function)

interacting region

--> potential

Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., PTP123, 89 (2010).

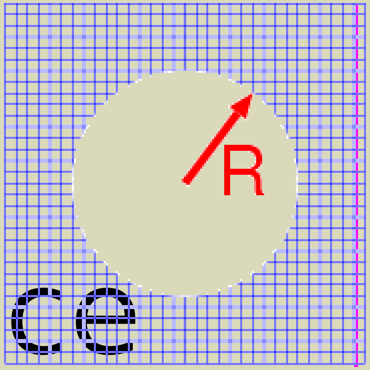
NOTE:
> Potential is not a direct experimental observable.
> Potential is a useful tool to give (and to reproduce) the physical quantities. (e.g., phase shift)

....

Multi-hadron on lattice

ii) HAL’s procedure:

make better use of the lattice output ! (wave function)



interacting region

--> potential

Ishii, Aoki, Hatsuda,
PRL99, 022001 (2007);
ibid., PTP123, 89 (2010).

=>

> Phase shift
> Nuclear many-body problems

An improved recipe for NY potential:

- cf. Ishii (HAL QCD), PLB712 (2012) 437.

- Take account of not only the spatial correlation but also the temporal correlation in terms of the R-correlator:

$$-\frac{1}{2\mu}\nabla^2 R(t,\vec{r}) + \int d^3r'\, U(\vec{r},\vec{r}\,')R(t,\vec{r}\,') = -\frac{\partial}{\partial t}R(t,\vec{r})$$

$$\to \frac{k^2}{2\mu}R(t,\vec{r})$$

- A general expression of the potential:

$$U(\vec{r},\vec{r}\,') = V_{NY}(\vec{r},\vec{\nabla})\delta(\vec{r}-\vec{r}\,')$$

$$V_{NY} = V_0(r) + V_\sigma(r)(\vec{\sigma}_N\cdot\vec{\sigma}_Y) + V_T(r)S_{12} + V_{LS}(r)(\vec{L}\cdot\vec{S}_+) + V_{ALS}(r)(\vec{L}\cdot\vec{S}_-) + O(\nabla^2)$$

The potential is obtained at moderately large imaginary time; no single state saturation is required.

[1] The potential is obtained from the NBS wave function at moderately large imaginary time; it would be $t - t_0 \gg 1/m_\pi \sim 1.4$ fm. In addition, no single state saturation between the ground state and the excited states with respect to the relative motion, e.g., $t - t_0 \gg (\Delta E)^{-1} = \left((2\pi)^2/(2\mu(La)^2)\right)^{-1} \simeq 8.0$ fm, is required for the HAL QCD method[13].

RECIPE:

Compute the 4pt correlator

$$F_{\alpha\beta,JM}^{\langle B_1B_2\overline{B_3B_4}\rangle}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t)B_{2,\beta}(\vec{X},t)\,\overline{\mathscr{J}_{B_3B_4}^{(J,M)}(t_0)} \right| 0 \right\rangle, \quad (2.3)$$

Take into account the threshold energy differences for coupled-channel system

$$R_{\alpha\beta,JM}^{\langle B_1B_2\overline{B_3B_4}\rangle}(\vec{r},t-t_0) = \mathrm{e}^{(m_{B_1}+m_{B_2})(t-t_0)} F_{\alpha\beta,JM}^{\langle B_1B_2\overline{B_3B_4}\rangle}(\vec{r},t-t_0)$$

$$= \sum_n A_n \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},0)B_{2,\beta}(\vec{X},0) \right| E_n \right\rangle \mathrm{e}^{-(E_n-m_{B_1}-m_{B_2})(t-t_0)} + O(\mathrm{e}^{-(E_{\rm th}-m_{B_1}-m_{B_2})(t-t_0)}) \quad (2.4)$$

elastic — inelastic

Obtain the potential by using the appropriate equation(s); For spin-singlet,

$$\left(\frac{\nabla^2}{2\mu_\lambda} - \frac{\partial}{\partial t}\right) R_{\lambda\varepsilon}(\vec{r},t) \simeq V_{\lambda\lambda'}^{(\mathrm{LO})}(\vec{r})\theta_{\lambda\lambda'}R_{\lambda'\varepsilon}(\vec{r},t), \text{ with } \theta_{\lambda\lambda'} = \mathrm{e}^{(m_{B_1}+m_{B_2}-m_{B_1'}-m_{B_2'})(t-t_0)}.$$

For spin-triplet, the "tensor force" becomes active

$$\begin{Bmatrix}\mathscr{P}\\ \mathscr{Q}\end{Bmatrix} \times \left\{ V_{\lambda\lambda'}^{(0)}(r) + V_{\lambda\lambda'}^{(\sigma)}(r) + V_{\lambda\lambda'}^{(T)}(r)S_{12} \right\} \theta_{\lambda\lambda'}R_{\lambda'\varepsilon}(\vec{r},t-t_0) = \begin{Bmatrix}\mathscr{P}\\ \mathscr{Q}\end{Bmatrix} \times \left\{ \frac{\nabla^2}{2\mu_\lambda} - \frac{\partial}{\partial t} \right\} R_{\lambda\varepsilon}(\vec{r},t-t_0) \quad (2.7)$$

Where

$$\begin{cases} R(\vec{r};\,^3S_1) = \mathscr{P}R(\vec{r};J=1) \equiv \frac{1}{24}\sum_{\mathscr{R}\in O}\mathscr{R}R(\vec{r};J=1), \\ R(\vec{r};\,^3D_1) = \mathscr{Q}R(\vec{r};J=1) \equiv (1-\mathscr{P})R(\vec{r};J=1). \end{cases} \quad (2.6)$$

In the lowest few orders, we have

$$V(\vec{r},\vec{\nabla}_r) = V^{(0)}(r) + V^{(\sigma)}(r)\vec{\sigma}_1\cdot\vec{\sigma}_2 + V^{(T)}(r)S_{12} + V^{(LS)}_{(ALS)}(r)\vec{L}\cdot(\vec{\sigma}_1 \pm \vec{\sigma}_2) + O(\nabla^2), \quad (2.5)$$

Almost physical point lattice QCD calculation using N_F=2+1 clover fermion + Iwasaki gauge action

- APE-Stout smearing (ρ=0.1, n_{stout}=6)
- Non-perturbatively O(a) improved Wilson Clover action at β=1.82 on $96^3 \times 96$ lattice
- $1/a$ = 2.3 GeV (a = 0.085 fm)
- Volume: $96^4 \rightarrow (8\text{fm})^4$
- m_ρ=145MeV, m_K=525MeV
- DDHMC(ud) and UVPHMC(s) with preconditioning
- K.-I.Ishikawa, et al., PoS LAT2015, 075; arXiv:1511.09222 [hep-lat].

- NBS wf is measured using wall quark source with Coulomb gauge fixing, spatial PBD and temporal DBC; #stat=207configs x 4rotation x 96src

In lattice QCD calculations, we compute the normalized four-point correlation function

$$R_{\alpha\beta}^{(J,M)}(\vec{r},t-t_0) = \sum_{\vec{X}} \left\langle 0 \left| B_{1,\alpha}(\vec{X}+\vec{r},t) B_{2,\beta}(\vec{X},t) \overline{\mathcal{J}_{B_3B_4}^{(J,M)}(t_0)} \right| 0 \right\rangle / \exp\{-(m_{B_1}+m_{B_2})(t-t_0)\},$$

$$p = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) u_c, \qquad n = -\varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) d_c, \tag{2}$$

$$\Sigma^+ = -\varepsilon_{abc}\left(u_a C\gamma_5 s_b\right) u_c, \qquad \Sigma^- = -\varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) d_c, \tag{3}$$

$$\Sigma^0 = \frac{1}{\sqrt{2}}\left(X_u - X_d\right), \qquad \Lambda = \frac{1}{\sqrt{6}}\left(X_u + X_d - 2X_s\right), \tag{4}$$

$$\Xi^0 = \varepsilon_{abc}\left(u_a C\gamma_5 s_b\right) s_c, \qquad \Xi^- = -\varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) s_c, \tag{5}$$

where

$$X_u = \varepsilon_{abc}\left(d_a C\gamma_5 s_b\right) u_c, \quad X_d = \varepsilon_{abc}\left(s_a C\gamma_5 u_b\right) d_c, \quad X_s = \varepsilon_{abc}\left(u_a C\gamma_5 d_b\right) s_c, \tag{6}$$

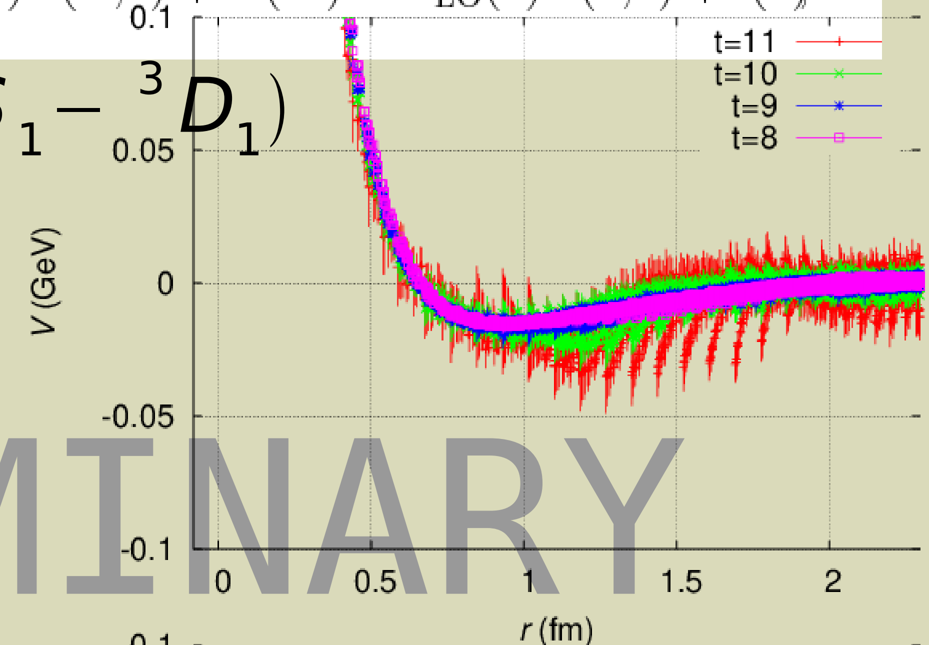
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots \tag{8}$$

Preliminary result of LN potential at the $(m_\pi, m_K) \approx (145, 525)$MeV

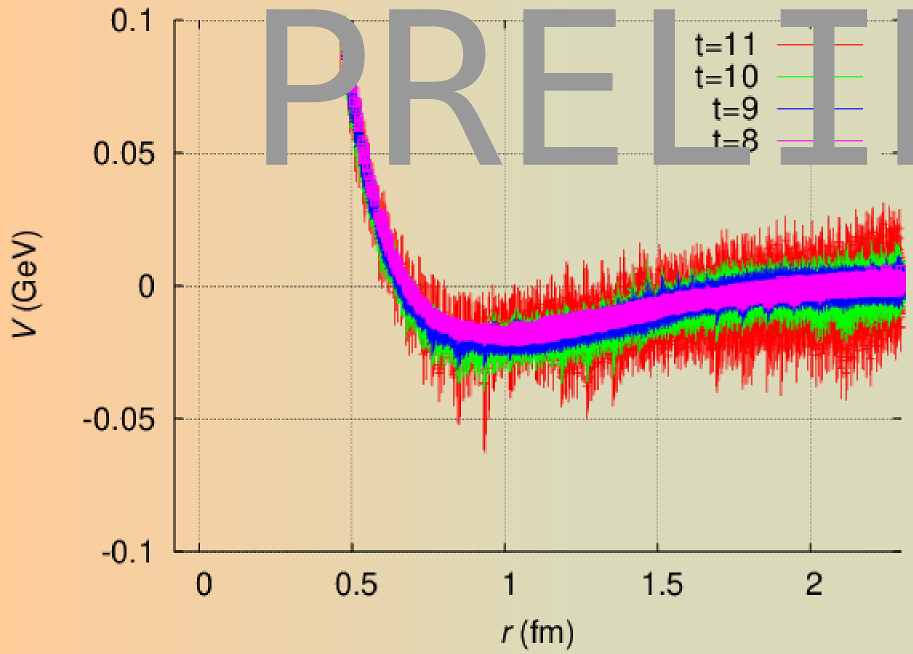
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) = \int d^3r'\, U(\vec{r},\vec{r}\,') R(\vec{r}\,',t) + O(k^4) = V_{\mathrm{LO}}(\vec{r}) R(\vec{r},t) + \cdots (8)$$

ΛN

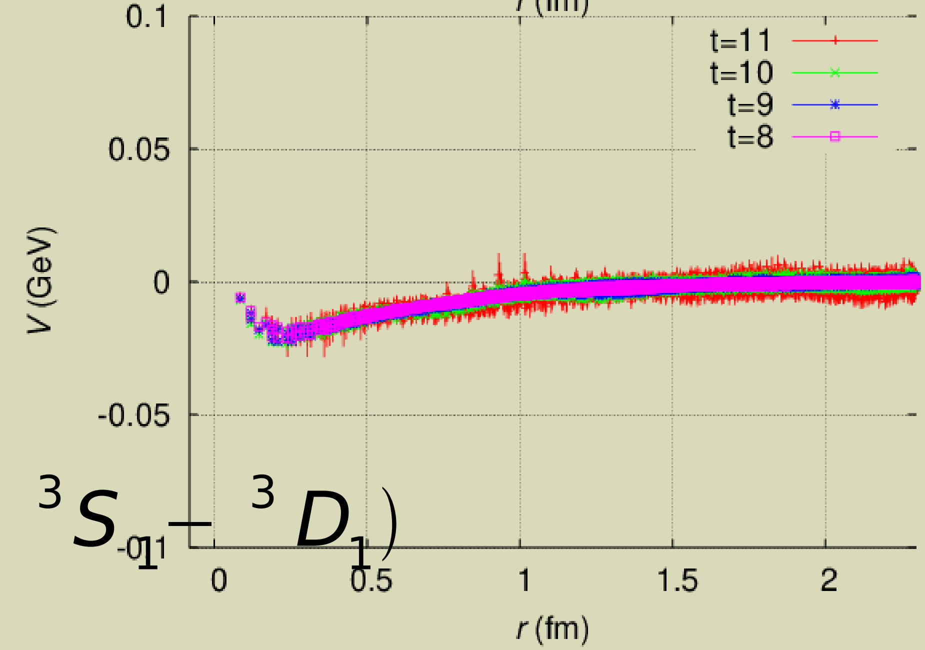
$V_C(\,^3S_1 - \,^3D_1)$



PRELIMINARY

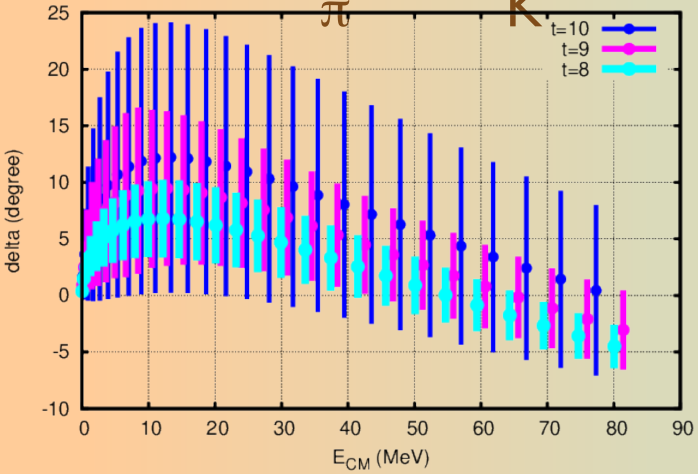


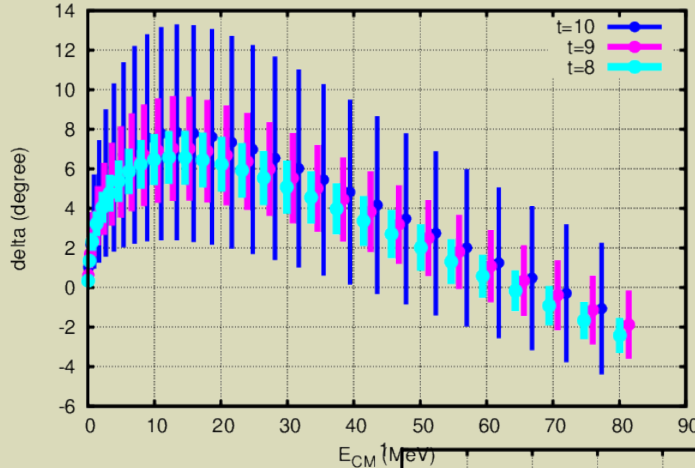
$V_C(\,^1S_0)$



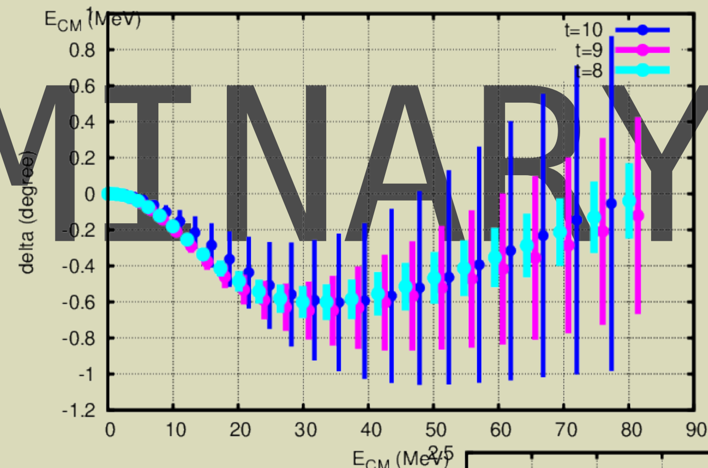
$V_T(\,^3S_1 - \,^3D_1)$

Preliminary results of the LN phase shift at (m_π, m_K)≈(145, 525)MeV

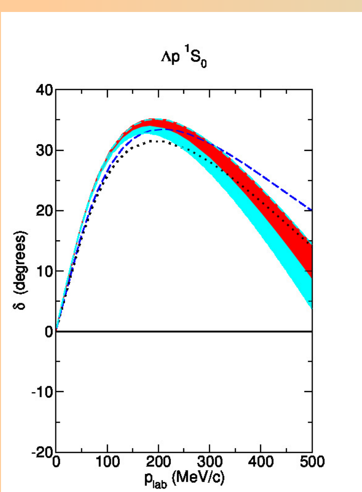


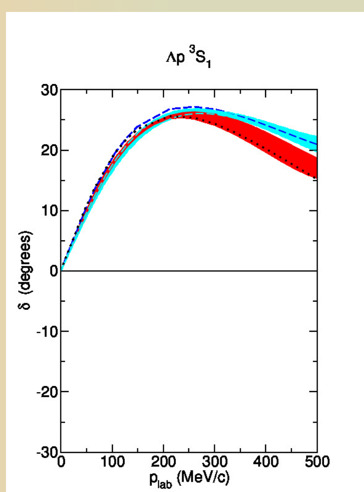


ΛN

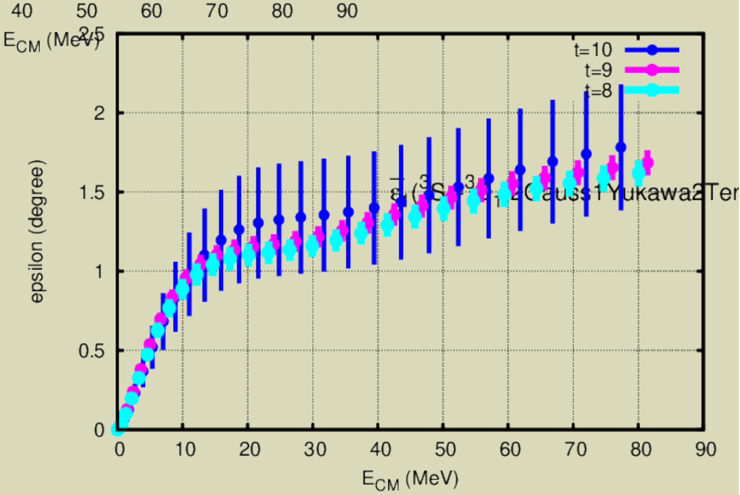


PRELIMINARY





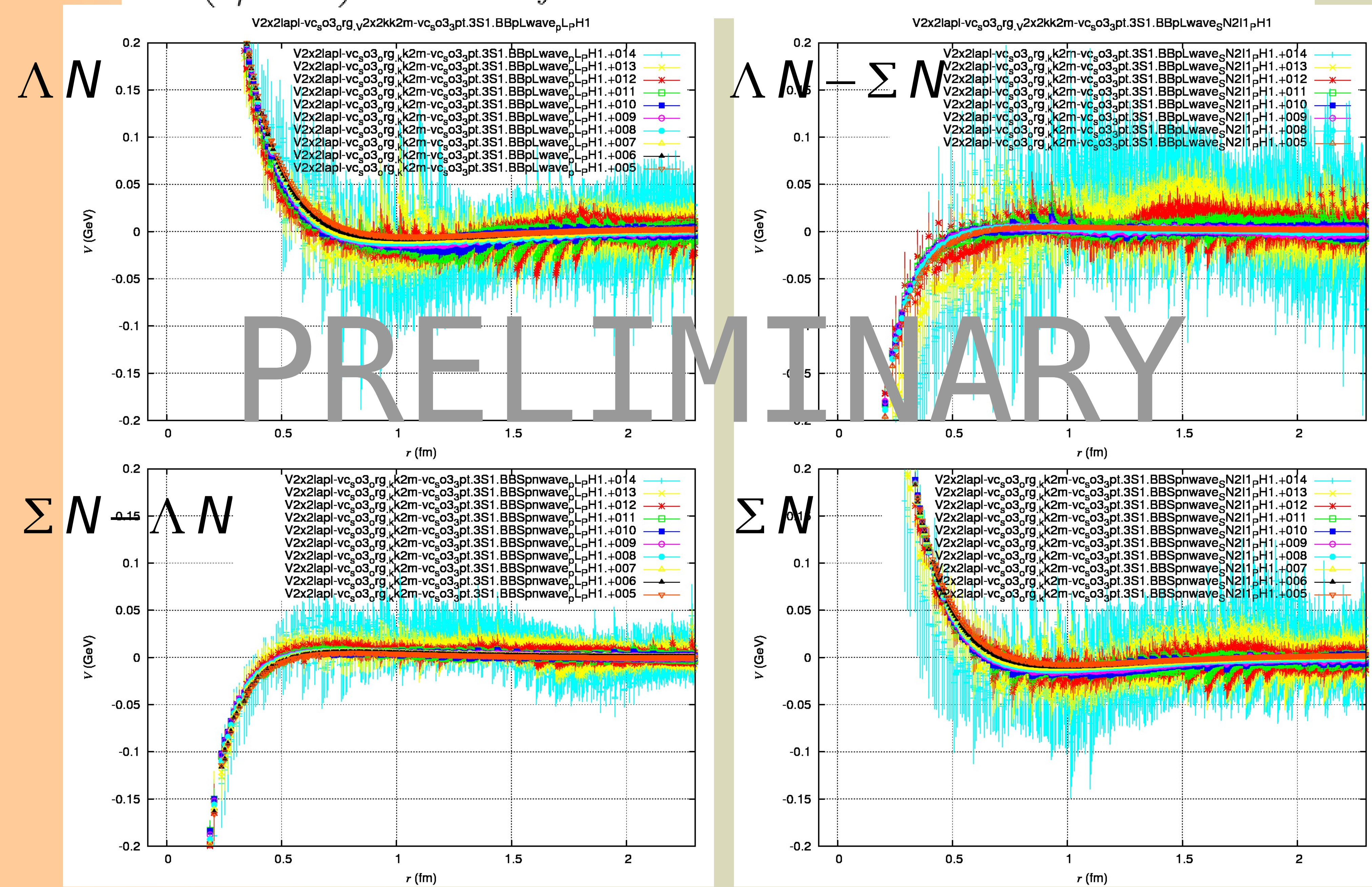
Haidenbauer et al., Eur.Phys.J.A(2020)56:91.



Very preliminary result of LN potential at the physical point

$$V_C(\,^3S_1 - \,^3D_1)$$

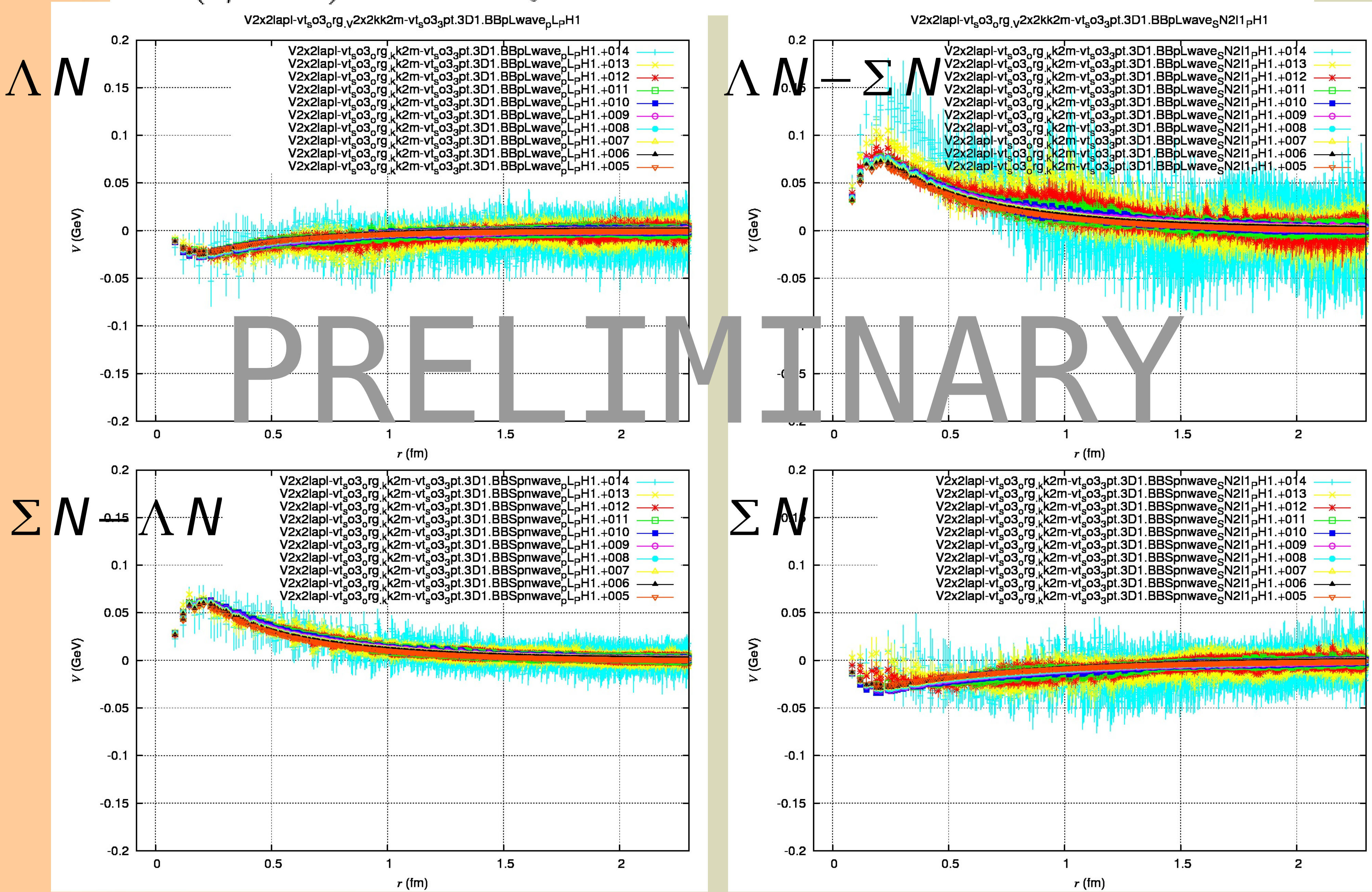
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$$V_T({}^3S_1 - {}^3D_1)$$

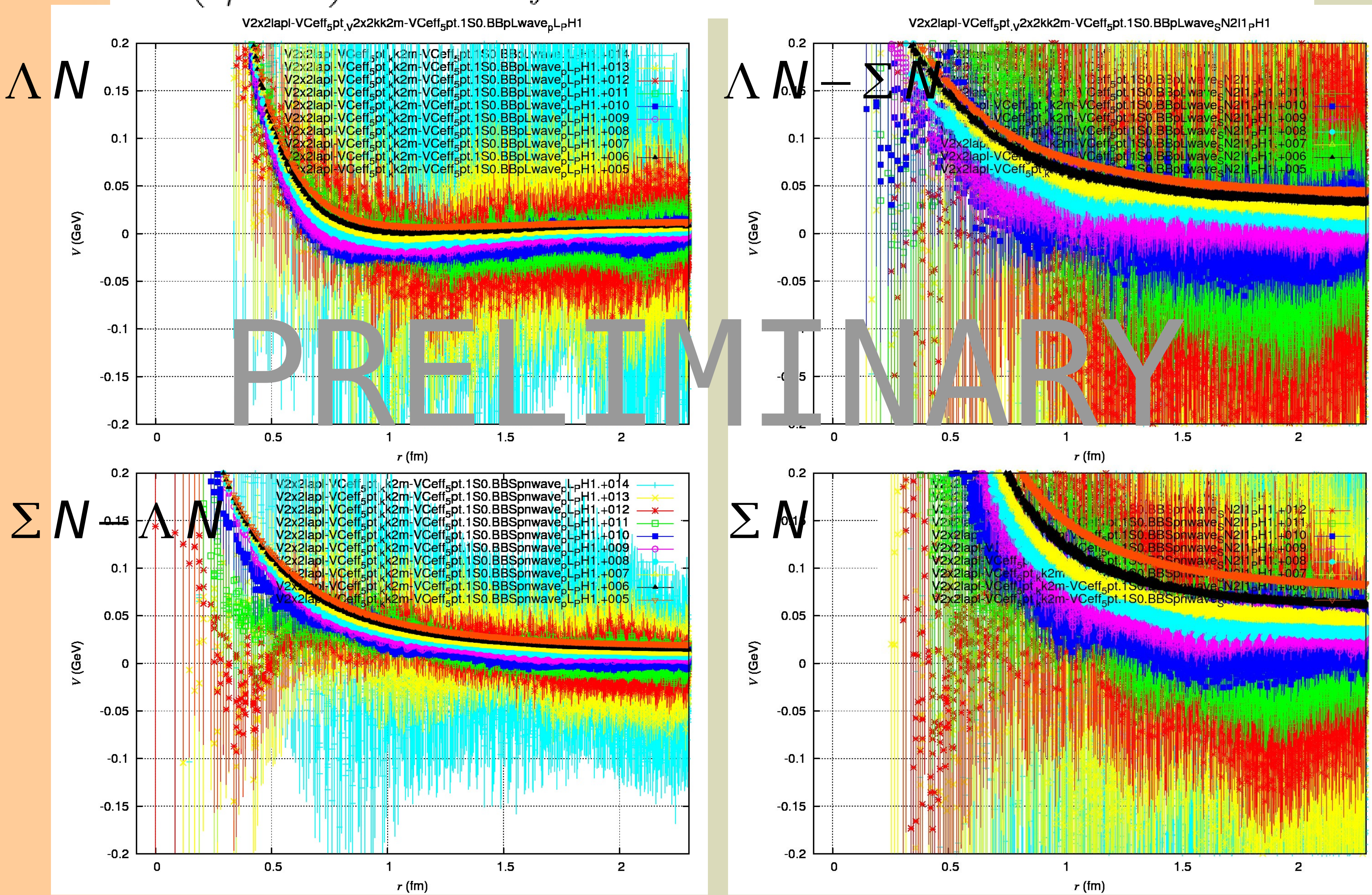
$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$



Very preliminary result of LN potential at the physical point

$V_C(\,^1S_0)$

$$\left(\frac{\nabla^2}{2\mu} - \frac{\partial}{\partial t}\right) R(\vec{r},t) \;=\; \int d^3r'\, U(\vec{r},\vec{r}')R(\vec{r}',t) + O(k^4) = V_{\rm LO}(\vec{r})R(\vec{r},t) + \cdots (8)$$



Generalization to the various baryon-baryon channels strangeness S=0 to -4 systems

$$\langle pn\overline{pn}\rangle, \tag{4.1}$$

$$\begin{aligned}
&\langle p\Lambda\overline{p\Lambda}\rangle,\ \langle p\Lambda\overline{\Sigma^+ n}\rangle,\ \langle p\Lambda\overline{\Sigma^0 p}\rangle,\\
&\langle \Sigma^+ n\overline{p\Lambda}\rangle,\ \langle \Sigma^+ n\overline{\Sigma^+ n}\rangle,\ \langle \Sigma^+ n\overline{\Sigma^0 p}\rangle,\\
&\langle \Sigma^0 p\overline{p\Lambda}\rangle,\ \langle \Sigma^0 p\overline{\Sigma^+ n}\rangle,\ \langle \Sigma^0 p\overline{\Sigma^0 p}\rangle,
\end{aligned} \tag{4.2}$$

$$\begin{aligned}
&\langle \Lambda\Lambda\overline{\Lambda\Lambda}\rangle,\ \langle \Lambda\Lambda\overline{p\Xi^-}\rangle,\ \langle \Lambda\Lambda\overline{n\Xi^0}\rangle,\ \langle \Lambda\Lambda\overline{\Sigma^+\Sigma^-}\rangle,\ \langle \Lambda\Lambda\overline{\Sigma^0\Sigma^0}\rangle,\\
&\langle p\Xi^-\overline{\Lambda\Lambda}\rangle,\ \langle p\Xi^-\overline{p\Xi^-}\rangle,\ \langle p\Xi^-\overline{n\Xi^0}\rangle,\ \langle p\Xi^-\overline{\Sigma^+\Sigma^-}\rangle,\ \langle p\Xi^-\overline{\Sigma^0\Sigma^0}\rangle,\ \langle p\Xi^-\overline{\Sigma^0\Lambda}\rangle,\\
&\langle n\Xi^0\overline{\Lambda\Lambda}\rangle,\ \langle n\Xi^0\overline{p\Xi^-}\rangle,\ \langle n\Xi^0\overline{n\Xi^0}\rangle,\ \langle n\Xi^0\overline{\Sigma^+\Sigma^-}\rangle,\ \langle n\Xi^0\overline{\Sigma^0\Sigma^0}\rangle,\ \langle n\Xi^0\overline{\Sigma^0\Lambda}\rangle,\\
&\langle \Sigma^+\Sigma^-\overline{\Lambda\Lambda}\rangle,\ \langle \Sigma^+\Sigma^-\overline{p\Xi^-}\rangle,\ \langle \Sigma^+\Sigma^-\overline{n\Xi^0}\rangle,\ \langle \Sigma^+\Sigma^-\overline{\Sigma^+\Sigma^-}\rangle,\ \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Sigma^0}\rangle,\ \langle \Sigma^+\Sigma^-\overline{\Sigma^0\Lambda}\rangle,\\
&\langle \Sigma^0\Sigma^0\overline{\Lambda\Lambda}\rangle,\ \langle \Sigma^0\Sigma^0\overline{p\Xi^-}\rangle,\ \langle \Sigma^0\Sigma^0\overline{n\Xi^0}\rangle,\ \langle \Sigma^0\Sigma^0\overline{\Sigma^+\Sigma^-}\rangle,\ \langle \Sigma^0\Sigma^0\overline{\Sigma^0\Sigma^0}\rangle,\\
&\langle \Sigma^0\Lambda\overline{p\Xi^-}\rangle,\ \langle \Sigma^0\Lambda\overline{n\Xi^0}\rangle,\ \langle \Sigma^0\Lambda\overline{\Sigma^+\Sigma^-}\rangle,\ \langle \Sigma^0\Lambda\overline{\Sigma^0\Lambda}\rangle,
\end{aligned} \tag{4.3}$$

$$\begin{aligned}
&\langle \Xi^-\Lambda\overline{\Xi^-\Lambda}\rangle,\ \langle \Xi^-\Lambda\overline{\Sigma^-\Xi^0}\rangle,\ \langle \Xi^-\Lambda\overline{\Sigma^0\Xi^-}\rangle,\\
&\langle \Sigma^-\Xi^0\overline{\Xi^-\Lambda}\rangle,\ \langle \Sigma^-\Xi^0\overline{\Sigma^-\Xi^0}\rangle,\ \langle \Sigma^-\Xi^0\overline{\Sigma^0\Xi^-}\rangle,\\
&\langle \Sigma^0\Xi^-\overline{\Xi^-\Lambda}\rangle,\ \langle \Sigma^0\Xi^-\overline{\Sigma^-\Xi^0}\rangle,\ \langle \Sigma^0\Xi^-\overline{\Sigma^0\Xi^-}\rangle,
\end{aligned} \tag{4.4}$$

$$\langle \Xi^-\Xi^0\overline{\Xi^-\Xi^0}\rangle. \tag{4.5}$$

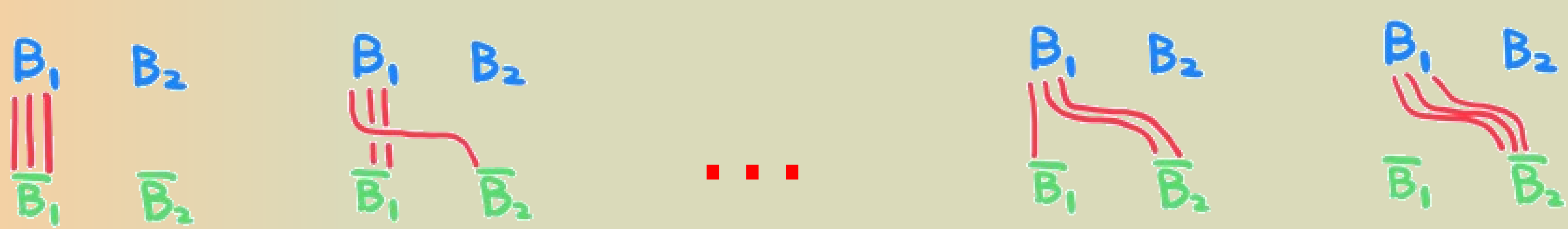
Make better use of the computing resources!

HN, CPC **207**, 91(2016) [arXiv:1510.00903[hep-lat]],
(See also arXiv:1604.08346)

Classification of baryon blocks in the effective block algorithm

- The number of declared blocks in terms of quark propagation form, i.e., from [111] to [222], in the simultaneous calculation of 4pt correlators from NN to ΞΞ

- Proton: 18+ 0+ 31+ 0+106+ 16+121+ 12 = 304
- Σ^+: 3+ 0+ 10+ 0+ 52+ 3+ 55+ 1 = 124
- Ξ^0: 16+ 19+ 0+ 0+118+102+ 29+ 14 = 298
- Λ(dsu): 242+318+436+408+290+266+376+248 = 2584
- Λ(sud): 94+164+102+132+130+164+102+ 96 = 984
- Λ(uds): 94+102+130+102+164+132+164+ 96 = 984



Summary

(I-1) LN potentials (central, tensor) at $(m_\pi, m_K) \approx (145, 525)$MeV.
phase shifts below the SN threshold

Both channels are attractive. (but weaker than empirical values)

Spin dependence is very weak. Relatively large statistical uncertainty.

(I-2) Effective block algorithm for the various baron-baryon interaction

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Simultaneous calcs (NN to XiXi) is the point we have to take into account for the comprehensive perspective as well as energy-computing-resource efficiency.

The algorithm will be applied to more wide range problems.

Future work:

(II-1) Physical quantities including the binding energies of few-body problem of light hypernuclei with the lattice YN (and NN) potentials

(II-2) New application of effective baryon block algorithm for the various baron-baryon interaction from NN to ΞΞ.

> Classification of baryon blocks from NN to ΞΞ, which comprises 52 4pt-correlators (2639 diagrams)

> Search for better approach to increase the accuracy.

> Spin-orbit force.