Progress on Meson-Baryon Scattering

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Introduction

- goal: determine meson-baryon and baryon-baryon scattering parameters in large number of flavor channels
- use of stochastic LapH method to handle slice-to-slice quark propagators
- analysis using "box matrix" B and scattering K-matrix
- analysis can include higher partial waves, multi-channels
- show preliminary results:
  - for $I = \frac{1}{2}, \frac{3}{2}$ $N\pi$ amplitudes including $\Delta(1232)$
  - $I = 0$ strangeness $S = 1$ s-wave amplitude relevant for $\Lambda(1405)$

- key collaborators: John Bulava, Ben Hörz, Drew Hanlon, Daniel Mohler, Amy Nicholson, Sarah Skinner, André Walker-Loud
- other collaborators: CalLat, Mainz
- acknowledgment: NSF, TACC Frontera

Excited states from correlation matrices

- energies from temporal correlations $C_{ij}(t) = \langle 0|\overline{O}_i(t) O_j(0)|0\rangle$
- in finite volume, energies are discrete (neglect wrap-around)

$$C_{ij}(t) = \sum_n Z_i^{(n)} Z_j^{(n)*}\, e^{-E_n t}, \qquad Z_j^{(n)} = \langle 0|\, O_j\, |n\rangle$$

- not practical to do fits using above form
- define new correlation matrix $\widetilde{C}(t)$ using a single rotation

$$\widetilde{C}(t) = U^\dagger\, C(\tau_0)^{-1/2}\, C(t)\, C(\tau_0)^{-1/2}\, U$$

- columns of U are eigenvectors of $C(\tau_0)^{-1/2}\, C(\tau_D)\, C(\tau_0)^{-1/2}$
- choose τ_0 and τ_D large enough so $\widetilde{C}(t)$ diagonal for $t > \tau_D$
- 2-exponential fits to $\widetilde{C}_{\alpha\alpha}(t)$ yield energies E_α and overlaps $Z_j^{(n)}$
- energy shifts from non-interacting using 1-exp fits to ratio of correlators

Building blocks for single-hadron operators

- operator construction from PRD 72, 094506 (2005) and PRD 88, 014511 (2013)
- building blocks: covariantly-displaced LapH-smeared quark fields
- stout links $\widetilde{U}_j(x)$
- Laplacian-Heaviside (LapH) smeared quark fields

$$\widetilde{\psi}_{a\alpha}(x) = \mathcal{S}_{ab}(x,y)\,\psi_{b\alpha}(y), \qquad \mathcal{S} = \Theta\left(\sigma_s^2 + \widetilde{\Delta}\right)$$

- 3d gauge-covariant Laplacian $\widetilde{\Delta}$ in terms of $\widetilde{U}$
- displaced quark fields:

$$q^A_{a\alpha j} = D^{(j)}\widetilde{\psi}^{(A)}_{a\alpha}, \qquad \overline{q}^A_{a\alpha j} = \overline{\widetilde{\psi}}^{(A)}_{a\alpha}\,\gamma_4\,D^{(j)\dagger}$$

- displacement $D^{(j)}$ is product of smeared links:

$$D^{(j)}(x,x') = \widetilde{U}_{j_1}(x)\,\widetilde{U}_{j_2}(x{+}d_2)\,\widetilde{U}_{j_3}(x{+}d_3)\dots\widetilde{U}_{j_p}(x{+}d_p)\delta_{x',\ x+d_{p+1}}$$

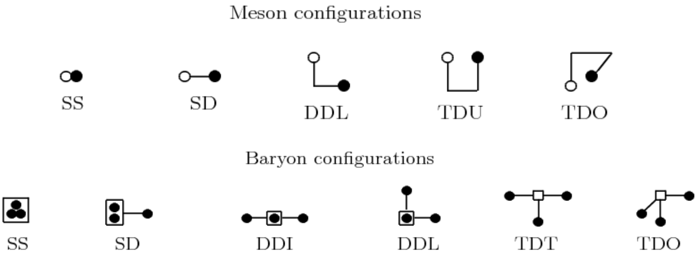
- to good approximation, LapH smearing operator is

$$\mathcal{S} = V_s V_s^\dagger$$

  - columns of matrix V_s are eigenvectors of $\widetilde{\Delta}$

Extended operators for single hadrons

- quark displacements build up orbital, radial structure



$$\overline{\Phi}^{AB}_{\alpha\beta}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot(\mathbf{x}+\frac{1}{2}(\boldsymbol{d}_\alpha+\boldsymbol{d}_\beta))}\delta_{ab}\,\overline{q}^B_{b\beta}(\boldsymbol{x},t)\,q^A_{a\alpha}(\boldsymbol{x},t)$$

$$\overline{\Phi}^{ABC}_{\alpha\beta\gamma}(\boldsymbol{p},t) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{p}\cdot\mathbf{x}}\varepsilon_{abc}\,\overline{q}^C_{c\gamma}(\boldsymbol{x},t)\,\overline{q}^B_{b\beta}(\boldsymbol{x},t)\,\overline{q}^A_{a\alpha}(\boldsymbol{x},t)$$

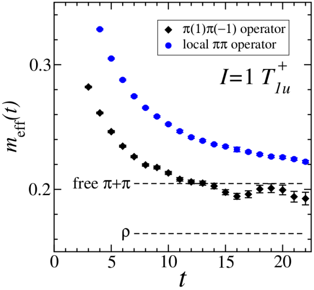
- group-theory projections onto irreps of lattice symmetry group

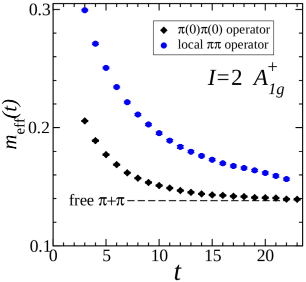
$$\overline{M}_l(t) = c^{(l)*}_{\alpha\beta}\,\overline{\Phi}^{AB}_{\alpha\beta}(t) \qquad \overline{B}_l(t) = c^{(l)*}_{\alpha\beta\gamma}\,\overline{\Phi}^{ABC}_{\alpha\beta\gamma}(t)$$

- definite momentum $\boldsymbol{p}$, irreps of little group of $\boldsymbol{p}$

Two-hadron operators

- comparison of $\pi(\boldsymbol{k})\pi(-\boldsymbol{k})$ and localized $\sum_{\boldsymbol{x}} \pi(\boldsymbol{x})\pi(\boldsymbol{x})$ operators





- important to use superposition of products of single-hadron operators of definite momenta
- efficient construction, generalizes to three or more hadrons

Stochastic estimation of quark propagators

- stochastic LapH method (PRD 83, 114505 (2011)) for time-slice to time-slice propagators
- introduce Z_4 noise vectors η in the LapH subspace

$$\eta_{\alpha k}(t), \qquad t = \text{time},\ \alpha = \text{spin},\ k = \text{eigenvector number}$$

- solve $D[U]X^{(r)} = \eta^{(r)}$ for each of N_R noise vectors $\eta^{(r)}$, then obtain a Monte Carlo estimate of all elements of D^{-1}

$$D^{-1}_{ij} \approx \frac{1}{N_R} \sum_{r=1}^{N_R} X_i^{(r)} \eta_j^{(r)*}$$

- variance reduction using noise dilution
- dilution introduces projectors $P^{(a)}$, then define

$$\eta^{[a]} = P^{(a)}\eta, \qquad X^{[a]} = D^{-1}\eta^{[a]}$$

to obtain Monte Carlo estimate with drastically reduced variance

$$D^{-1}_{ij} \approx \frac{1}{N_R} \sum_{r=1}^{N_R} \sum_{a} X_i^{(r)[a]} \eta_j^{(r)[a]*}$$

Stochastic LapH method

- introduce Z_N noise in the LapH subspace

$$\rho_{\alpha k}(t), \qquad t = \text{time},\ \alpha = \text{spin},\ k = \text{eigenvector number}$$

- four dilution schemes:

$$\begin{aligned}
P_{ij}^{(a)} &= \delta_{ij} && a = 0 && \text{(none)} \\
P_{ij}^{(a)} &= \delta_{ij}\delta_{ai} && a = 0,1,\ldots,N-1 && \text{(full)} \\
P_{ij}^{(a)} &= \delta_{ij}\delta_{a,Ki/N} && a = 0,1,\ldots,K-1 && \text{(interlace-}K\text{)} \\
P_{ij}^{(a)} &= \delta_{ij}\delta_{a,i \bmod k} && a = 0,1,\ldots,K-1 && \text{(block-}K\text{)}
\end{aligned}$$

- apply dilutions to
  - time indices (full for fixed src, interlace for relative src)
  - spin indices (full)
  - LapH eigenvector indices (interlace-16 mesons)

Current ensemble and software

- currently using CLS D200 ensemble
- size: $64^3 \times 128$ lattice, $a \sim 0.065$ fm
- open boundary conditions in time
- number of configs = 2000
- quark masses: $m_\pi \sim 200$ MeV, $m_K \sim 480$ MeV
- smearing: $N_{\text{ev}} = 448$
- sources:
  $t_0 = 35$ forward,
  $t_0 = 64$ forward and backward,
  $t_0 = 92$ backward
- software: common subexpression elimination with tensor contractions (Ben Hörz)
- heavy use of batched BLAS routines

Flavor channels

Isospin channel	D200 Number of Correlators
$I=0,\ S=0,\ NN$	8357
$I=0,\ S=-1,\ \Lambda, N\overline{K}, \Sigma\pi$ (45 SH)	8143
$I=\frac{1}{2},\ S=0, N\pi$	696
$I=\frac{1}{2},\ S=-1, N\Lambda, N\Sigma$	17816
$I=1,\ S=0, NN$ (66 SH)	7945
$I=\frac{3}{2},\ S=0, \Delta, N\pi$	3218
$I=\frac{3}{2},\ S=-1, N\Sigma$	23748
$I=0,\ S=-2, \Lambda\Lambda, N\Xi, \Sigma\Sigma$ (66 SH)	16086
$I=2,\ S=-2, \Sigma\Sigma$ (66 SH)	4589
Single hadrons	33

Scattering parameters from finite-volume energies

- extract scattering parameters from finite-volume energies using NPB924, 477 (2017) implementation (modified) of Lüscher method
- parametrize K-matrix or its inverse
- find best-fit values of parameters from quantization condition

$$\det(1 - \widetilde{K}^{-1} B^{-1}) = 0 \quad \text{or} \quad \det(B^{-1} - \widetilde{K}) = 0$$

- $\widetilde{K}$ is K-matrix with threshold factors removed
- B is so-called box matrix
- minimization using determinant residual method or spectrum method
- use of function of matrix A with real parameter μ:

$$\Omega(\mu, A) \equiv \frac{\det(A)}{\det[(\mu^2 + AA^\dagger)^{1/2}]}$$

- use $A = 1 - \widetilde{K}^{-1} B^{-1}$

Motivation

- meson-baryon amplitudes useful for pheno. at $m_\pi^{\rm phys}$ and for chiral EFT's at varying $m_\pi^{\rm phys}$.
- $\Delta(1232) \to N\pi$ used as a d.o.f. in some EFT's
- scattering lengths $a_{N\pi}^{I=3/2}$ and $a_{N\pi}^{I=1/2}$ impact lattice-pheno. discrepancy for $\sigma_{\pi N}$, relevant for dark matter direct detection. (see arxiv:1602.07688)
- lattice QCD is good laboratory to study $\Lambda(1405)$ by varying quark masses.
- preliminary results shown: 2000 D200 configs, one source time
- all four source times to be completed soon

Additional difficulties

- correlator construction involves rank 3 tensors
- non-zero total spin requires additional partial waves

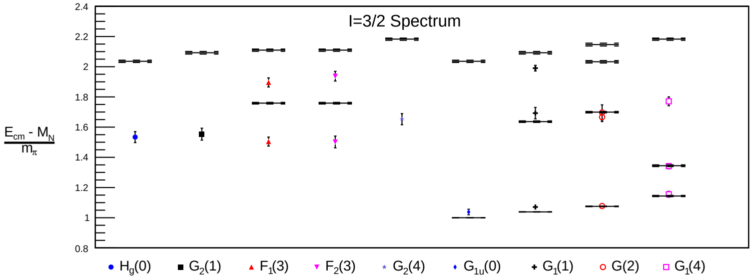
$$(J, L) = (1/2, 0), (3/2, 1), (1/2, 1), (5/2, 2), (3/2, 2)$$

- current analysis neglects all $L >= 2$ waves
- for $\Lambda(1405)$, coupled channels with $\Sigma\pi,\ NK,\ \Lambda\eta$
- mixing with stable hadrons in $I = 1/2$ and $I = 0$, $S = 1$
- relevant stable hadron for $I = 1/2$ is nucleon, for $I = 0, S = 1$ is $\Lambda(1115)$

Analysis details

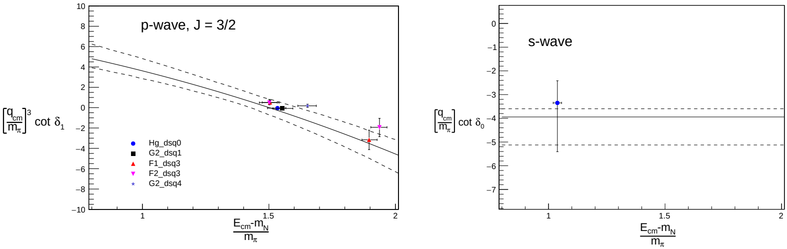
- spectrum from ratio fits + GEVP
- determination of energy shifts Δ_E from non-interacting energies stable against variations of (τ_0, τ_D) and $n_{\rm ops}$
- parametrize resonant amplitudes with BW form, non-resonant with a constant (LO eff. range)
- parameters estimated from determinant-residual fits, using $1 - \widetilde{K}^{-1}B^{-1}$ as input to Ω-function with $\mu = 1$.
- covariance matrix and all statistical errors estimated using bootstrap with $N_B = 800$ samples
- all elastic levels are included. A level within 1-sigma of an inelastic threshold not considered
- parametrizations provided for all s- and p-waves. Higher partial waves are ignored for now

$I = 3/2$ spectrum determination



- irreps with leading $(2J, L) = (3, 1)$ wave: $H_g(0)$, $G_2(1)$, $F_1(3)$, $G_2(4)$.
- irrep with leading $(1, 0)$ wave: $G_{1u}(0)$.
- irrep with leading $(1, 1)$ wave: $G_{1g}(0)$ not included because ground state is inelastic.
- irreps with s- and p-wave mixing: $G_1(1)$, $G(2)$, $G_1(4)$.

$I = 3/2$ scattering amplitudes



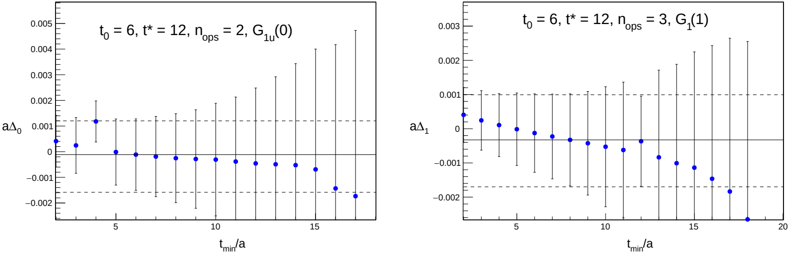
- 17 levels across $H_g(0)$, $G_{1u}(0)$, $G_1(1)$, $G(2)$, $F_1(3)$, $F_2(3)$, $G_1(4)$, $G_2(4)$:
- BW-form for $\widetilde{K}_1^{J=3/2}(E)$, constants for $\widetilde{K}_0^{J=1/2}(E)$ and $\widetilde{K}_1^{J=1/2}(E)$.

$$\frac{m_\Delta}{m_\pi} = 6.380(20), \quad g_{\Delta N\pi} = 13.7(1.5), \quad \chi^2/\text{d.o.f.} = 1.74$$

$$m_\pi a_0^{J=1/2} = -0.254(41), \quad (m_\pi a_1^{J=1/2})^{-1} = 2.61(44)$$

- resonance parameters consistent with fit to p-wave only irreps.

$I = 1/2$ results



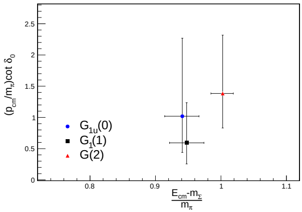
- energy shifts need increased statistics
- global fit to 6 levels in irreps $G_{1u}(0)$, $G_1(1)$, $G(2)$, $G_1(2)$:
- both p-waves modelled as constants:

$$\tilde{K}_1^{J=1/2}(E) = -0.1, \qquad \tilde{K}_1^{J=3/2}(E) = 0.1$$

- results insensitive to variations in constants.
- scattering length:

$$m_\pi a_0^{I=1/2} = 0.386(64), \qquad \chi^2/\text{d.o.f.} = 2.74$$

$I = 0, \; S = 1$ results



- d-wave mixing cannot be ignored due to low-lying $\Lambda^*(1520)$ resonance in $(3,2)$ wave.
- single-hadron operators with derivatives needed to capture the orbital structure: future work. (See Meinel+Rendon '21)
- assumption: $(3,2)$ wave is negligible for lowest $\Sigma\pi$ state in $G_1(1)$ and $G(2)$ irreps, which also contain $(1,0)$ and $(1,1)$ waves.
- in $G_1(1)$ and $G(2)$ irreps ground state Λ must be discarded.

Conclusion

- goal: determine meson-baryon and baryon-baryon scattering parameters in large number of flavor channels
- showed preliminary results:
  - for $I = \frac{1}{2}, \frac{3}{2}$ $N\pi$ amplitudes including $\Delta(1232)$
  - $I = 0$ strangeness $S = 1$ s-wave amplitude relevant for $\Lambda(1405)$
- CLS D200 ensemble $64^3 \times 128$, $a \sim 0.065$ fm, $m_\pi \sim 200$ MeV, $m_K \sim 480$ MeV
- results for full statistics ready in a few months