Pion Pole Contribution to HLbL from Twisted Mass Lattice QCD at the physical point

Sebastian Burri

Flavor Singlet Project for ETMC

Georg Bergner, Andrew Gasbarro, Konstantin Ottnad, Bartosz Kostrzewa, Marcus Petschlies, Ferenc Pittler, Fernanda Steffens, Carsten Urbach, Urs Wenger

Institute for Theoretical Physics, University of Bern

Lattice21

Project overview

Goal

Computing $\mathcal{F}_{P\to\gamma^*\gamma^*}$, $P = \pi_0, \eta, \eta'$ to determine the corresponding contributions to HLbL in the muon $g-2$.

Using

- Twisted mass clover improved lattice QCD at maximal twist
- Four dynamical flavors ($N_f = 2+1+1$) [C. Alexandrou et al., arXiv:2104.06747]
- Analysis on

ensemble	$L^3 \cdot T/a^4$	m_π [MeV]	a [fm]	$a \cdot L_x$ [fm]	$m_\pi \cdot L_x$
cA30.32	$32^3 \cdot 64$	260	0.0927	2.97	4.02
cB072.64	$64^3 \cdot 128$	135	0.0800	5.12	3.63
cC06.80	$80^3 \cdot 160$	135	0.069	5.52	3.78

- Two ensembles at the physical point (m_s also physical to within a few percent)

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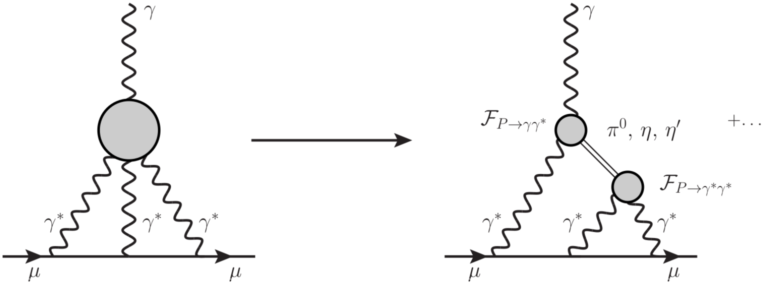
Hadronic light-by-light



Anomalous muon magnetic moment

- Extracted from the muon electromagnetic vertex function
- Hadronic contributions: HVP, HLbL and higher order electroweak corrections
- Dispersive approach to HLbL [G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP 91 (2014)]

Following [M. Knecht, A. Nyffeler, Phys. Rev. D65, 073034 (2002)], the transition form factors are defined via the matrix element

$$M_{\mu\nu}(p, q_1) = i \int d^4x\, e^{iq_1 x} \langle 0| T\{j_\mu(x) j_\nu(0)\}|P(p)\rangle$$
$$= \varepsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \mathcal{F}_{P\gamma^*\gamma^*}(q_1^2, q_2^2).$$

Defining

$$\tilde{A}_{\mu\nu}(\tau) = \langle 0| T\{j_\mu(\vec{q}_1, \tau) j_\nu(\vec{p} - \vec{q}_1, 0)\}|P(p)\rangle\,,$$

the matrix element is recovered by integration:

$$M^E_{\mu\nu} = \int_{-\infty}^{\infty} d\tau\, e^{\omega_1 \tau} \tilde{A}_{\mu\nu}(\tau), \quad i^{n_0} M^E_{\mu\nu}(p, q_1) = M_{\mu\nu}(p, q_1).$$

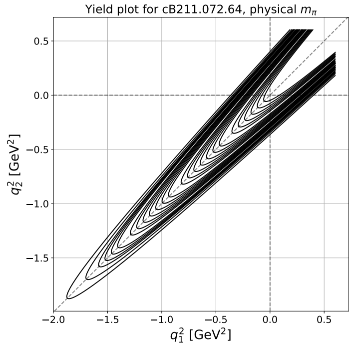
On the lattice, starting from the amplitude

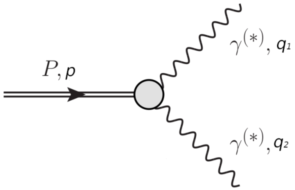
$$C_{\mu\nu}(\tau, t_P) = a^6 \sum_{\vec{x},\vec{z}} \langle j_\mu(\vec{x}, t_i) j_\nu(\vec{0}, t_f) P^\dagger(\vec{z}, t_0) e^{i\vec{p}\vec{z}} e^{-i\vec{x}\vec{q}_1}\rangle,$$

one constructs

$$\tilde{A}_{\mu\nu}(\tau) = \frac{2E_P}{Z_P} \lim_{t_P \to \infty} e^{E_P(t_f - t_0)} C_{\mu\nu}(\tau, t_P).$$

Kinematics





Pseudoscalar at rest, i.e. $\vec{p} = \vec{0}$

$$\Rightarrow q_1^2 = \omega_1^2 - \vec{q}_1^2$$
$$q_2^2 = (m_P - \omega_1)^2 - \vec{q}_1^2$$

Threshold for hadron production

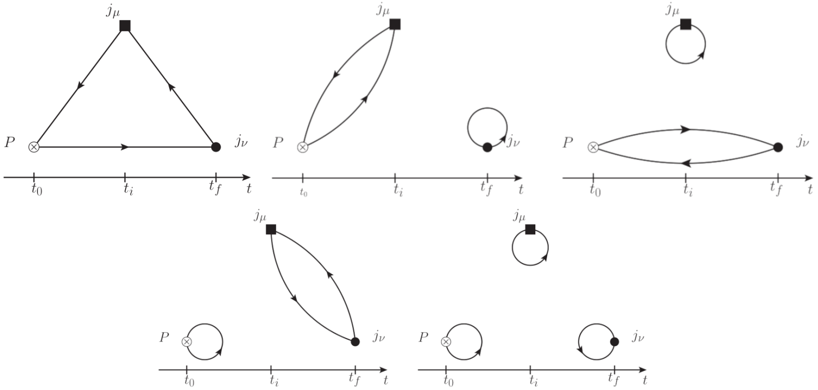
$$\Rightarrow q_{1,2}^2 < m_V^2 = \min(m_\rho^2, 4m_P^2)$$
$$-\sqrt{m_V^2 + \vec{q}_1^2} + m_P < \omega_1 < \sqrt{m_v^2 + \vec{q}_1^2}$$

Define

$$\tilde{A}(\tau) = i m_P^{-1} \varepsilon_{ijk} \frac{\vec{q}_1^{\,i}}{\vec{q}_1^{\,2}} \tilde{A}_{jk}(\tau)$$

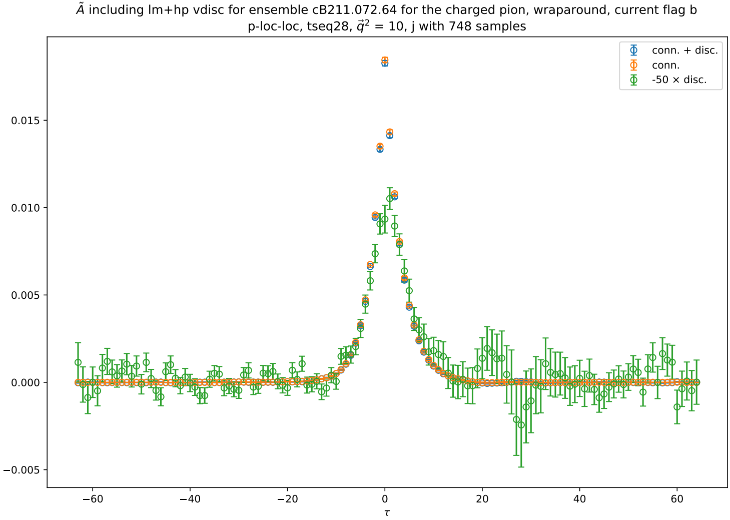
Considered diagrams

The amplitude $C_{\mu\nu}$ contains connected, pseudoscalar disconnected, doubly disconnected and vector current disconnected diagrams.



Isospin rotation $\pi_0 \to \pi_+$ and π_-, no disconnected pseudoscalar contribution to $\pi_\pm$ amplitude.

Vector current disconnected contribution



Well under control on the coarser physical ensemble, depending on orbit around 0.5 - 2 % magnitude compared to connected with opposite sign.

Tail fits

We need

$$\mathcal{F}(q_1^2, q_2^2) = \int_{-t_{cut}}^{t_{cut}} d\tau\, \tilde{A}^{(latt.)}(\tau) e^{\omega\tau} + \int_{\pm t_{cut}}^{\pm\infty} d\tau\, \tilde{A}^{(fit)}(\tau) e^{\omega\tau}.$$

Following [Gerardin et al., Phys. Rev. D94, 074507 (2016) and refs. therein], consider

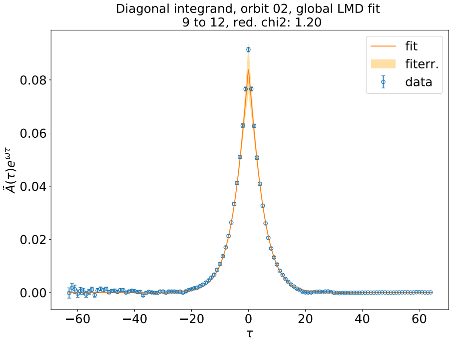
- Vector meson dominance (VMD) model:

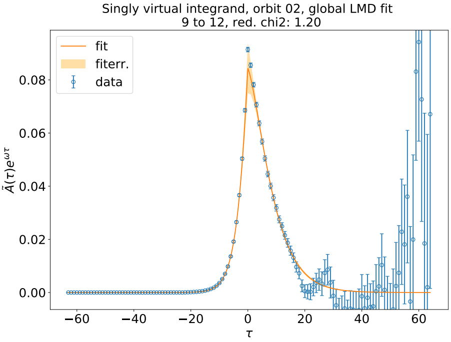
$$\mathcal{F}^{VMD}_{\pi_0\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{\alpha M_V^4}{(M_V^2 - q_1^2)(M_V^2 - q_2^2)} \rightarrow \tilde{A}^{(fit,VMD)}(\tau).$$

- Lowest meson dominance (LMD) model:

$$\mathcal{F}^{LMD}_{\pi_0\gamma^*\gamma^*}(q_1^2, q_2^2) = \frac{\alpha M_V^4 + \beta(q_1^2 + q_2^2)}{(M_V^2 - q_1^2)(M_V^2 - q_2^2)} \rightarrow \tilde{A}^{(fit,LMD)}(\tau).$$

Integrands



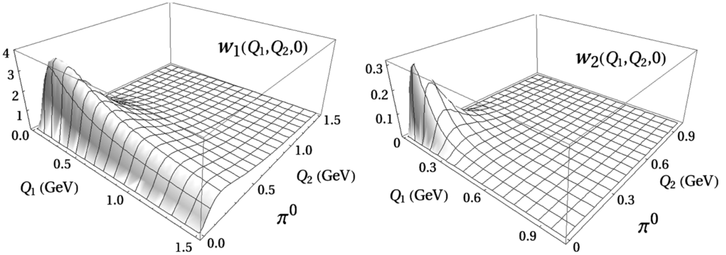


- Diagonal kinematics: $q_1^2 = q_2^2 \Rightarrow \omega = m_\pi/2$.
- Singly virtual kinematics: $q_1^2 = 0 \Rightarrow \omega = |\vec{q}_1^2|$.
- Global fit to model, i.e. simultaneous for all orbits $1 \leq \vec{q}^2(2\pi/L_x)^2 \leq 32$.

Pseudoscalar pole contribution

3d integral representation [A. Nyffeler, Phys. Rev. D94, 053006 (2016) and refs. therein]

$$a_\mu^{P-pole} = \left(\frac{\alpha}{\pi}\right)^3 \int_0^\infty dQ_1 \int_0^\infty dQ_2 \int_{-1}^{+1} d\tau$$
$$\Big[w_1(Q_1, Q_2, \tau) \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -(Q_1+Q_2)^2) \mathcal{F}_{P\gamma^*\gamma^*}(-Q_2^2, 0)$$
$$+ w_2(Q_1, Q_2, \tau) \mathcal{F}_{P\gamma^*\gamma^*}(-Q_1^2, -Q_2^2) \mathcal{F}_{P\gamma^*\gamma^*}(-(Q_1+Q_2)^2, 0) \Big]$$



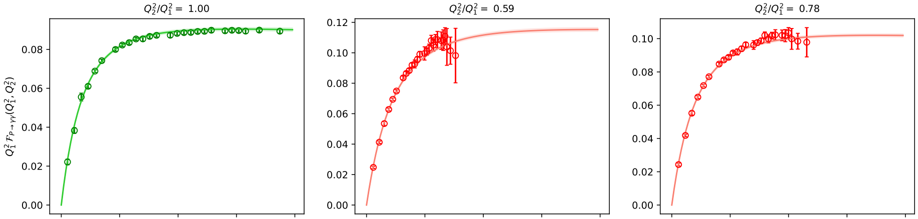
z-expansion

Extract $\mathcal{F}_{\pi_0\gamma^*\gamma^*}$ over the whole kinematical range using the modified z-expansion [Gerardin et al., Phys. Rev. D100, 034520 (2019) and refs. therein]:

$$P(Q_1^2,Q_2^2)\mathcal{F}_{\pi_0\gamma^*\gamma^*}(Q_1^2,Q_2^2) = \sum_{m,n=0}^{N} c_{nm}\left(z_1^n - (-1)^{N+n+1}\frac{n}{N+1}z_1^{N+1}\right)\left(z_2^m - (-1)^{N+m+1}\frac{m}{N+1}z_2^{N+1}\right)$$

- $z_k = z_k(Q_k^2)$, $P(Q_1^2,Q_2^2)$ a polynomial in four-momenta
- Sampling in momentum plane by fixing Q_2^2/Q_1^2 using continuous free parameter ω.
- Correlated order $N \in 1,2,3$ fits.

Example TFFs



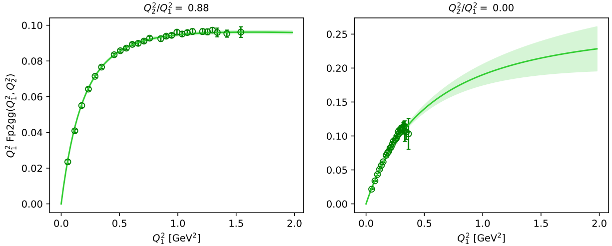
Fitted parameters with errors:
0.43793
-0.15149   -0.23170
0.06629   0.18400   -0.47719

0.00342
0.00535   0.26756
0.12740   0.10524   0.28895

χ^2/dof: 1.362 with $N_{dof} =$ 72
including full covariance matrix.

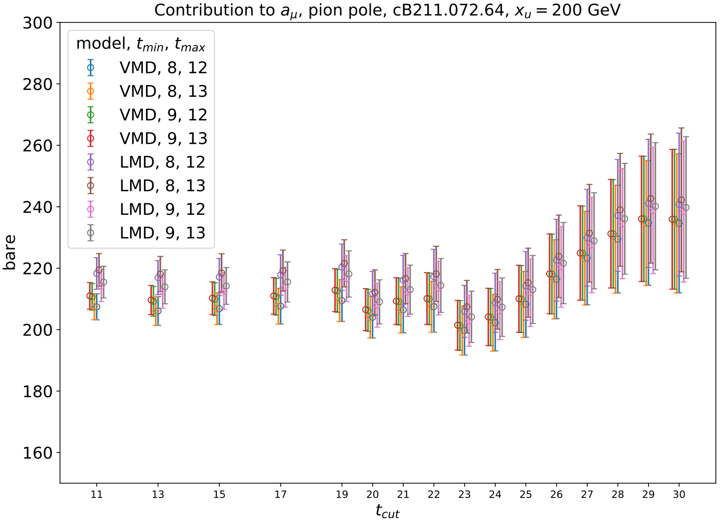
$F_{P\to\gamma\gamma}(0,0) =$ 0.5110 +/- 0.0109 (rel. err. 0.0213)

LVC RC from $F^{exp.}_{P\to\gamma\gamma}(0,0) = 0.276(4)$ GeV^{-1}:
$\sqrt{Z_V \cdot Z_A} =$ 0.7349 +/- 0.0078 (rel. err. 0.0107)



Global LMD fit, $\{t_{min}, t_{max}\} = \{9, 12\}$, reduced $\chi^2 = 1.20$, integration with $t_{cut} = 15$.

Pion pole contribution to a_μ

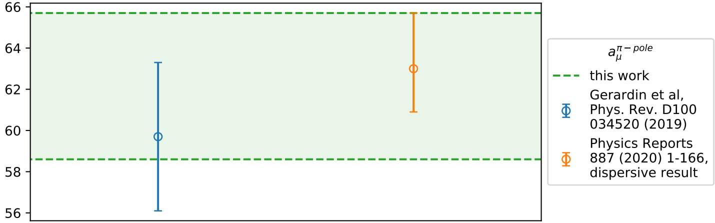


Stable results for $t_{cut} \in [0.88, 2.08]$ fm, consistent within errors up to 2.4 fm.

Preliminary results

- We use local iso-singlet vector currents $V_\mu^{(0,0)}$ and local iso-vector axial vector currents $A_\mu^{(1,\pm)}$ instead of conserved currents → introduces renormalization factor $Z_V^2 \cdot Z_A^2$ in $a_\mu^{\pi-pole}$.
- Z_V^2 and Z_A^2 are available for our setup from an independent calculation from first principles from within ETMC - see talk by Matteo di Carlo on Friday.
- From the data shown on the previous slide with $t_{cut} \in [0.88, 2.08]$ fm we get without considering the systematic error a preliminary

$$a_\mu^{\pi-pole} \in [58.6, 65.7] \times 10^{-11}$$ with a 2%-6% statistical error.



Conclusion & Outlook

Summary

Our setup allows the calculation of the pion pole contribution to a_μ from lattice QCD directly at the physical point. We get promising preliminary results, compatible with other calculations.

Next steps

- Characterization of the systematic error.
- Analysis on the second physical point ensemble, estimate continuum limit.
- Exploring different kinematic frames, i.e. with non-zero pion momentum.
- η/η' pole contributions.