



Neutron Electric Dipole Moment Induced by the θ Term from Chiral Fermions

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(χ QCD collaboration)

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EDM and CP-violation

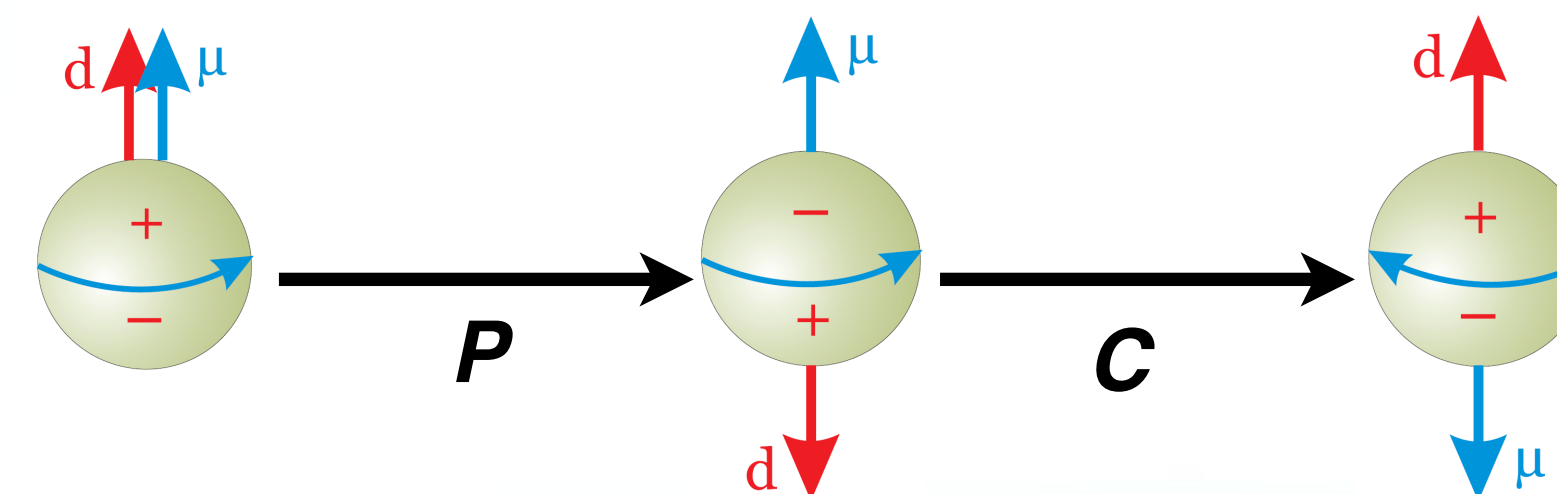
- ◆ The CP violation allowed in the SM (the CKM phase) is **insufficient for Baryogenesis** under Sakharov conditions, BSM interactions?

A. D. Sakharov, JETP Lett. 5 24-27 (1967)

- ◆ A non-zero intrinsic electric dipole moment (EDM) of a fundamental particle violates the CP(T) symmetry.

- ◆ Nucleon EDM (nEDM) is a sensitive probe of BSM: the contribution to the nEDM from the weak CP-violating (CPV) phase is $\sim 10^{-31}$ e·cm, **10^{-5} of the current experimental limit.**

- ◆ **Lattice QCD: model-independent connection between the CPV interactions (theta term and BSM interactions) and the nEDM.**



Experiments

First experiment: $|d_n| < 5 \times 10^{-20} \text{ e} \cdot \text{cm}$

Smith et al., RP108:120-122 (1957)

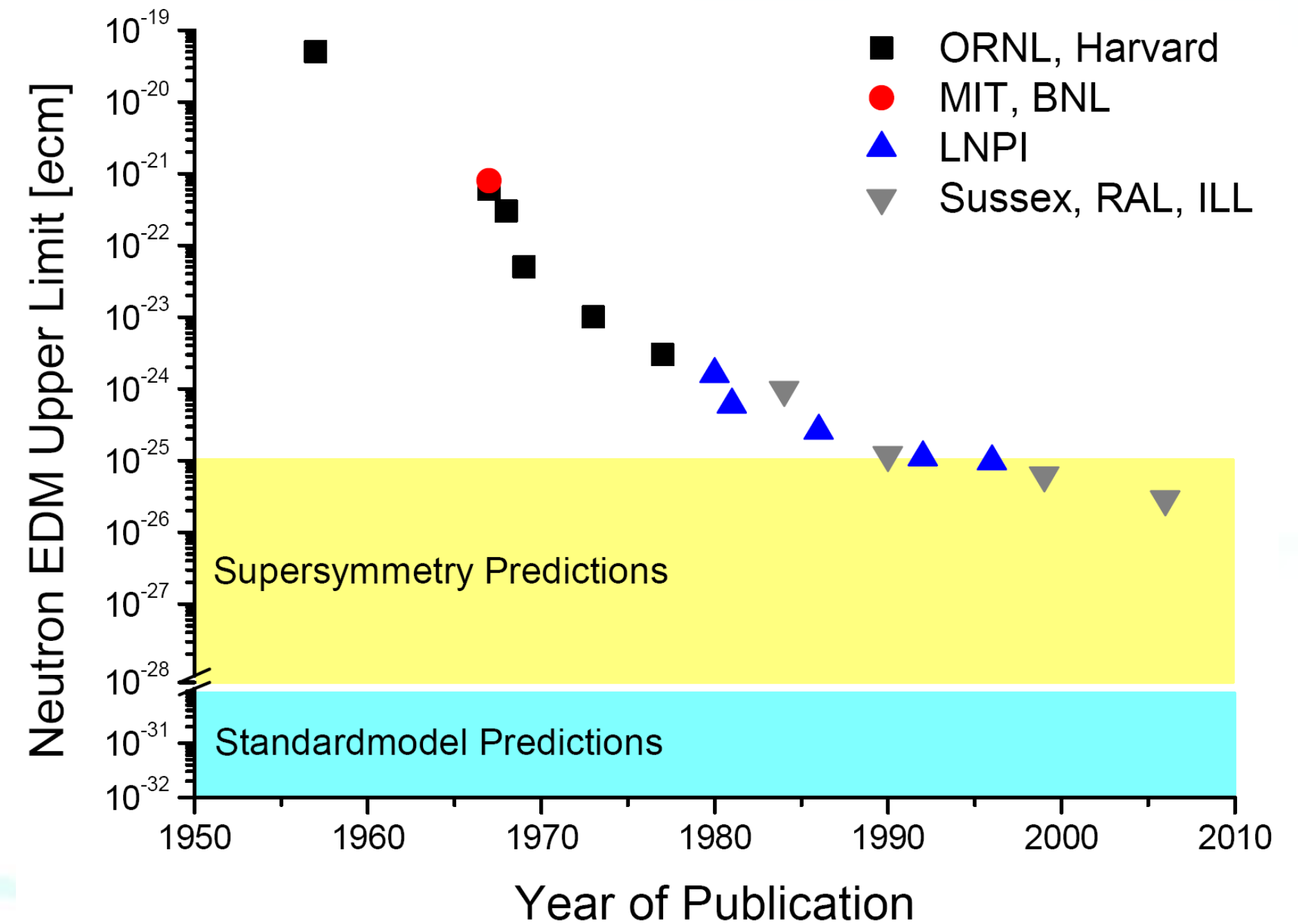
Current neutron EDM limits: $\sim 10^{-26} \text{ e} \cdot \text{cm}$

Baker et al, PRL97:131801 (2006)

Graner et al, PRL116:161601 (2016)

Most recent result: $0.0(1.1)(0.2) \times 10^{-26} \text{ e} \cdot \text{cm}$

C. Abel et al., PRL124:081803 (2020)



During the past 60 years of experiments, **six orders of magnitude** have been covered.

Many experiments are aiming at improving the limit down to $10^{-28} \text{ e} \cdot \text{cm}$ in the next ~ 10 years. It is still a long way to trek to detect a non-zero nEDM, while it leaves plenty of room for the strong CPV and/or BSM physics.

Lattice Efforts

$$F_3 = \tilde{F}_3 + 2\alpha^1 F_2$$

		m_π [MeV]	m_N [GeV]	F_2	α	\tilde{F}_3	F_3
PRD93:074503 (2016) [10]	n	373	1.216(4)	$-1.50(16)^a$	$-0.217(18)$	$-0.555(74)$	$0.094(74)$
PRD72:014504 (2005) [5]	n	530	1.334(8)	$-0.560(40)$	$-0.247(17)^b$	$-0.325(68)$	$-0.048(68)$
	p	530	1.334(8)	$0.399(37)$	$-0.247(17)^b$	$0.284(81)$	$0.087(81)$
PRD73:054509 (2006) [6]	n	690	1.575(9)	$-1.715(46)$	$-0.070(20)$	$-1.39(1.52)$	$-1.15(1.52)$
	n	605	1.470(9)	$-1.698(68)$	$-0.160(20)$	$0.60(2.98)$	$1.14(2.98)$
PRL115:062001 (2015) [8]	n	465	1.246(7)	$-1.491(22)^c$	$-0.079(27)^d$	$-0.375(48)$	$-0.130(76)^d$
	n	360	1.138(13)	$-1.473(37)^c$	$-0.092(14)^d$	$-0.248(29)$	$0.020(58)^d$

watershed: Abramczyk et al., PRD96:014501 (2017)

	Neutron $\bar{\Theta}$ e · fm	Proton $\bar{\Theta}$ e · fm
This Work	$d_n = -0.003(7)(20)$	$d_p = 0.024(10)(30)$
This Work with $N\pi$	$d_n = -0.028(18)(54)$	$d_p = 0.068(25)(120)$
PRD103:054501 (2021) ETMC [66]	$ d_n = 0.0009(24)$	–
PRC103:015202 (2021) Dragos et al. [44]	$d_n = -0.00152(71)$	$d_p = 0.0011(10)$
arXiv:1901.05455 Syritsyn et al. [67]	$d_n \approx 0.001$	–

T. Bhattacharya et. al., PRD103:114507 (2021)

Theta QCD With Chiral Fermions

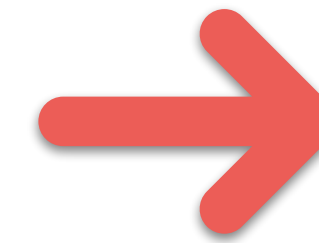
For overlap fermions, **the anomalous Ward identity (AWI) has been proven (with chiral axial vector current) and numerically checked** (with local axial current plus a normalization constant the same as the iso-vector case) at finite lattice spacings.

P. Hasenfratz, et. al., NPB643:280 (2002)

J. Liang et. al., PRD98:074505 (2018)

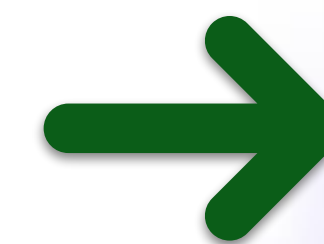
With the AWI, it can be shown that the topological charge term can be replaced with the $2mP$ term, which guarantees that **$d_n \rightarrow 0$ when $m_q \rightarrow 0$ even at finite lattice spacings.**

D. Guadagnoli, et. al., JHEP 0304, 019 (2003)



Relatively heavy pion masses and chiral extrapolation at finite lattice spacings

Topological charge can be defined from the overlap operator: $\frac{1}{2} \text{Tr}[\gamma_5 D_{\text{ov}}]$



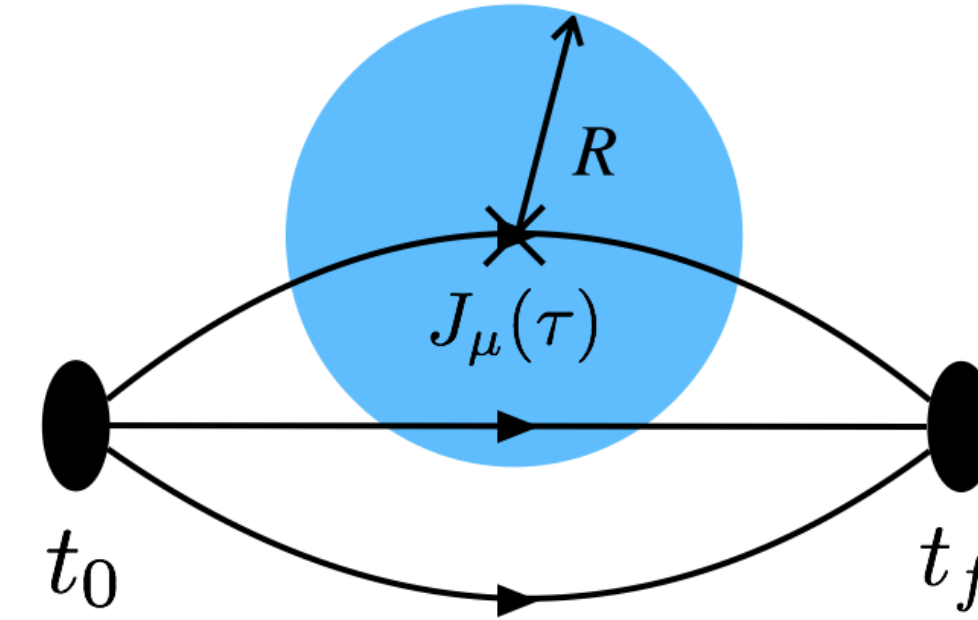
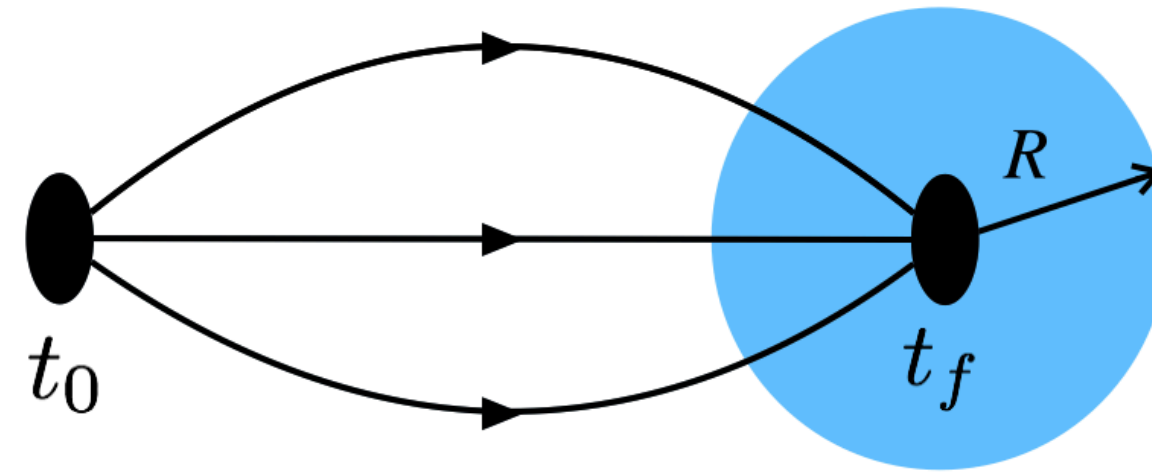
Index theorem

Error Reduction

The cluster decomposition error reduction (CDER)

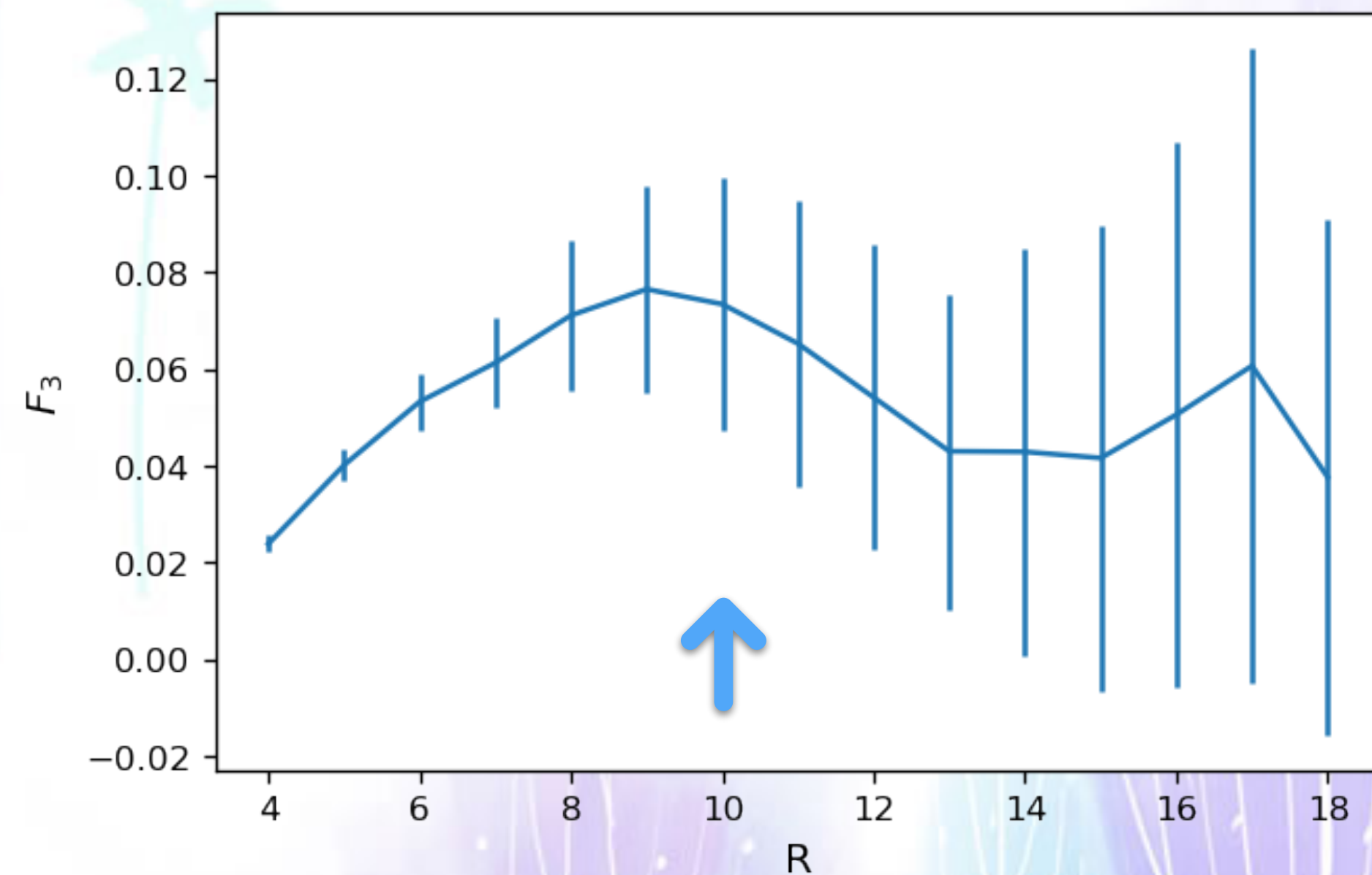
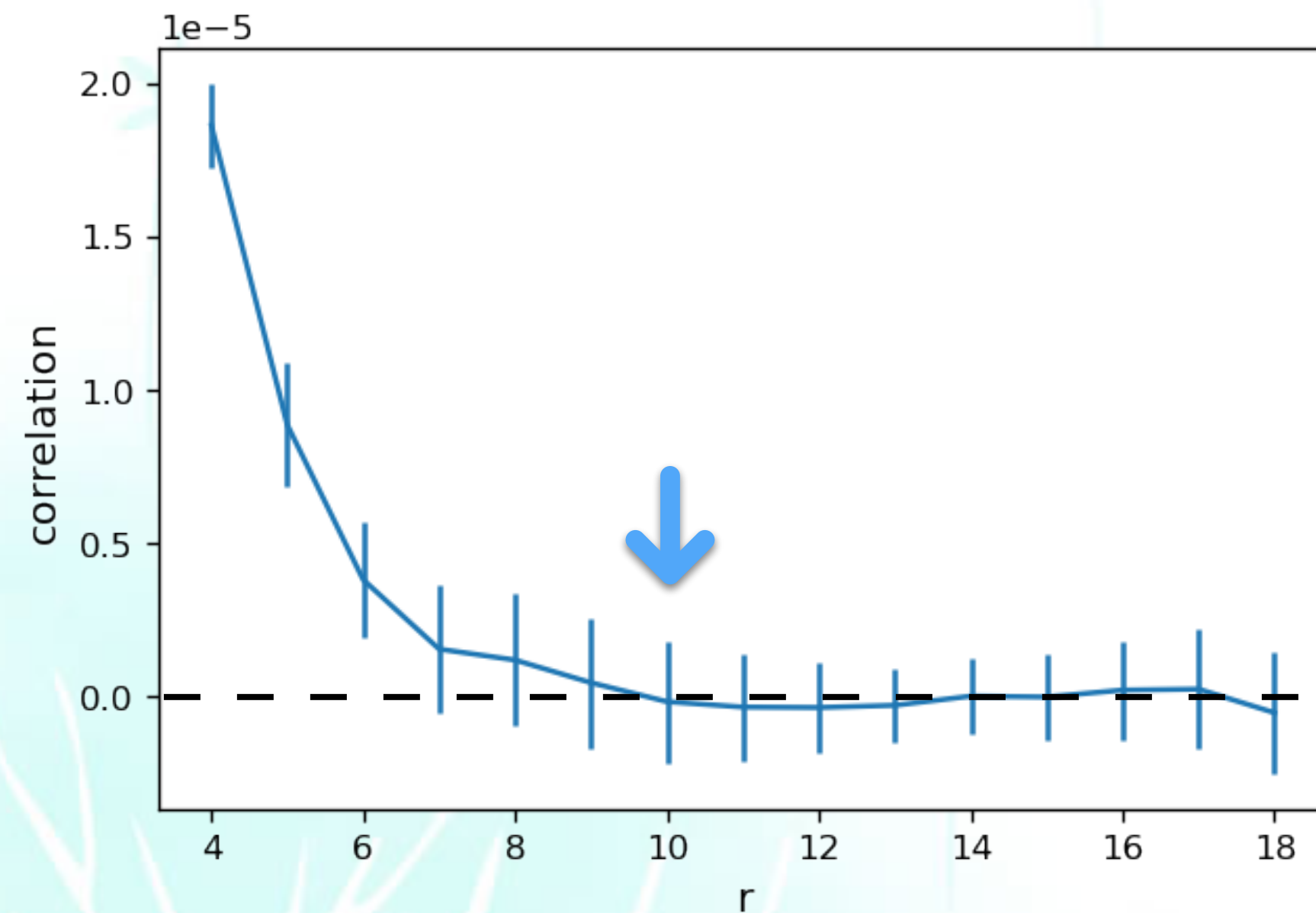
Liu, **Liang** and Yang, PRD97:034507 (2018)

$$Q \rightarrow \sum_x q(x)$$



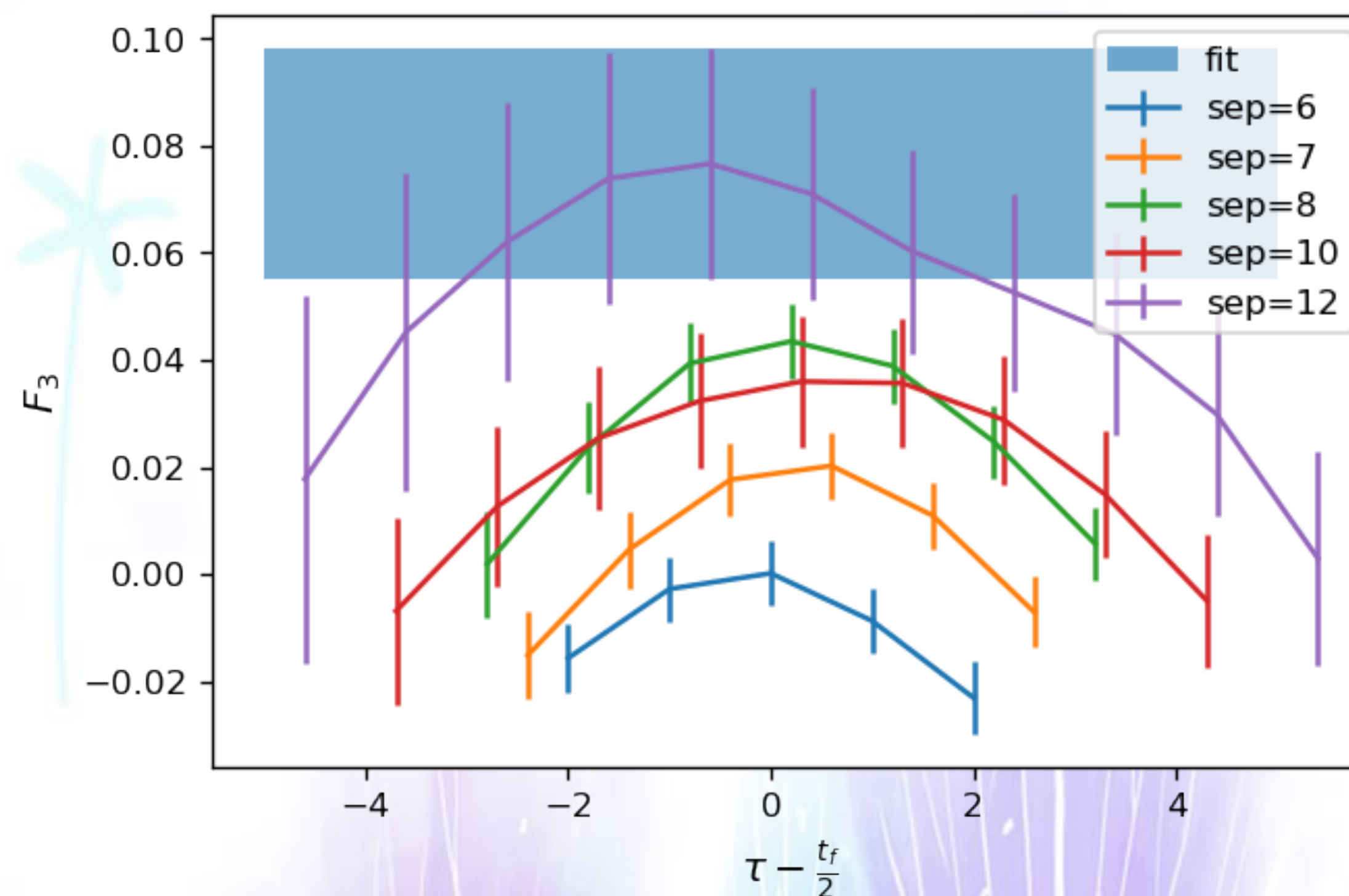
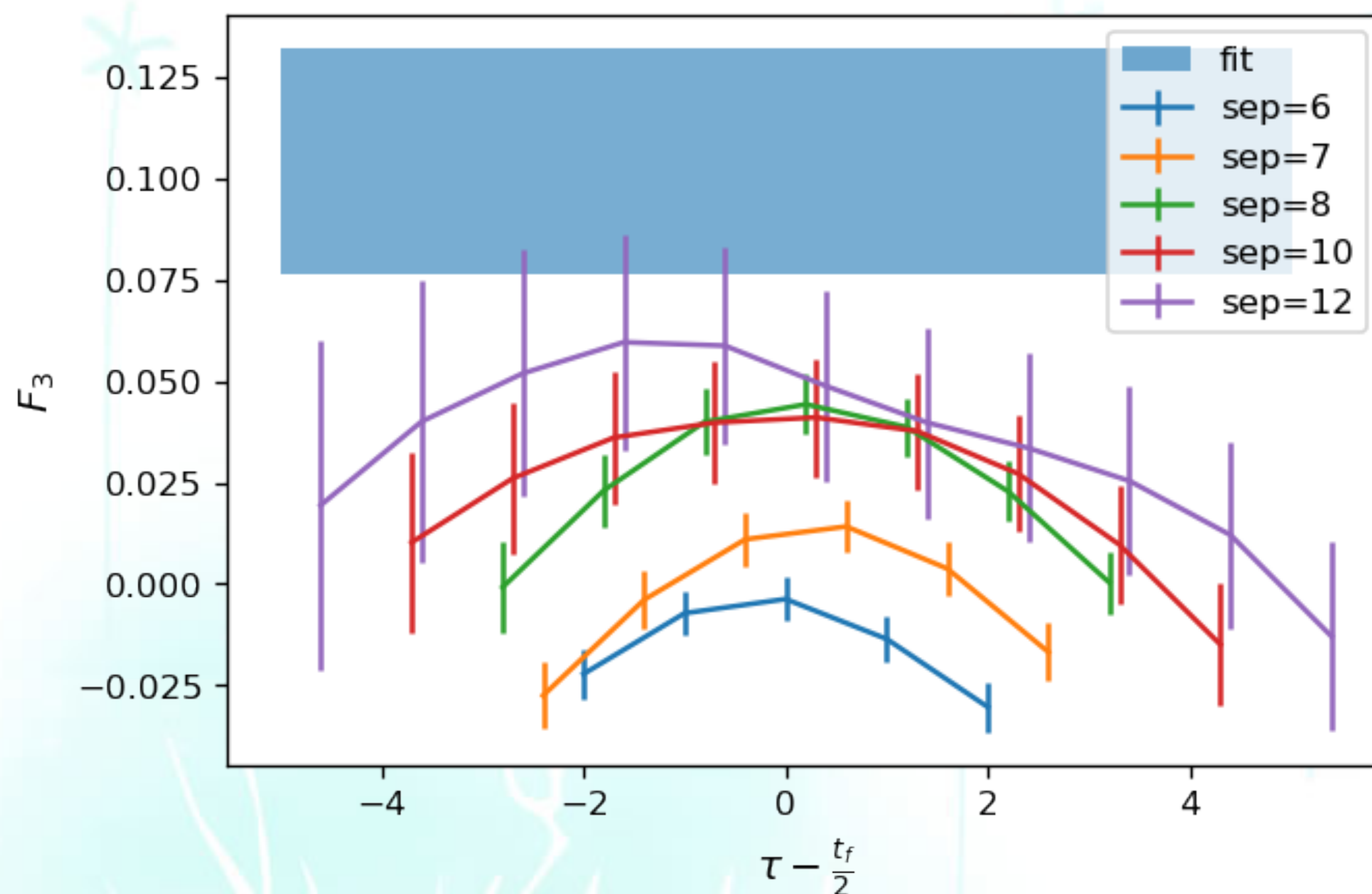
$$C_3^Q(t_f, R) = \sum_{\vec{x}} \left\langle \sum_{|r| \leq R} q(x+r) \chi(x) \bar{\chi}(0) \right\rangle$$

$$C_4^Q(t_f, \tau, R) = \sum_{\vec{x}, \vec{y}} e^{-i\vec{p} \cdot \vec{x}} e^{i\vec{q} \cdot \vec{y}} \left\langle \chi(x) \sum_{|r| \leq R} q(y+r) J_\mu(y) \bar{\chi}(0) \right\rangle$$



Lattice Setups

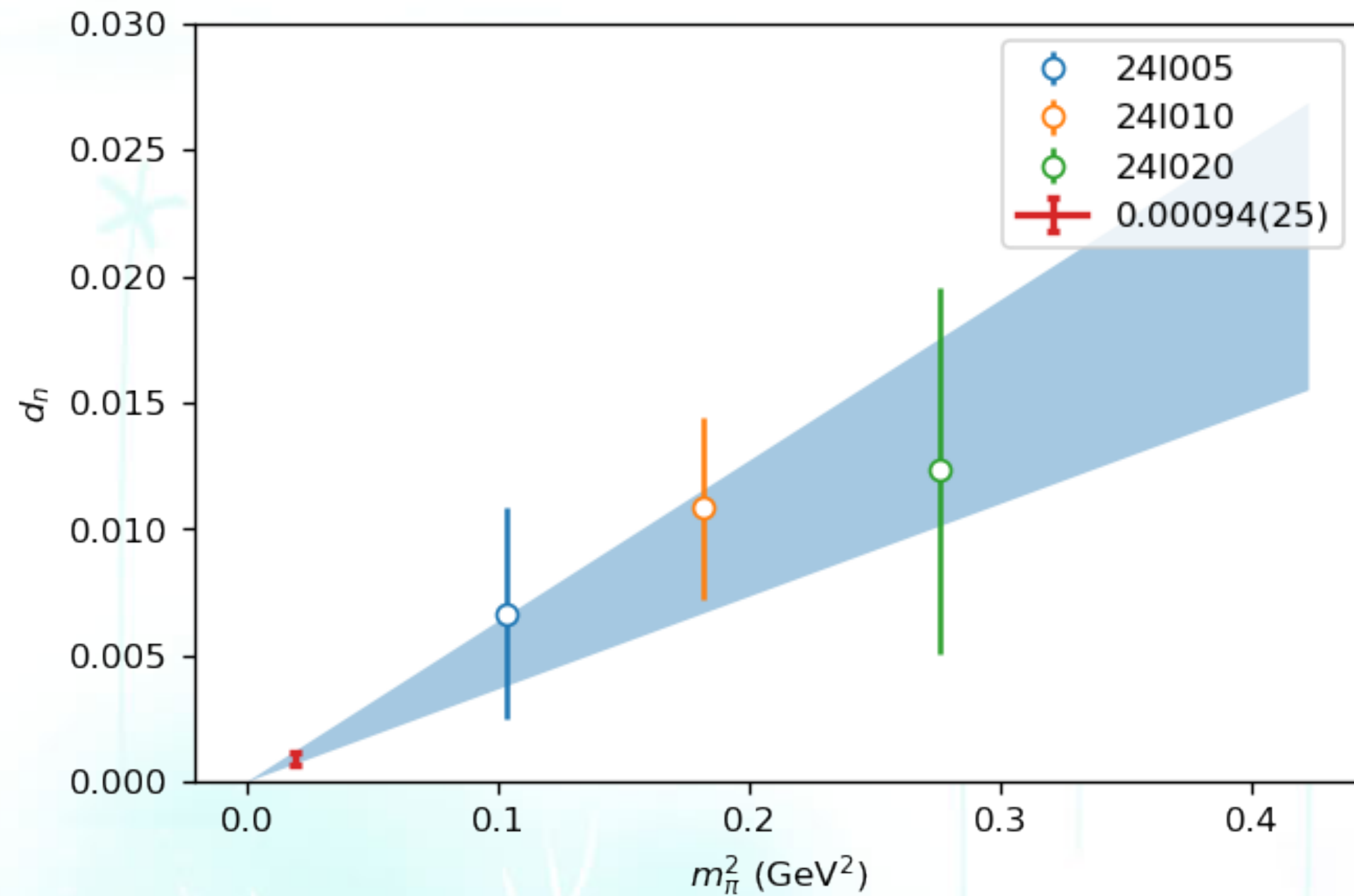
- ◆ Overlap fermions on three domain wall seas (by RBC/UKQCD) with lattice spacing 0.11 fm.
- ◆ Sea pion masses (valence pion masses) are **339 MeV** (282 MeV, 321 MeV, 348 MeV, 389 MeV), **432 MeV** (391 MeV, 426 MeV, 519 MeV, 560 MeV), **576 MeV** (432 MeV, 525 MeV, 606 MeV)
- ◆ Five source-sink separations are 6a, 7a, 8a, 10a, 12a. Standard two-state fits are used to control the excited-state effects.



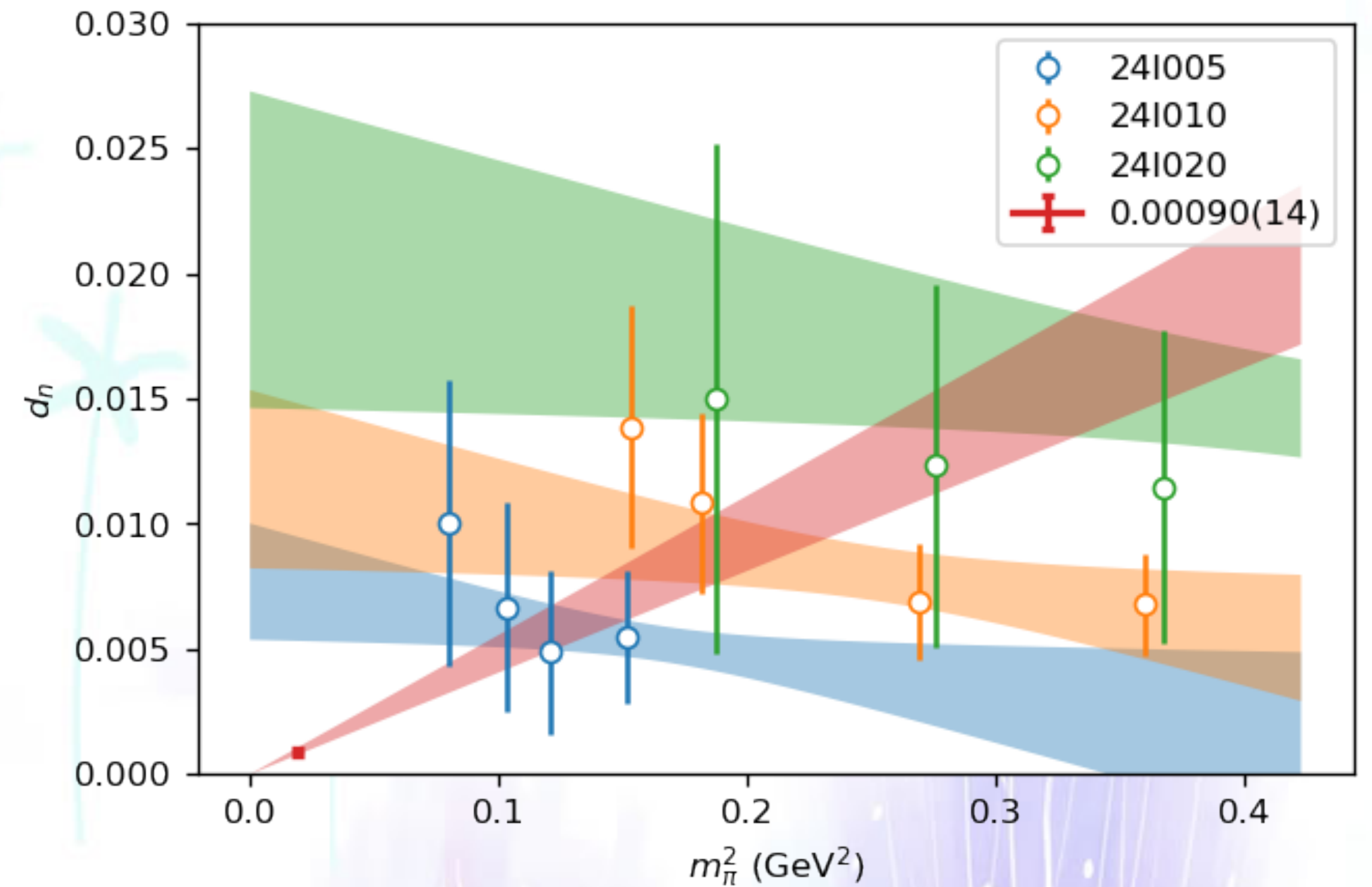
Preliminary Result

$$d_n^{(PQ)} = \frac{e \bar{\theta} m_{\text{sea}}}{4\pi^2 f^2} \left[F_\pi \log \left(\frac{m_\pi^2}{\mu^2} \right) + F_J \log \left(\frac{m_J^2}{\mu^2} \right) \right] + \bar{\theta} \frac{e}{\Lambda_\chi^2} \left[\frac{m_{\text{sea}}}{2} c(\mu) + \underline{d(m_{\text{sea}} - m_{\text{val}})} + \underline{f q_{jl} (m_{\text{sea}} - m_{\text{val}})} \right]$$

D.O'Connell and M. J. Savage, PLB633:319 (2006)



Without partially quenched points



With partially quenched points

Summary and Outlook

CP Violation is an important physical topic, but getting the nucleon (neutron) EDM on the lattice (especially directly at the physical point) is really hard.

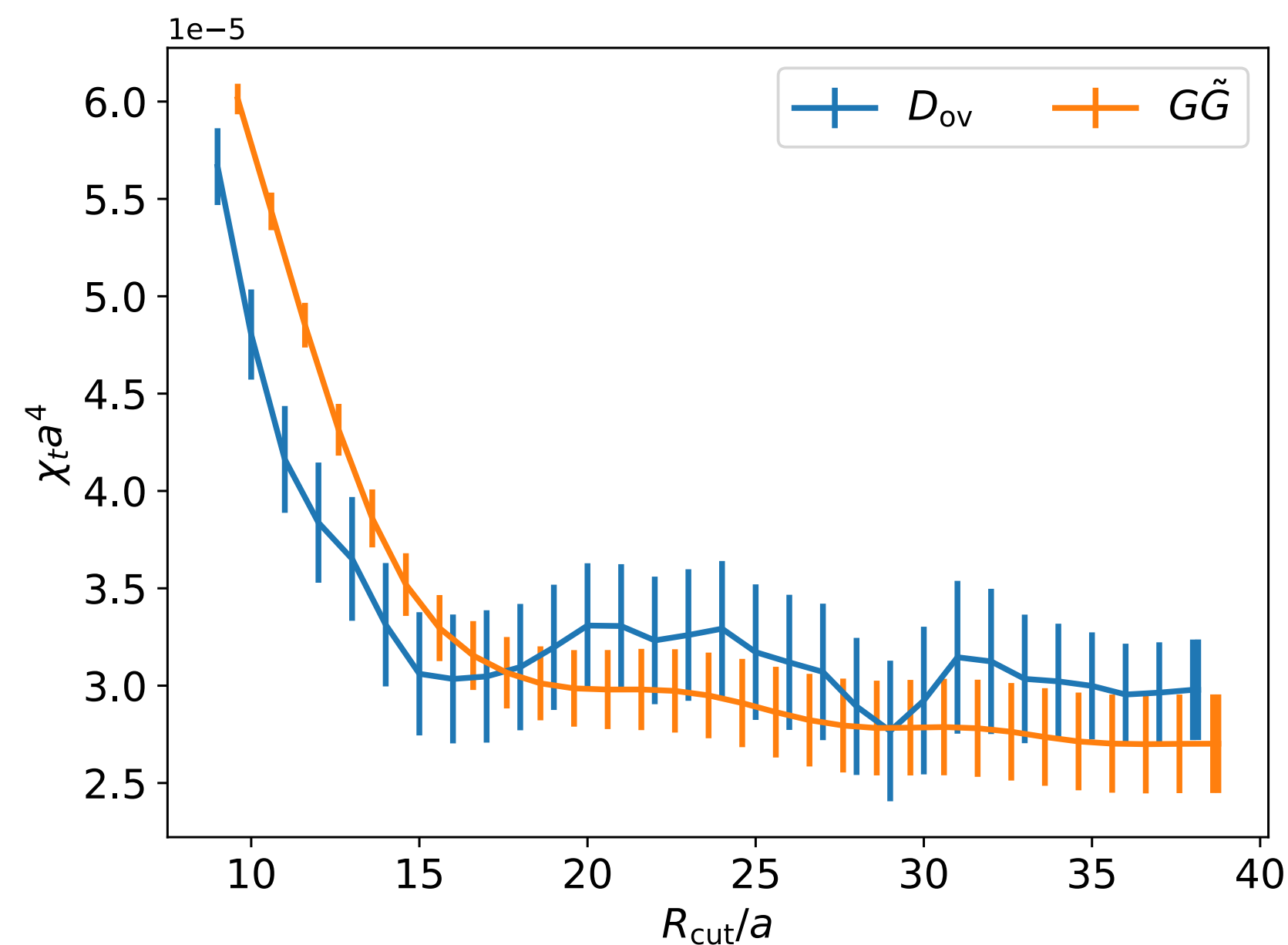
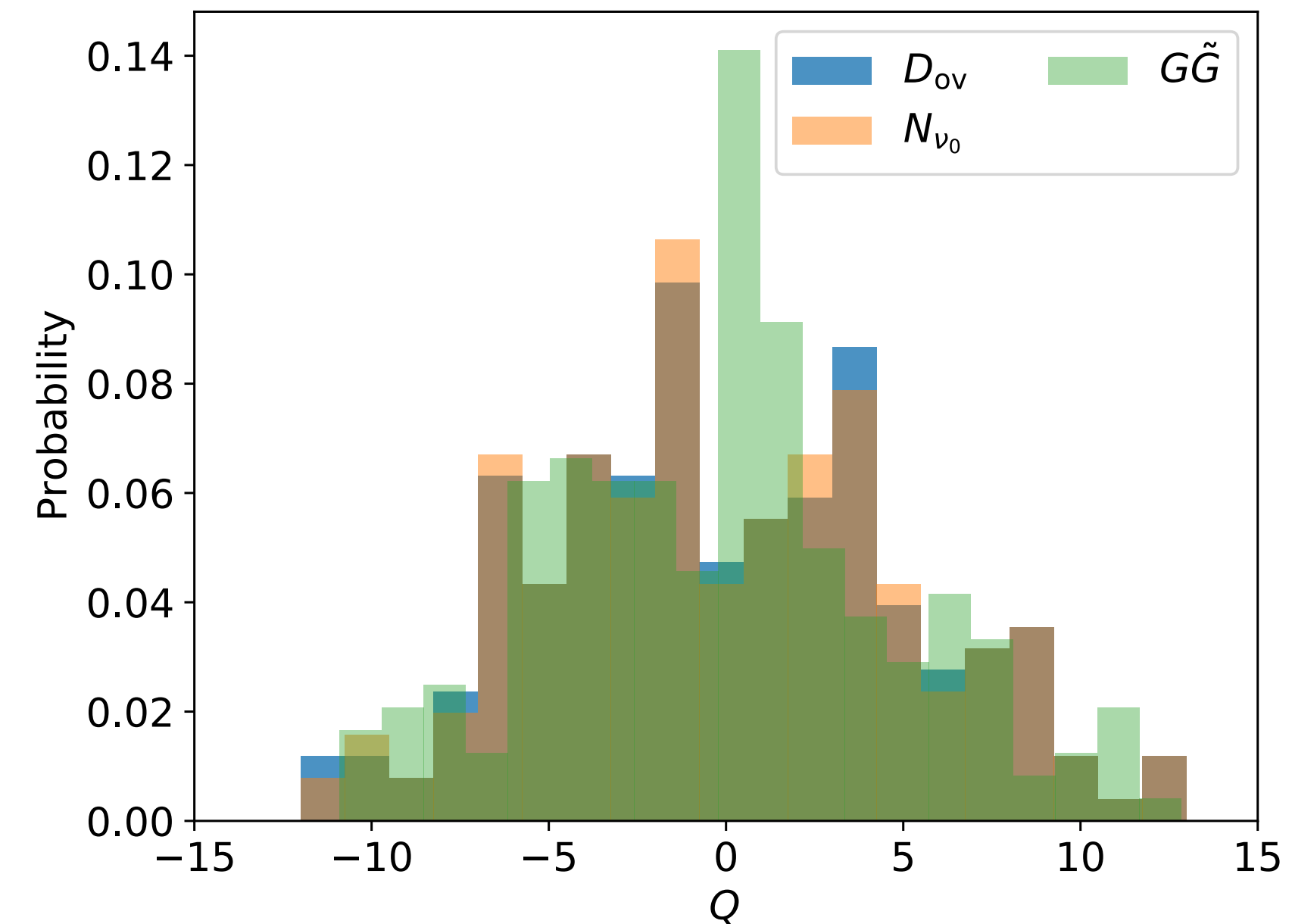
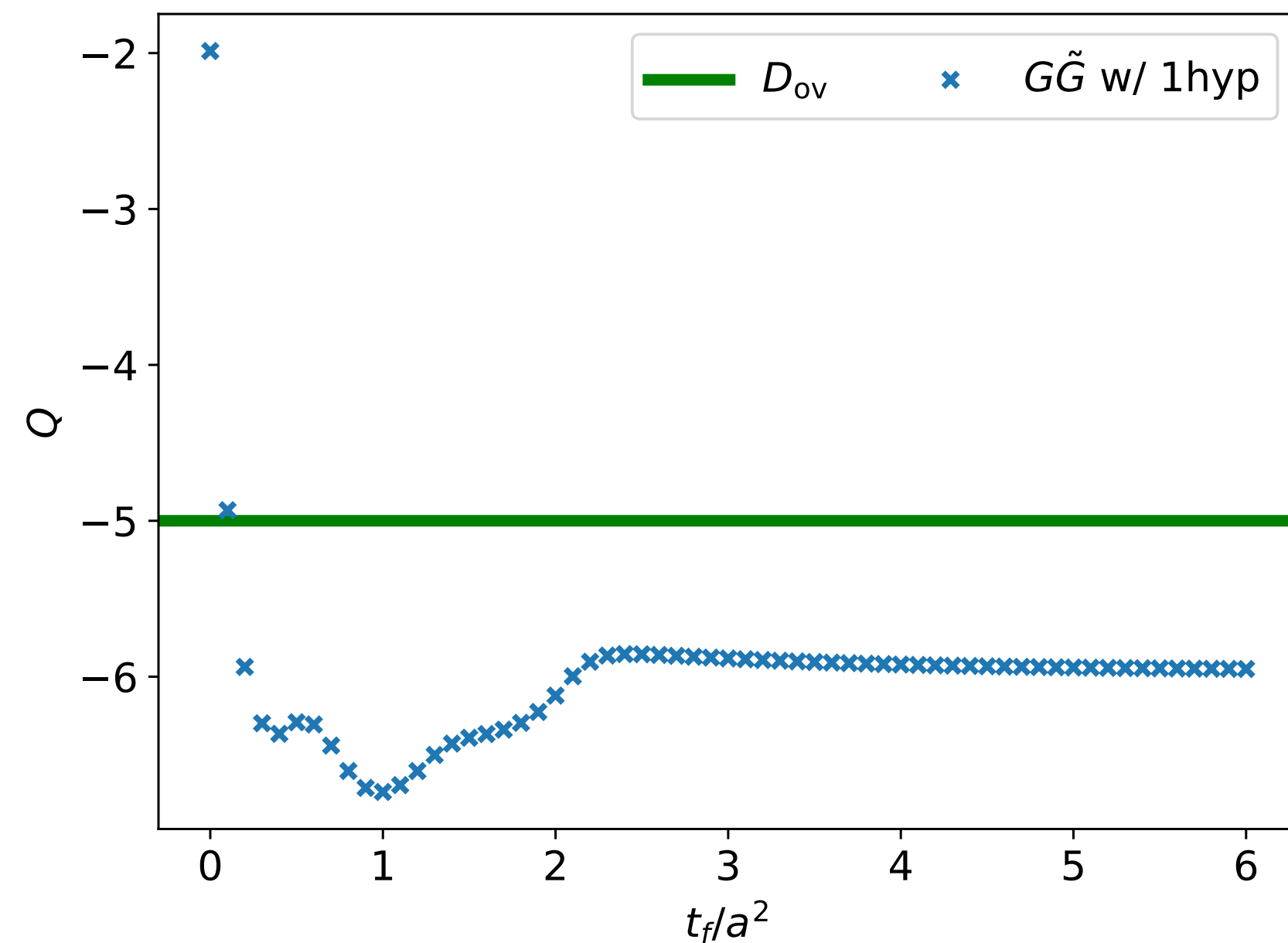
With the help of chiral fermions, we will be able to have reliable non-zero results at the physical pion mass limit.

Our current preliminary result of neutron EDM is $0.00090(14) \theta$ e.fm.

Different lattice spacings and lighter sea pion masses will be included in the next stage.

Thank you for your attention!

More about Topological Charges



The topological charges of individual configurations with different definitions are different, while the distributions are similar.

For physical observables such as the topological susceptibility, **different definitions agree within statistical errors.**

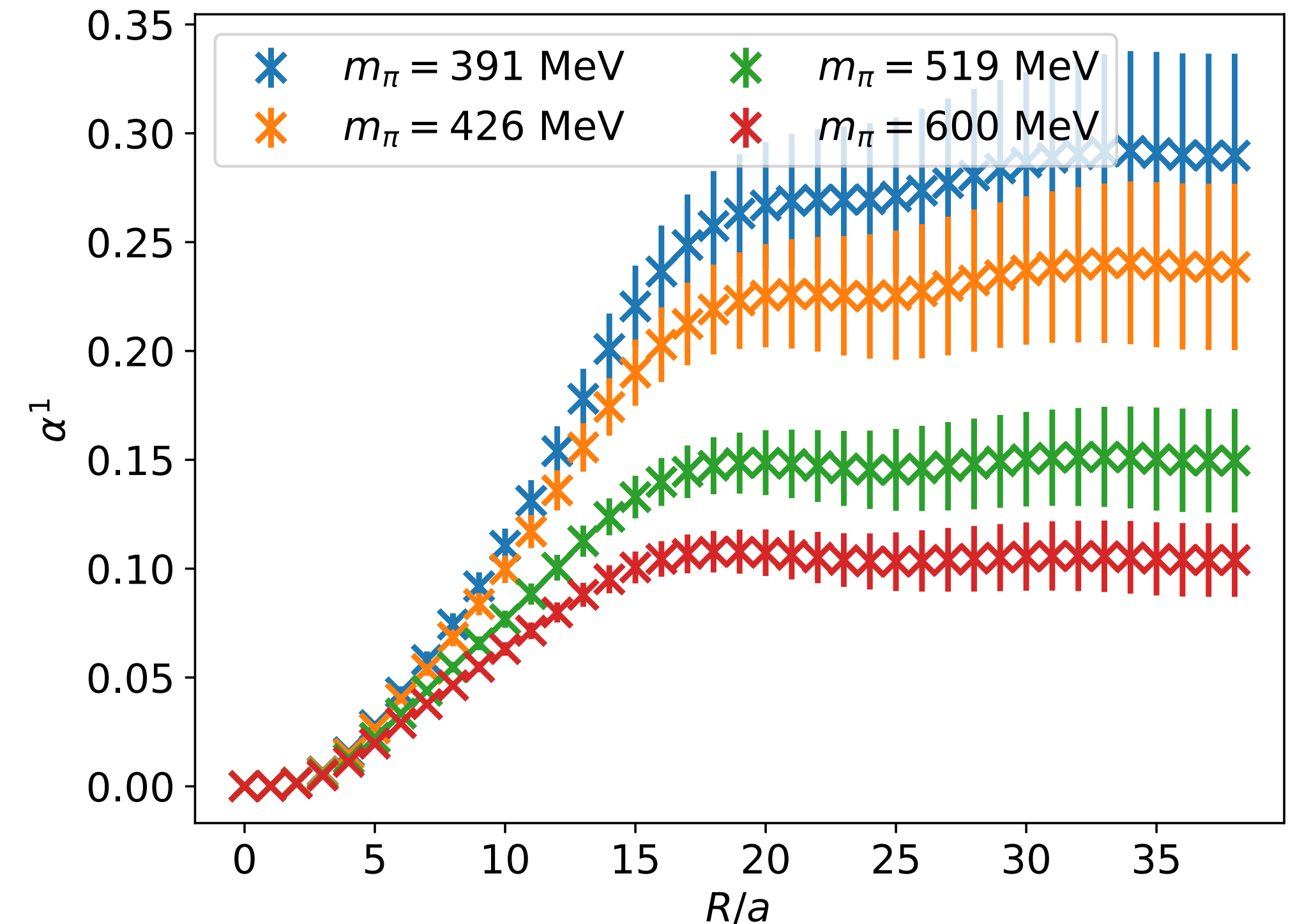
Topological Charges

$$\gamma_5 v_0 = \pm v_0 ,$$

$$Q = -\frac{1}{2} \sum_{\lambda} (2 - a \lambda) (v_{\lambda}, \gamma_5 v_{\lambda}) = n_- - n_+$$

$$Q = \frac{1}{2} \text{Tr} [\gamma_5 D] = \text{Tr} \left[\gamma_5 \frac{\rho D}{2\rho} \right] = Q = \frac{1}{3} \text{Tr} [\gamma_5 \rho D]$$

Using the zeros modes to fix the sign of the topological charge definition

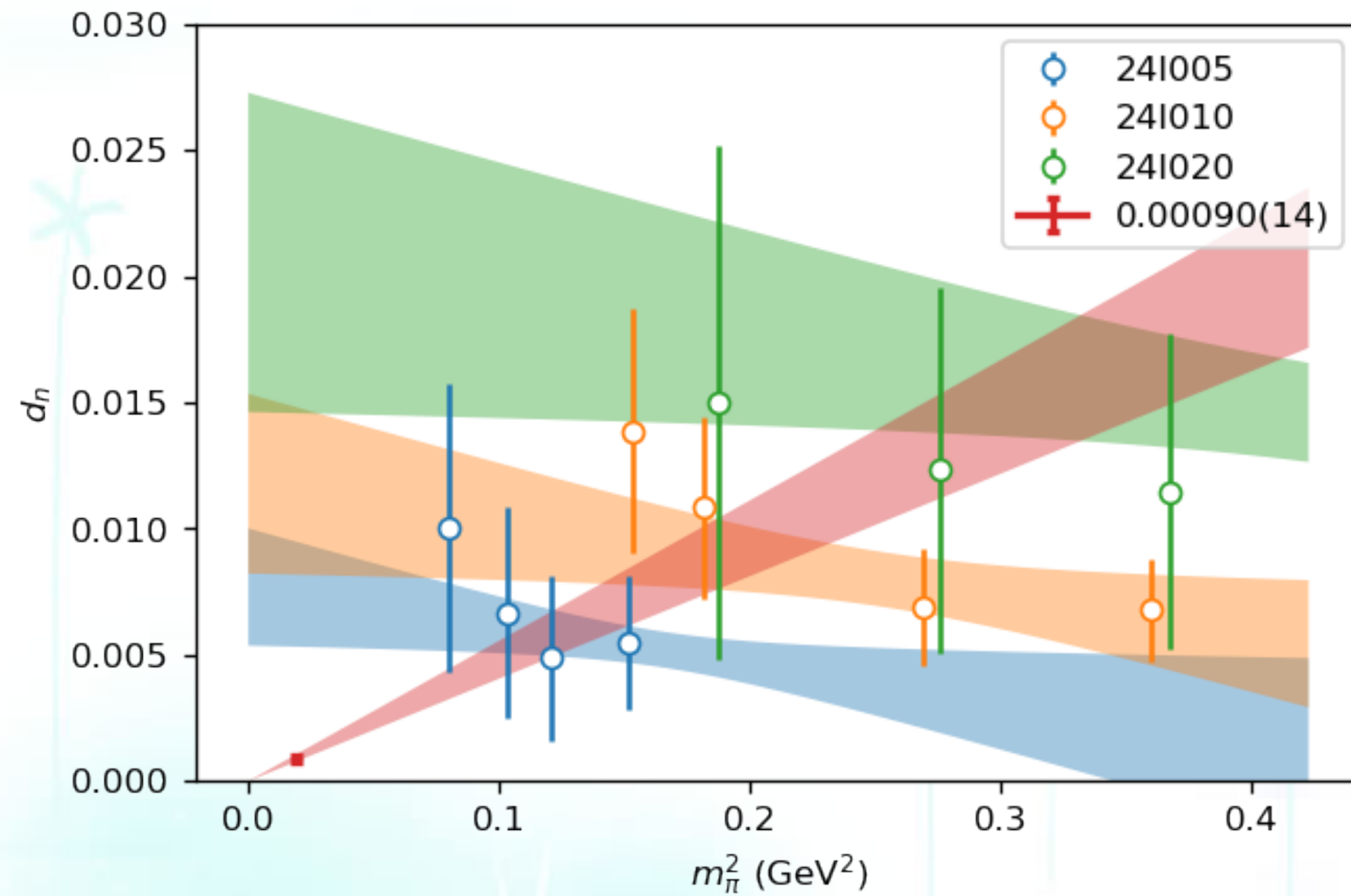


$$u^\theta = e^{i\alpha^1 \theta \gamma_5} u. \quad u^\theta(p) \bar{u}^\theta(p) = \frac{-i\not{p} + m e^{i2\alpha^1 \gamma_5 \theta}}{2m}.$$

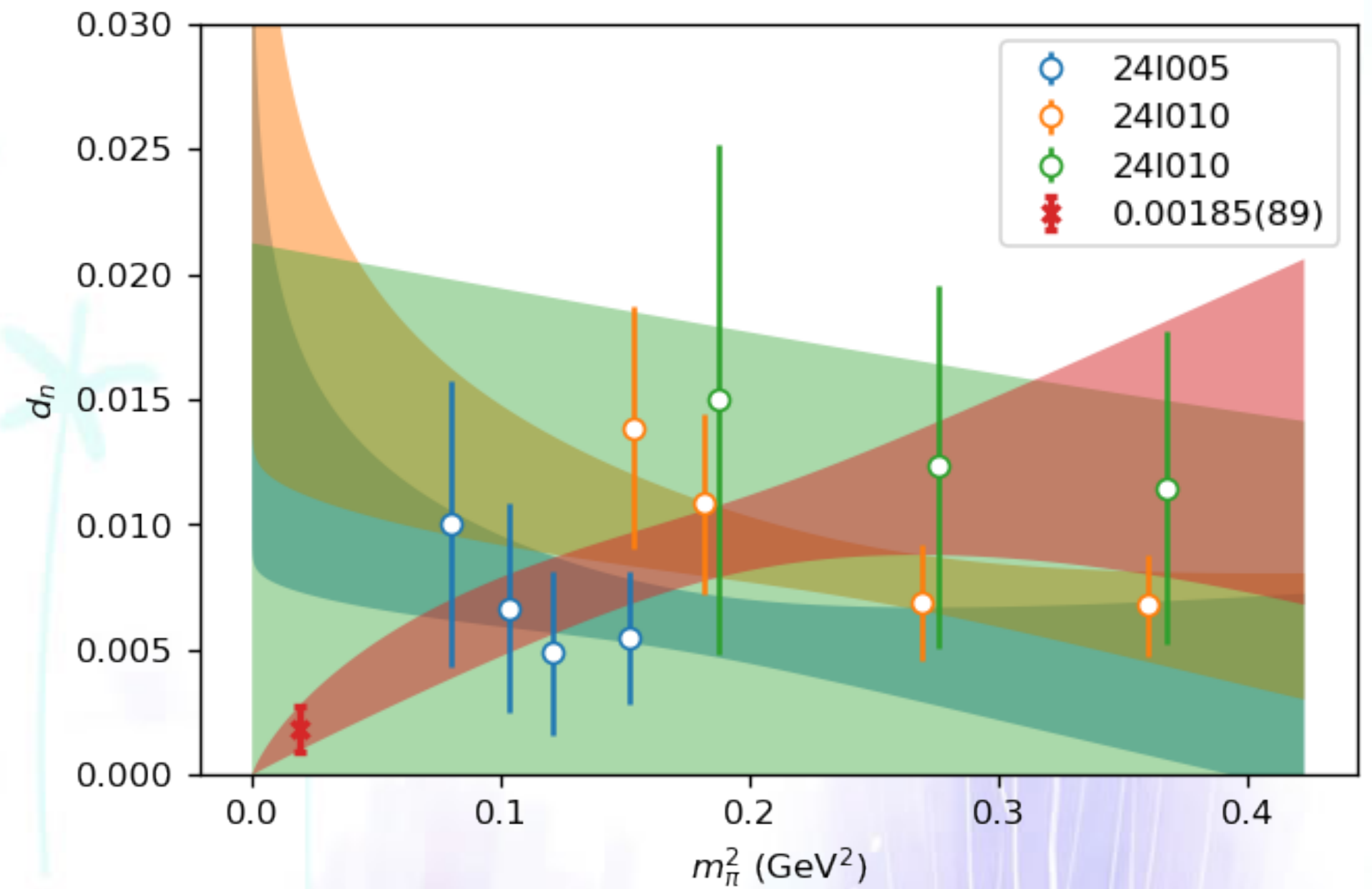
Preliminary Result

$$d_n^{(PQ)} = \frac{e \bar{\theta} m_{\text{sea}}}{4\pi^2 f^2} \left[F_\pi \log\left(\frac{m_\pi^2}{\mu^2}\right) + F_J \log\left(\frac{m_J^2}{\mu^2}\right) \right] + \bar{\theta} \frac{e}{\Lambda_\chi^2} \left[\frac{m_{\text{sea}}}{2} c(\mu) + \underline{d(m_{\text{sea}} - m_{\text{val}})} + f q_{jl} \underline{(m_{\text{sea}} - m_{\text{val}})} \right]$$

D.O'Connell and M. J. Savage, PLB633:319 (2006)



Without log term



With log term