

Consistency of lattice and R-ratio determinations
of the HVP: Update from RBC/UKQCD

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Calculation of the Hadronic Vacuum Polarization Contribution to the Muon Anomalous Magnetic Moment

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We present a first-principles lattice QCD + QED calculation at physical pion mass of the leading-order hadronic vacuum polarization contribution to the muon anomalous magnetic moment. The total contribution of up, down, strange, and charm quarks including QED and strong isospin breaking effects is $a_{\mu}^{\text{HVP LO}} = 715.4(18.7) \times 10^{-10}$. By supplementing lattice data for very short and long distances with R -ratio data, we significantly improve the precision to $a_{\mu}^{\text{HVP LO}} = 692.5(2.7) \times 10^{-10}$. This is the currently most precise determination of $a_{\mu}^{\text{HVP LO}}$.

Pure lattice result and dispersive result with reduced $\pi\pi$ dependence (window method)

Aaron Meyer (BNL → LBNL) & Mattia Bruno (BNL → CERN → Milano) joined since this 2018 paper

Lattice QCD – Time-Moment Representation

Starting from the vector current $J_\mu(x) = i \sum_f Q_f \bar{\Psi}_f(x) \gamma_\mu \Psi_f(x)$ we may write

$$a_\mu^{\text{HVP LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

with

$$C(t) = \frac{1}{3} \sum_{\vec{x}} \sum_{j=0,1,2} \langle J_j(\vec{x}, t) J_j(0) \rangle$$

and w_t capturing the photon and muon part of the HVP diagrams ([Bernecker-Meyer 2011](#)).

The correlator $C(t)$ is computed in lattice **QCD+QED** at **physical pion mass** with **non-degenerate** up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

Window method (introduced in RBC/UKQCD 2018)

We also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

$$a_\mu = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}$$

with

$$a_\mu^{\text{SD}} = \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)],$$

$$a_\mu^{\text{W}} = \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)],$$

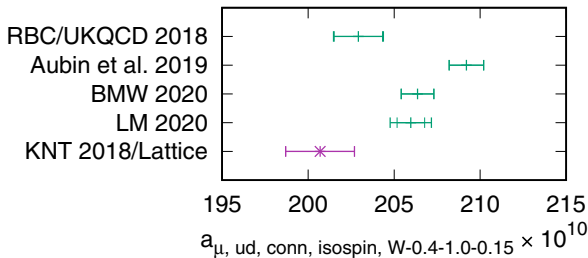
$$a_\mu^{\text{LD}} = \sum_t C(t) w_t \Theta(t, t_1, \Delta),$$

$$\Theta(t, t', \Delta) = [1 + \tanh [(t - t')/\Delta]] / 2.$$

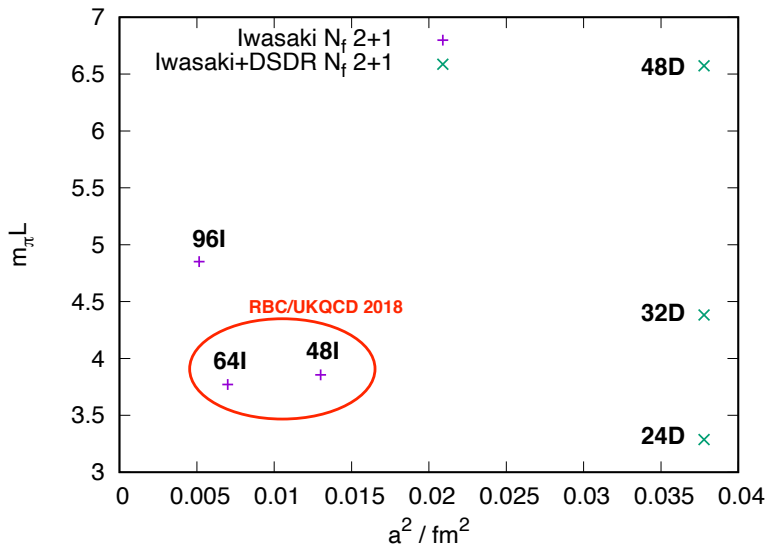
All contributions are well-defined individually and can be computed from lattice or R-ratio via $C(t) = \frac{1}{12\pi^2} \int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2} \sigma(s, e^+ e^- \rightarrow \text{had})$.

a_μ^{W} has small statistical and systematic errors on lattice!

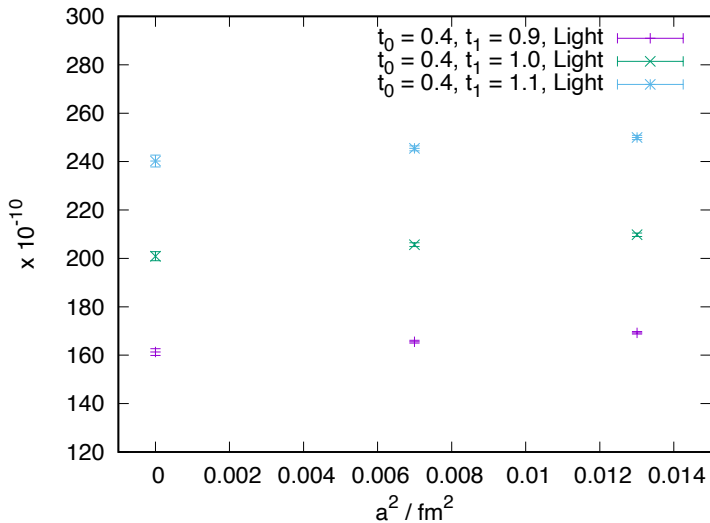
- ▶ In last few years, we reported on our progress for the complete result (improved bounding method, $(\pi\pi)_{I=1}$ phase shift study, improvements for disconnected/QED/SIB diagrams), this talk focuses entirely on progress on the Euclidean time window (RBC/UKQCD 2018) in the isospin symmetric limit with $t_0 = 0.4$ fm, $t_1 = 1.0$ fm, $\Delta = 0.15$ fm.
- ▶ This quantity promises reduced systematic lattice uncertainties, however, currently exhibits tensions between different lattice and R-ratio results:



Our extended list of ensembles, all with $m_\pi = 135 \pm 5$ MeV:

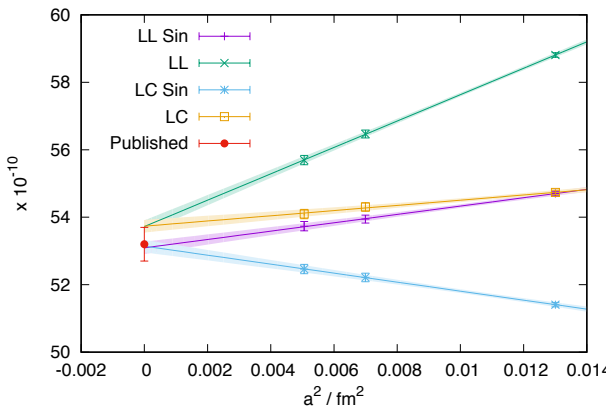


Previous continuum extrapolation mild (plot from [RBC/UKQCD 18](#)), will now check with even finer lattice spacing:



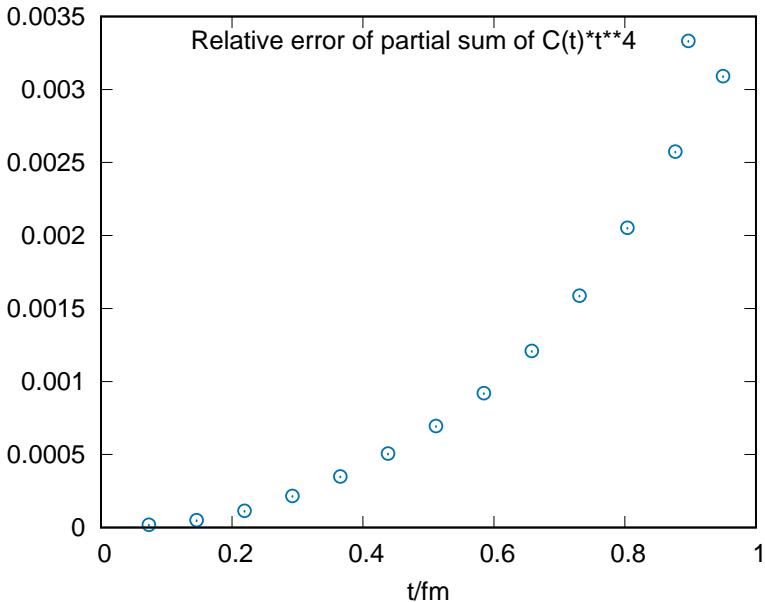
Previously already added $a^{-1} = 2.77$ GeV ensemble for strange quark:

- ▶ Third lattice spacing for strange data ($a^{-1} = 2.77$ GeV with $m_\pi = 234$ MeV with sea light-quark mass corrected from global fit):



- ▶ For light quark use new 96l ensemble at physical pion mass. Data still being generated on Summit in USA and Booster in Germany ($a^{-1} = 2.77$ GeV with $m_\pi = 139$ MeV)

We are still blinded, show only statistical relative error for new 96l:



Other improvements relevant for window:

- ▶ 4x statistics on 48l and 64l
- ▶ Explicit calculation of parametric derivatives at physical point
- ▶ Concluding study of missing charm determinant ($2+1 \rightarrow 2+1+1$)
- ▶ All of the above based on master-field-type study, see next slides

Master-field calculation of gradients

For a local observable

$$O = \frac{1}{V} \sum_y O_y \quad (1)$$

we can define the truncated master-field covariance

$$\text{Cov}_R(O, A) \equiv \frac{1}{V} \sum_{x,y,|y|\leq R} (\langle O_x A_{x+y} \rangle_\beta - \langle O_x \rangle_\beta \langle A_{x+y} \rangle_\beta) \quad (2)$$

such that, e.g., the β -derivative of O is given by

$$\frac{\langle O \rangle_{\beta+\varepsilon} - \langle O \rangle_\beta}{\varepsilon} = 6 \lim_{R \rightarrow \infty} \text{Cov}_R(O, A). \quad (3)$$

In practice use exponential approach to plateau for $R \rightarrow \infty$.

We isolate the dependence on sea-quark mass m of an observable O by studying

$$\langle O \rangle_m \equiv \frac{\int \det(D(m)) O P}{\int \det(D(m)) P} \quad (4)$$

with Dirac matrix $D(m)$ and residual weight P . Can show that

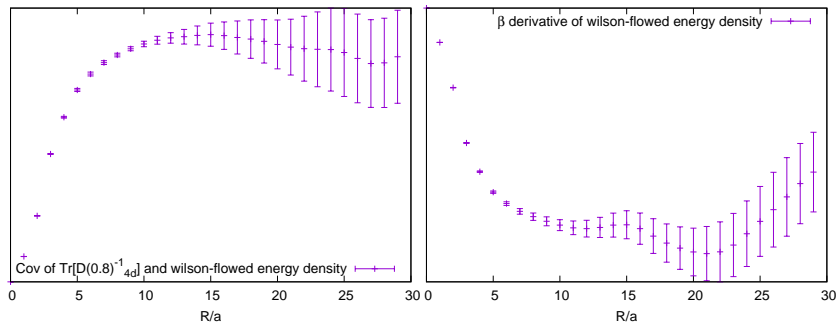
$$\frac{\langle O \rangle_{m+\varepsilon} - \langle O \rangle_m}{\varepsilon} = \text{Cov}(O, \text{Tr}[D_{4d}^{-1}(m)]) + \mathcal{O}(\varepsilon). \quad (5)$$

Finally, for DWF an additional flavor enters as

$$\det(D(m)D^{-1}(1)) \quad (6)$$

such that for $m = 1$ the factor is trivial and we can view adding an additional flavor as changing the sea-quark mass down from $m = 1$ to the target value.

Example for wilson-flowed energy density (96l, $t_0 \approx 2$)



Computed in similar way also derivatives of, e.g., VV and PP correlators.

Conclusions and Outlook

- ▶ 4x statistics on 48l and 64l
- ▶ Third lattice spacing (96l) at $a^{-1} \approx 2.7$ GeV
- ▶ $2+1 \rightarrow 2+1+1$ and parametric gradients using master-field approach
- ▶ Still blinded, data set almost complete, hope to publish window results before fall