Proton Decay Amplitudes
with Physical Chirally-Symmetric Quarks

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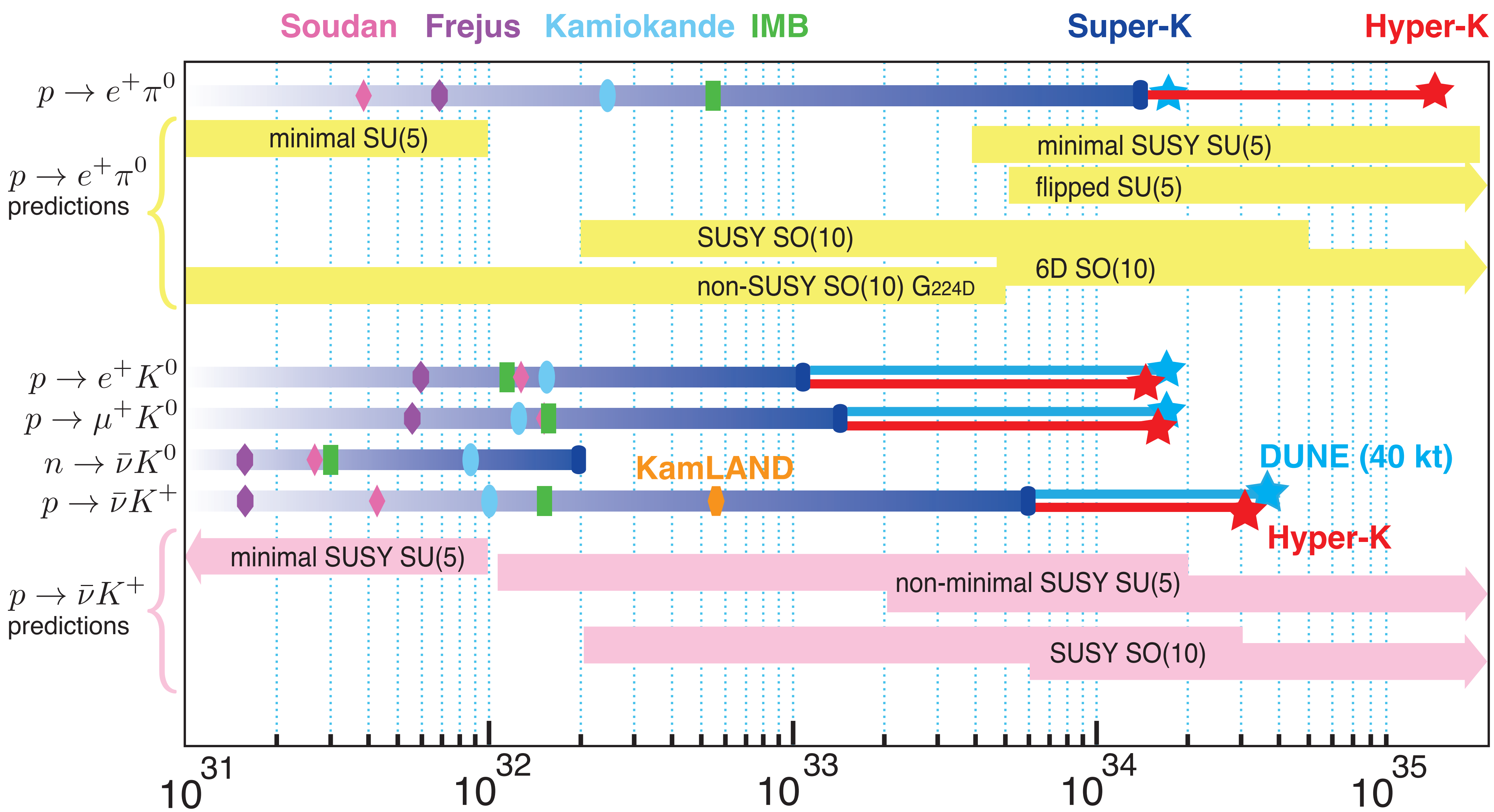
- Proton decay basics
  - *Experimental lifetime limits & outlook*
  - *Motivation and theory status*
  - *Effective nucleon decay operators and matrix elements*
- Need for lattice calculations
  - *Past calculations and model uncertainty*
  - *Summary of the present calculation*
- Lattice calculation and analysis
  - *Hadron masses and energies*
  - *Extraction of matrix elements*
  - *Operator renormalization*
  - *Momentum & continuum extrapolations*
- Results
  - *Comparison to earlier calculations*
  - *Nucleon annihilation amplitudes*
  - *Conclusions*

Proton Stability: Status and Outlook



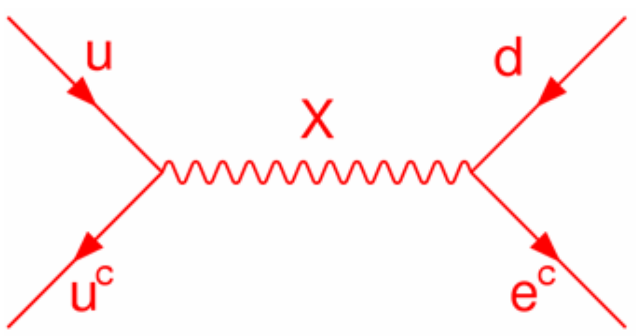
[LBNF and DUNE CDR, R.Acciarri et al (2015)]

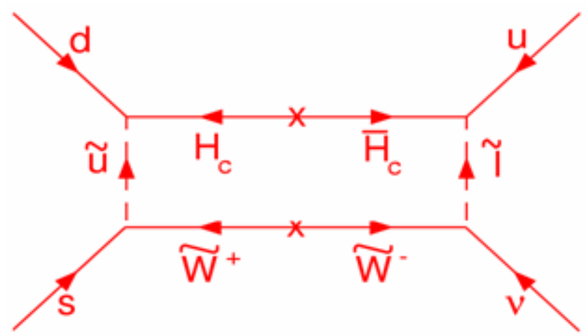
- Expect x10 improvement on lifetime limit from Hyper-K and DUNE
- Better sensitivity to $p \to \bar{\nu}K^+$ that affects supersymmetric GUT models

Motivation and Theory Status

- Proton lifetime is a test of baryon number conservation –
  – accidental symmetry of SM
  - violated by sphalerons
  - has to be violated for baryogenesis
- Missing piece of Grand-Unified Theories
- Probes scales inaccessible to colliders: Limits on GUT, extra dim., etc
- Limits on stability of nuclear matter

[Sakai,Yanagida '82; Weinberg '82]





ordinary GUT
- min.$SU(5)$ ruled out by $\tau(p \to e^+\pi^0)$
- $SO(10)$ probed by next-gen exp.

supersymmetric GUT
- min.SUSY-$SU(5)$ ruled out by $\tau(p \to \bar{\nu}K^+)$
- SUSY-$SO(10)$ probed by next-gen exp.

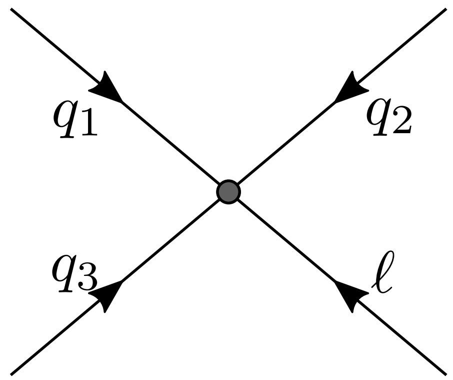
How Nucleon Structure Affects GUT Limits

Effective interaction

$$\mathcal{L}_{\rm eff} = \sum_I C_I \mathcal{O}_I \ + \text{h.c.}$$

$$\mathcal{O}_I = \epsilon^{abc}(\bar{q}_1^{aC} P_{\chi_I} q_2^b)(\bar{\ell}^C P_{\chi'_I} q_3^c) = \bar{\ell}^C_\alpha \, \mathcal{O}^{3q}_{I,\alpha}$$

$$q_{1,2,3} \in \{u, d, s\}\,, \quad P_{\chi_I^{(\prime)}} = \frac{1 \pm \gamma_5}{2}$$



Decay width $p \to \Pi\bar{\ell}$ $(\Pi = \pi, \mathrm{K}, \eta)$

$$\Gamma(p \to \Pi\bar{\ell}) = \frac{m_N}{32\pi}\left[1 - \left(\frac{m_\Pi}{m_N}\right)^2\right]^2 \Big|\sum_I C_I W^I_{\bar{\ell}}\Big|^2$$

Decay matrix elements $(W_{0,1})_I$ [S.Aoki et al, PRD62:014506 (200)]

$$\langle\bar{\ell}(q)\Pi(p)|\mathcal{O}^{\chi'}|N(k)\rangle = \bar{v}^C_{\ell\alpha}(q)\,P_{\chi'}\left[W_0(-q^2) - \frac{i\not{q}}{m_N}W_1(-q^2)\right]u_N(k)$$

and $W_{\bar{\ell}} = \left[W_0 + W_1 \cdot O(m_{\bar{\ell}}/m_N)\right]_{q^2 = m^2_{\bar{\ell}}}$

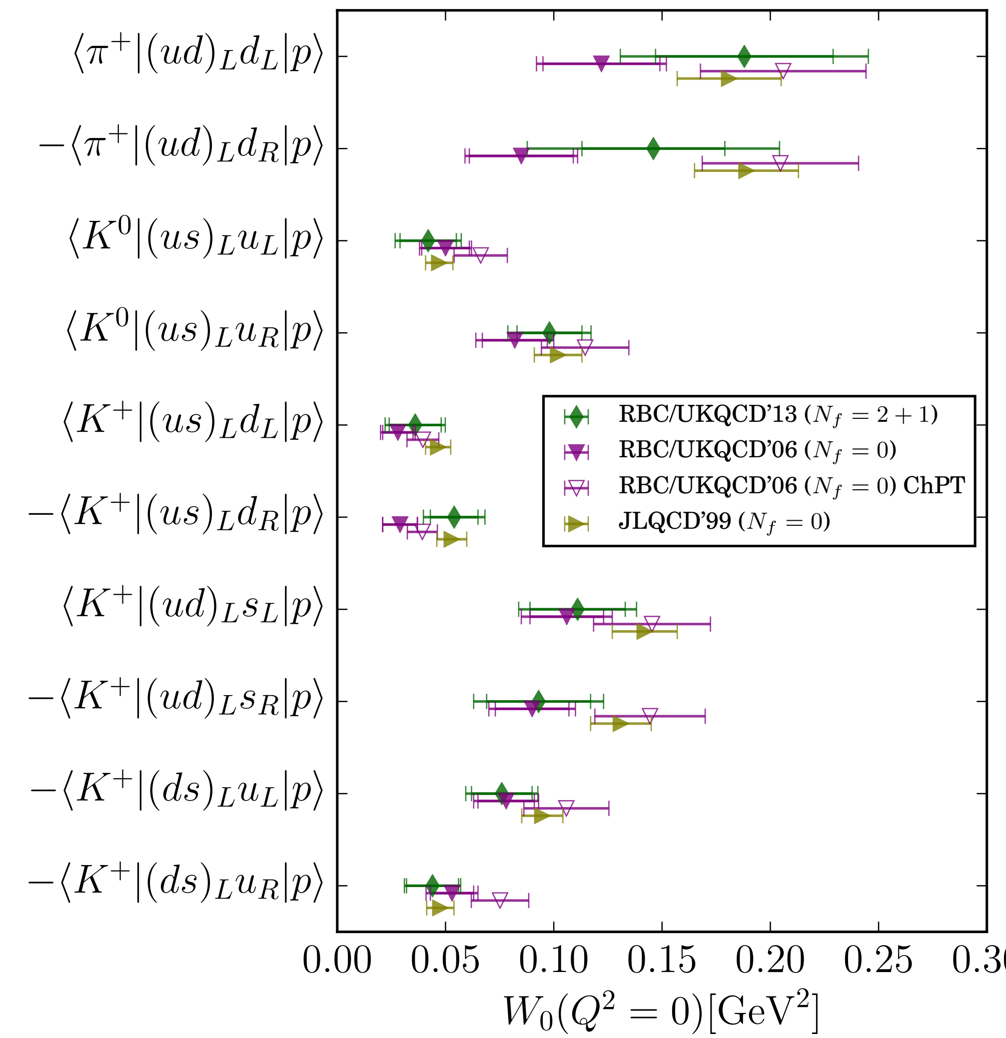
negligible for e^+
≈10% for μ^+

Nucleon Decay Matrix Elements

Nonperturbative matrix elements [form factors]

$$\langle \Pi(k-q)|\mathcal{O}^{3q}_\alpha|N(k)\rangle = \left[P_{\chi'}\left(W_0^{\mathcal{O}} - \frac{i\not{q}}{m_N}W_1^{\mathcal{O}}\right)u_N(k)\right]_\alpha$$

[S.Aoki et al, PRD62:014506 (200)]



Two methods to calculate $W_{0,1}$

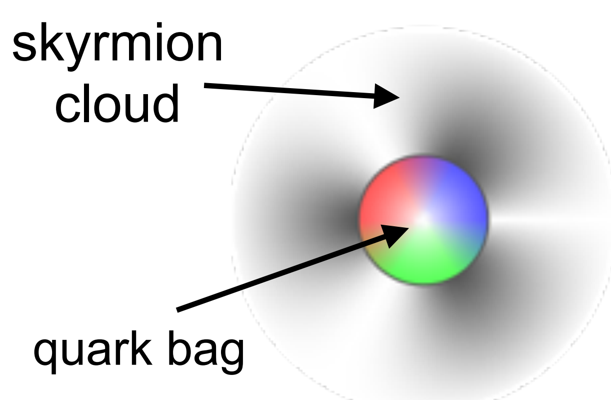
- Direct calculation on lattice
- Low-energy theory (soft-pion thm.) requires annihilation amplitude $\langle \mathrm{vac}|O^{3q}|N\rangle$ (also needed for $p \to 3\bar{\ell}$ decays)

Order-of-magnitude estimate

$$\langle \mathrm{vac}|\mathcal{O}^{3q}|N\rangle \sim \rho_q^{3/2}\sqrt{V_N} \sim \frac{1}{V_N} \approx 0.004\,\mathrm{GeV}^3$$

$$\langle \Pi|\mathcal{O}^{3q}|N\rangle \sim \langle \mathrm{vac}|\mathcal{O}^{3q}|N\rangle/f_\pi \approx 0.03\,\mathrm{GeV}^2$$

[Martin, Stavenga '12]
Suppression of $\langle \mathrm{vac}|O^{3q}|N\rangle$ in Chiral Bag model due skyrmion topology



Alternative explanation for the observed proton stability

This Work: Lattice Setup

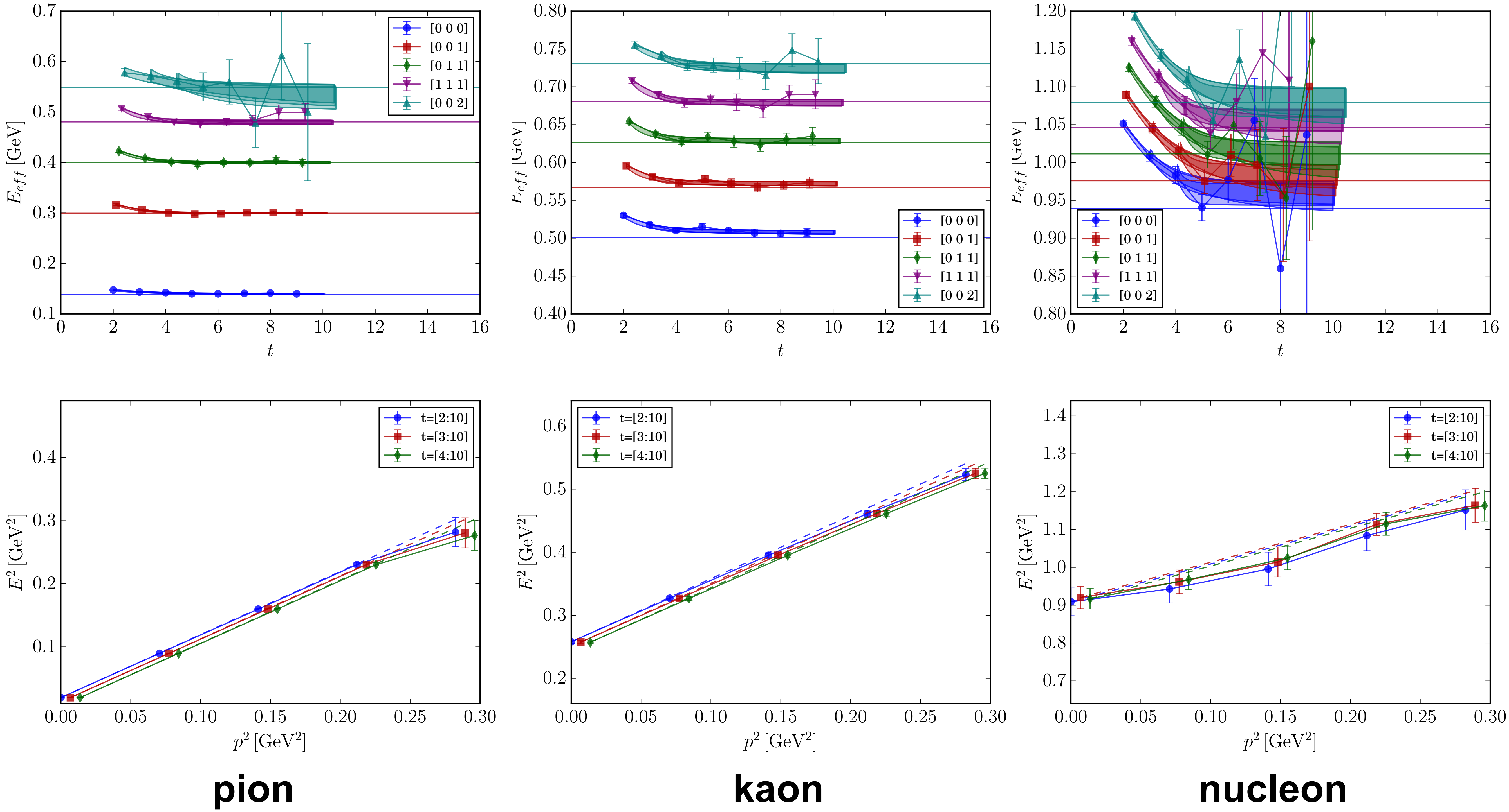
- Two ensembles: [32ID] $32^3 \times 64 (a = 0.14$ fm) and [24ID] $24^3 \times 64 (a = 0.20$ fm)
- Iwasaki gauge action+ Dislocation-supp. det.ratio (DSDR)
- $N_f = 2+1$ Chirally-symmetric (Mobius-)Domain Wall fermion action with physical light and strange quark masses
- Multigrid deflation of z-Mobius operator + AMA
- "Direct" ($p \to \pi, K$ matrix elements) and "Indirect" ($p \to$ vacuum + ChPT)
- Nonperturbative renormalization
- Two state-fit analysis of π, K, N spectrum and $p \to \pi, K$ matrix elements
- a^2 Continuum extrapolation

	24ID $24^3 \times 64$	32ID $32^3 \times 64$
β	1.633	1.75
a, fm	0.20	0.14
a^{-1}, GeV	1.02	1.37
$m_\pi L$	3.4	3.3
N_{conf}	134	94
N_{samp}	4288	3008

- three kinematic (Q^2) points to interpolate matrix elements to decay kinematic $Q^2 = -(m_{\bar{e}})^2$

Π	$\vec{n}_\Pi$	$\vec{n}_N$	$Q^2(\mathrm{GeV}^2)$ (24c)	$Q^2(\mathrm{GeV}^2)$ (32c)
π	[1 1 1]	[0 0 0]	0.010	−0.012
	[1 1 1]	[0 1 0]	0.113	0.095
	[0 0 2]	[0 0 0]	−0.116	−0.140
K	[0 1 1]	[0 0 0]	−0.034	−0.042
	[0 1 1]	[0 1 0]	0.058	0.056
	[0 0 1]	[0 0 0]	0.075	0.074

Proton and Meson Spectrum

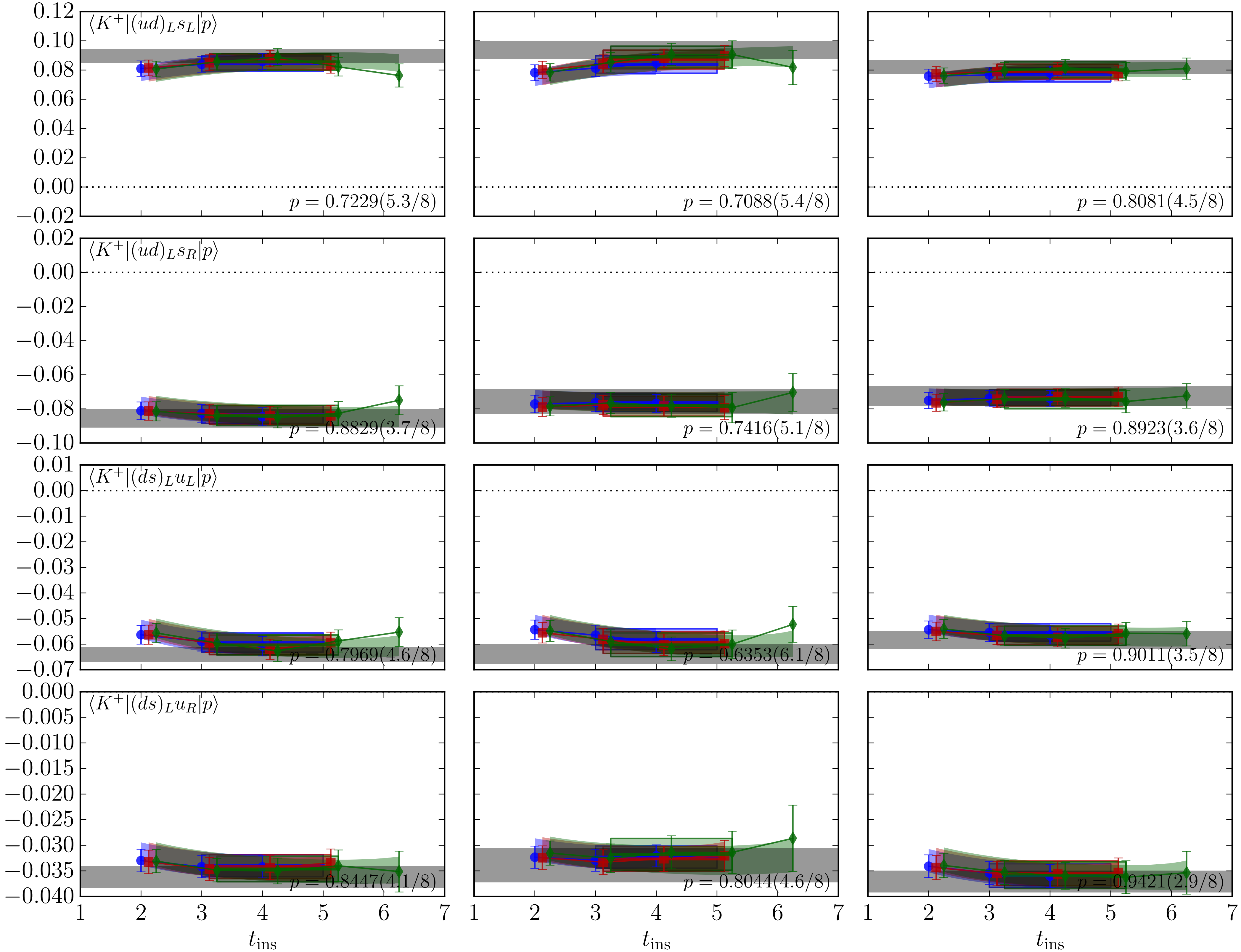


- 24ID ensemble (a=0.20 fm)
- Two-state fits + priors from large-t_{min} one-state fits

Extraction of Matrix Elements

32ID

W_0



- Two-state fits with energies fixed from spectrum fits

Nonperturbative Renormalization

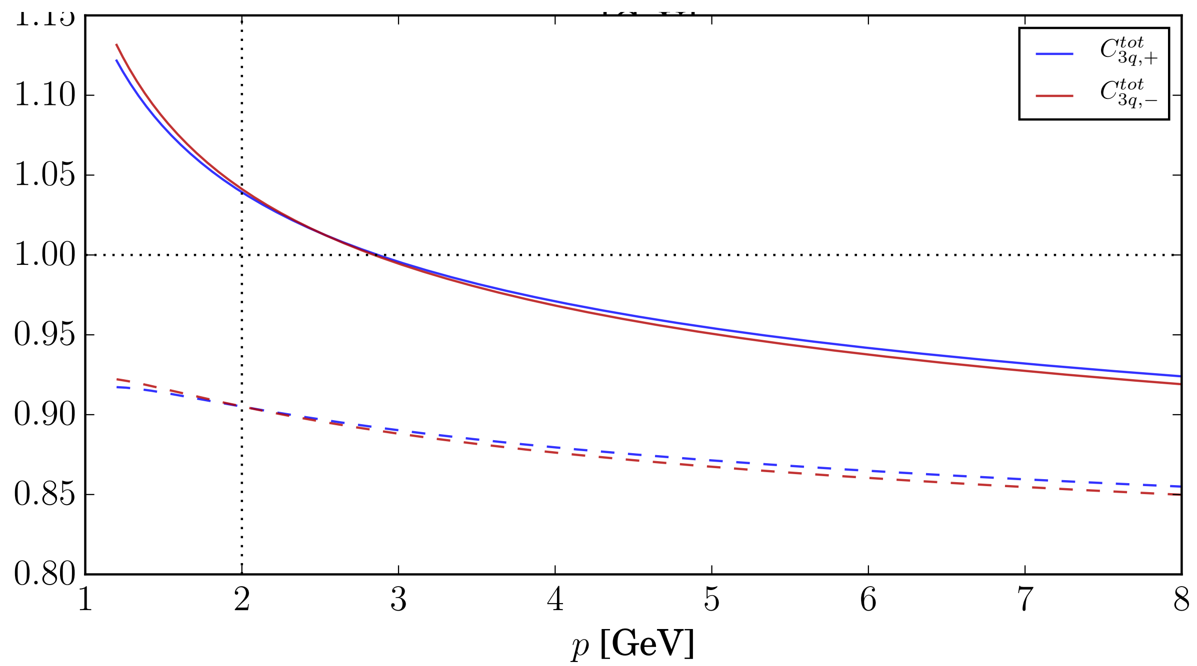
- symmetry-allowed mixing

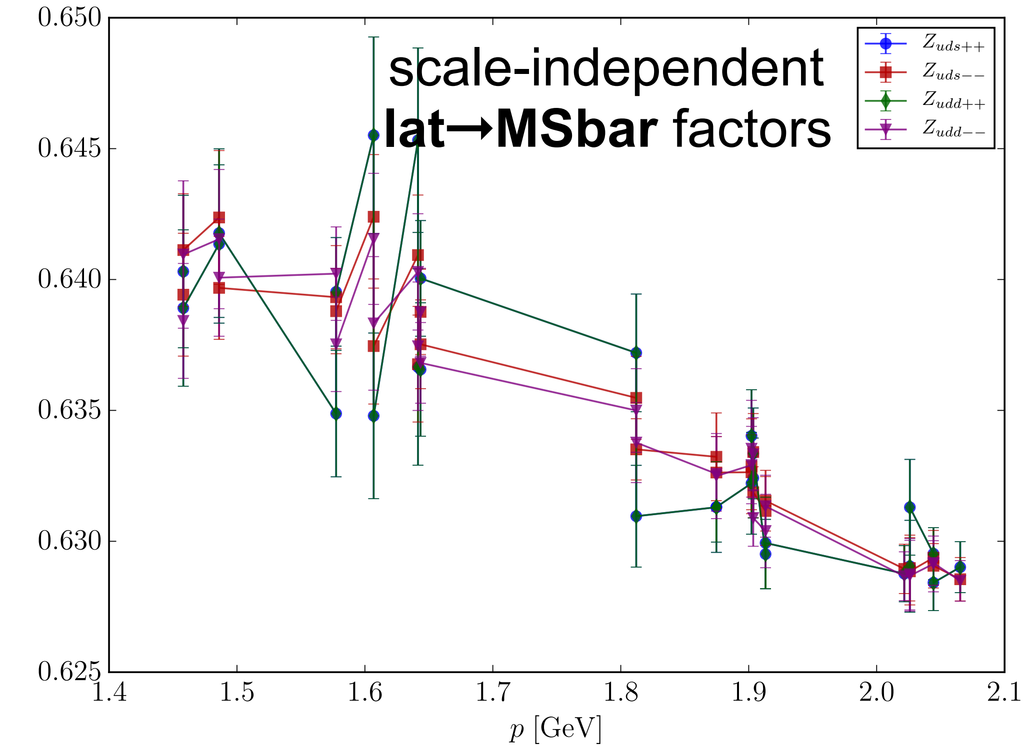
	$\mathcal{S}=-1$	$\mathcal{S}=+1$
$\mathcal{P}=-1$	SS, PP, AA	VV, TT
$\mathcal{P}=+1$	SP, PS, AV	VA, TQ

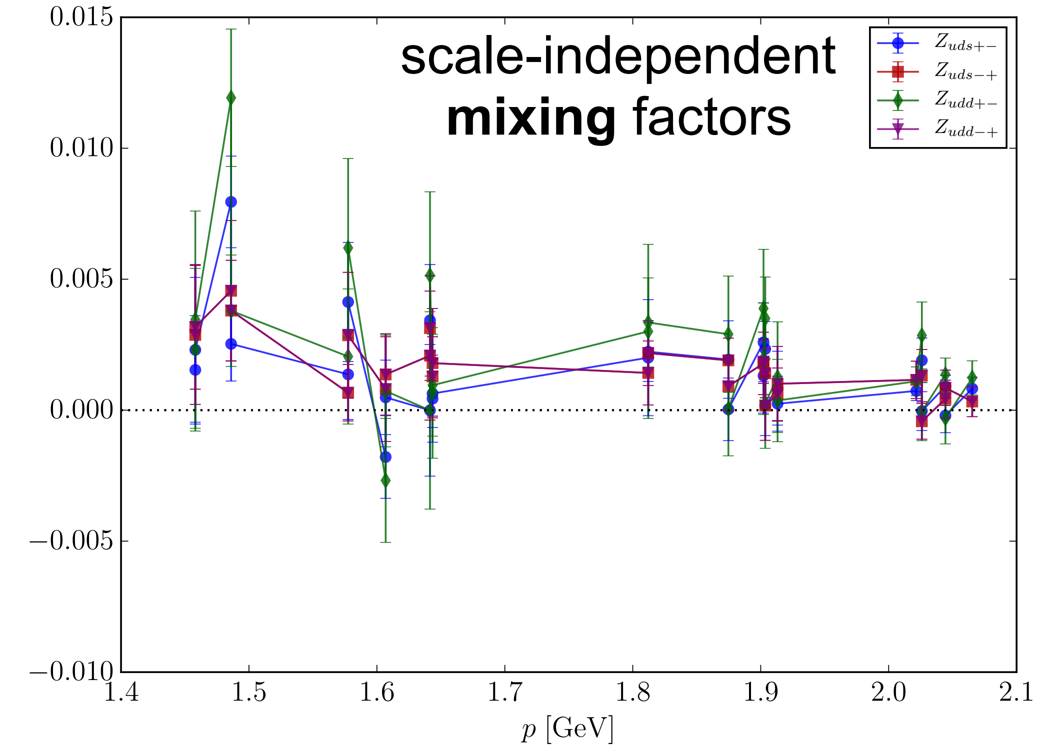
- *symmMOM* scheme : $p+q+r=0,\ p^2=q^2=r^2=\mu^2$

$$Z^{3q}_{IK}(\mu)\ \mathrm{Proj}_J\left[\langle \bar{q}_1(p)\bar{q}_2(q)\bar{q}_3(r)\,\mathcal{O}^{3q}_K\rangle_{\rm amp}\right]=\delta_{IJ}$$

- symmMOM(p)→MSbar(2 GeV) perturbative conversion at $O(\alpha^3)$ [J.Gracey, JHEP09:052 (2012)]

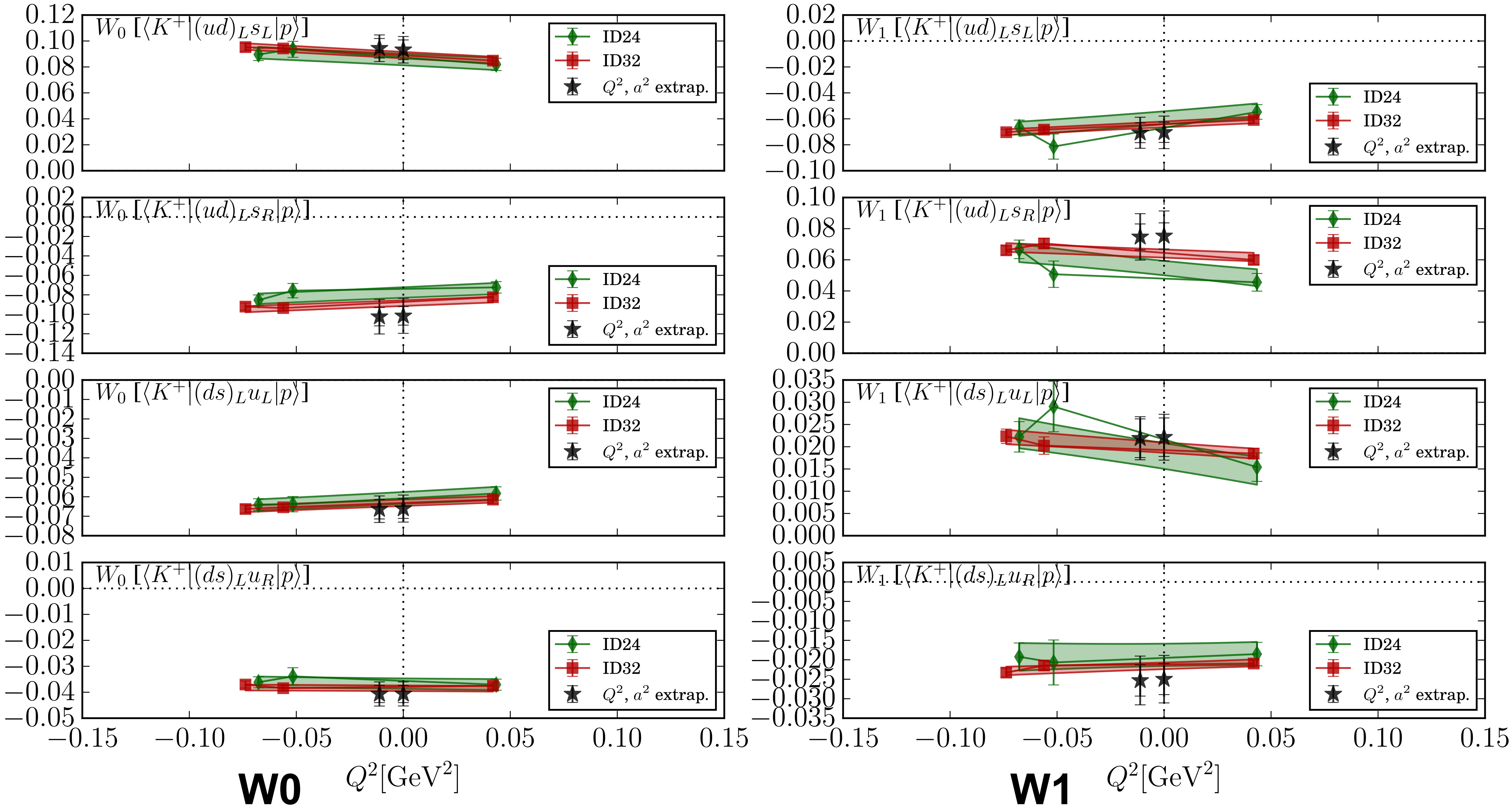






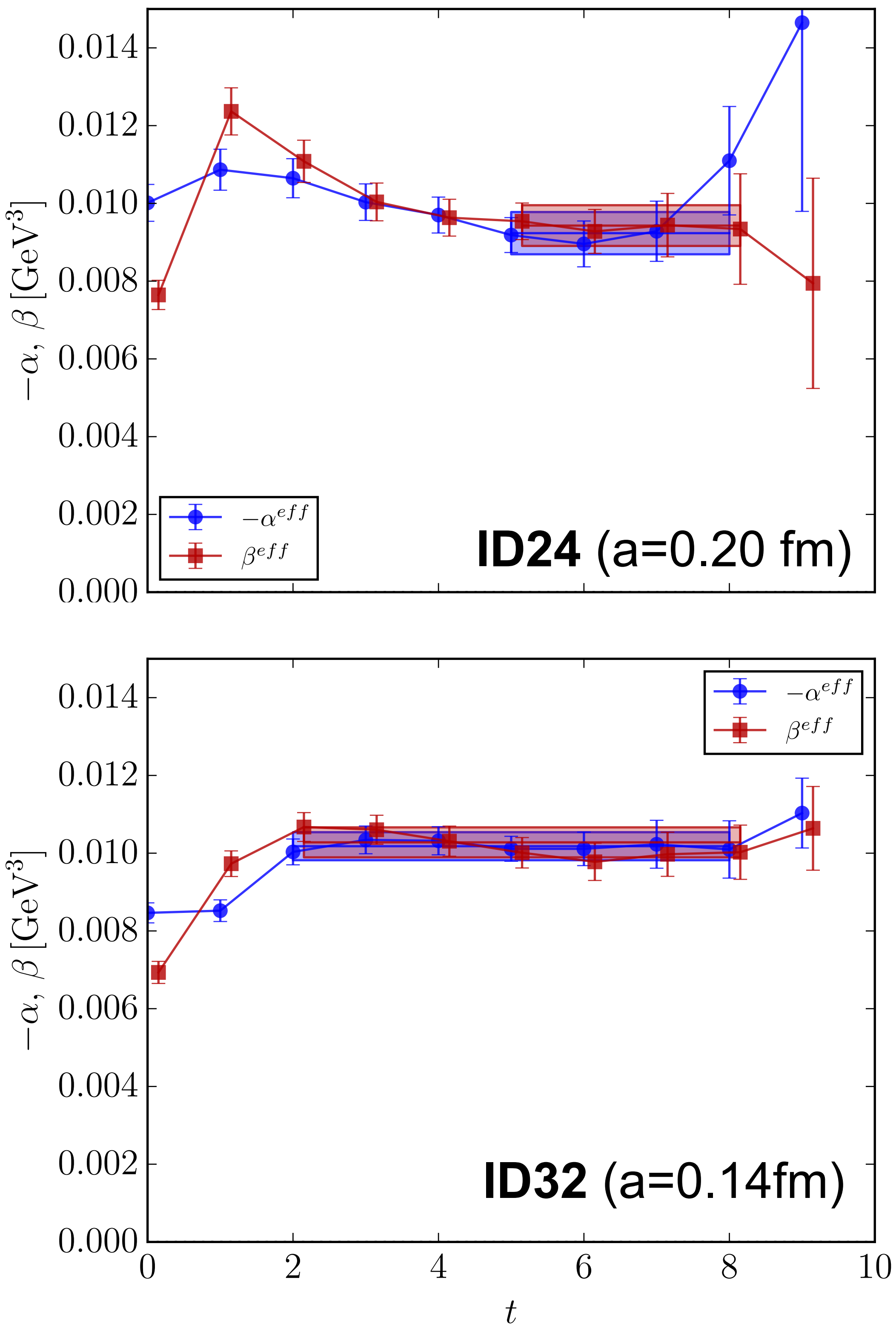
- chiral symmetry suppresses mixing of L⟺R fields & operators

Momentum and Continuum Extrapolation



- linear momentum extrapolation $Q^2 \rightarrow m_e^2, m_\mu^2$ to the decay kinematics
- Continuum extrapolation $A(a^2) \sim (A_0 + A_2\, a^2)$; sys.error = $|A_0 - A_{[a=0.14\mathrm{fm}]}|$

Proton Annihilation Amplitudes

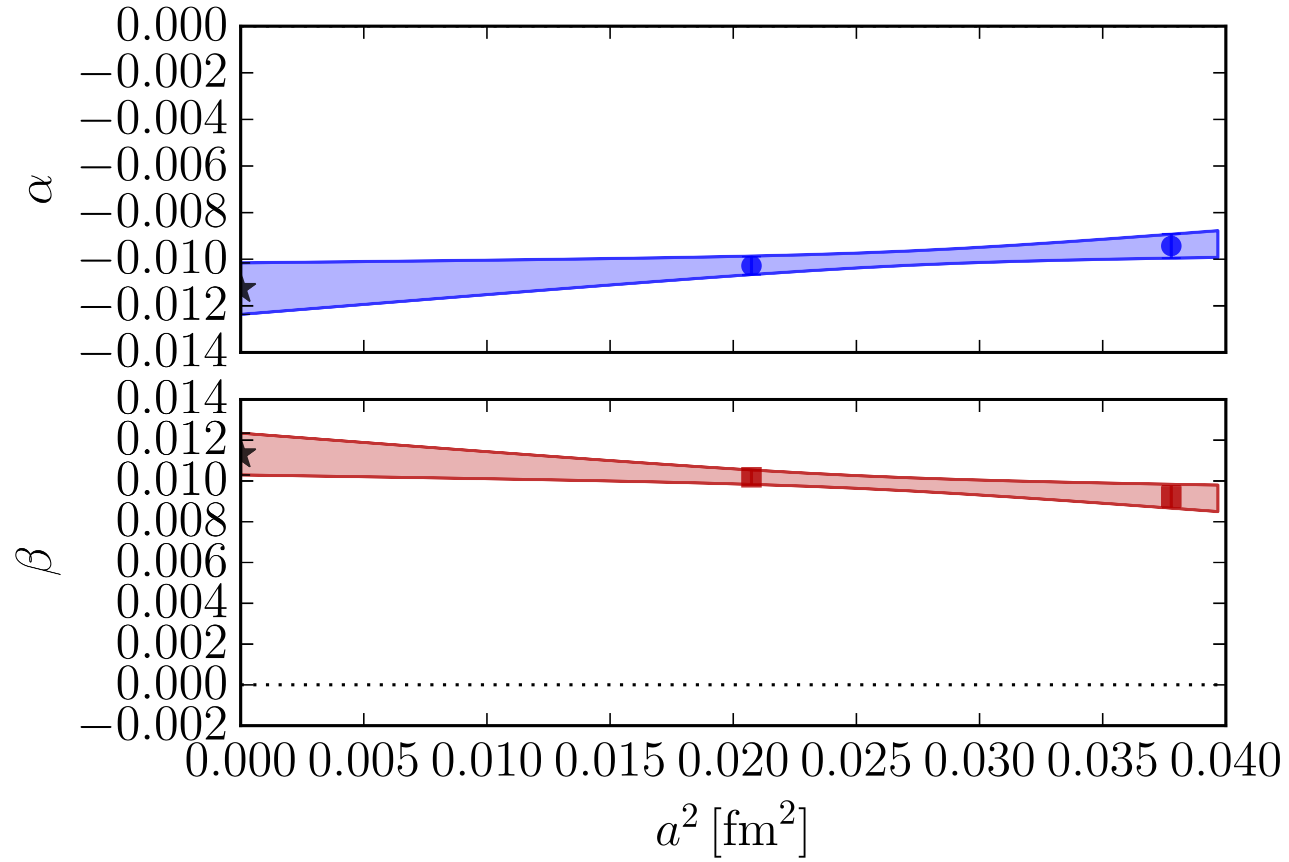


$$\langle \mathrm{vac}|\epsilon^{abc}(\bar{u}^{aC}d^b)_R u^c_L|N\rangle = \alpha\, P_L U_N$$
$$\langle \mathrm{vac}|\epsilon^{abc}(\bar{u}^{aC}d^b)_L u^c_L|N\rangle = \beta\, P_L U_N$$

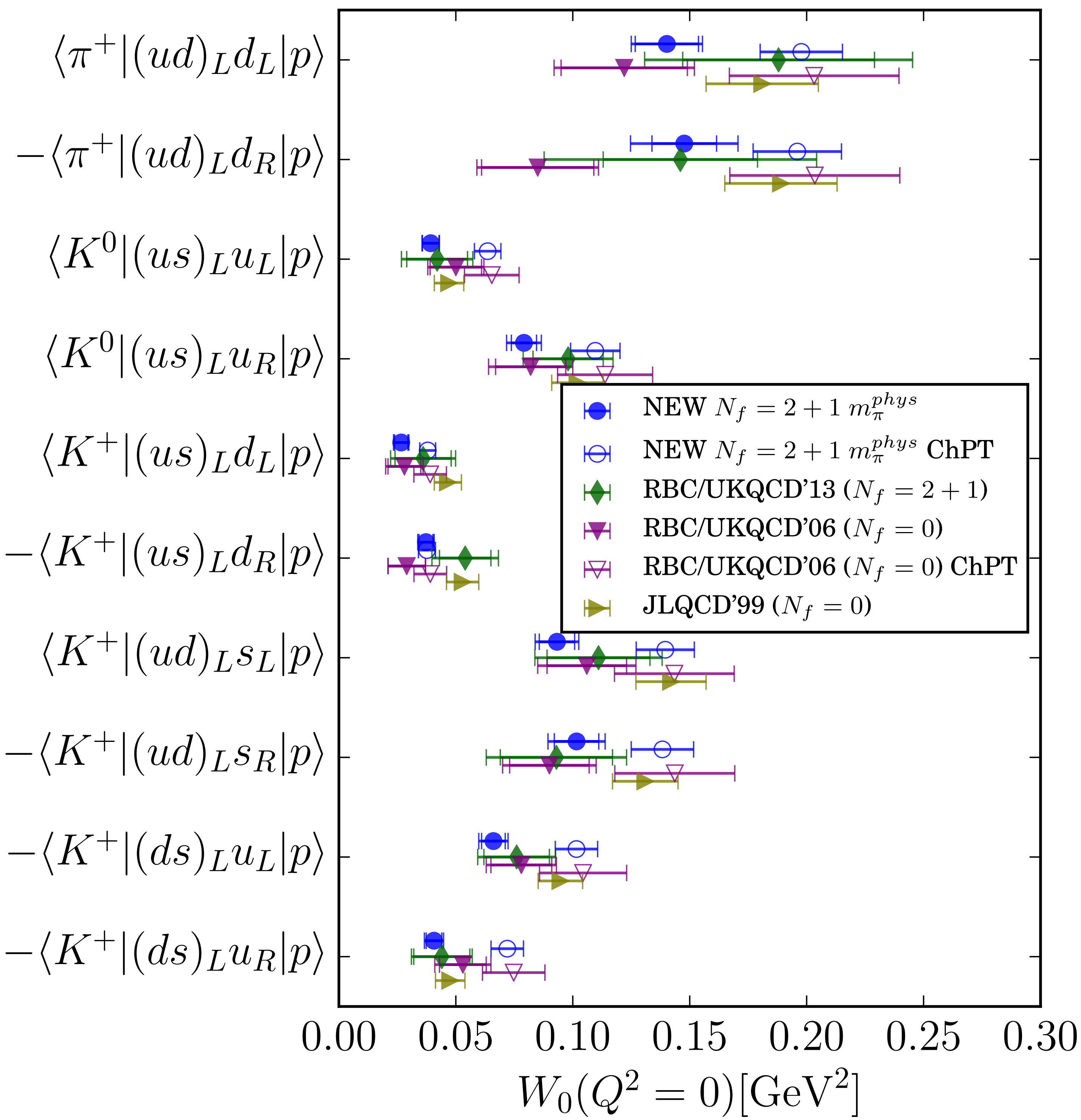
- connected to $\langle \pi/K|O^{3q}|N\rangle$ by soft-pion theorem
- $(\alpha+\beta)=0$ [within errorbars] implying

$$\epsilon^{abc}(\bar{u}^{aC}d^b)\gamma_5 u^c\,|N\rangle \overset{?}{\approx} 0$$

parity (–) (–) (+)



Comparison to Previous Work



- New results:
  (stat+sys) precision ~ 10-20%
- No FVE study, mpi*L~3.4
- physical-point results agree with prev. calculations at mπ ≳300 MeV

  [S.Aoki et al (2000)]
  [Y.Aoki et al (2006)]
  [Y.Aoki et al (2013)]

NO suppression of nucleon decay due to chiral skyrmion topology

Summary & Conclusions

- *Proton decay amplitudes at the physical point with chiral symmetric quarks and continuum extrapolation*
- *Sys+Stat. precision O(10-20%) ; may be improved with more statistics, finer lattice spacing, finite-volume study*
- *No topological suppression of nucleon decay found; limits on Grand-Unified Theories* ***stand***

BACKUP