Window contributions to the muon HVP with twisted-mass fermions

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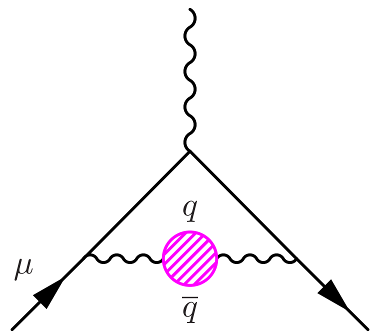
ArXiv:1707.03019; ArXiv:1808.00887;

ArXiv:1901.10462; ArXiv:1910.03874; ArXiv:2003.12086

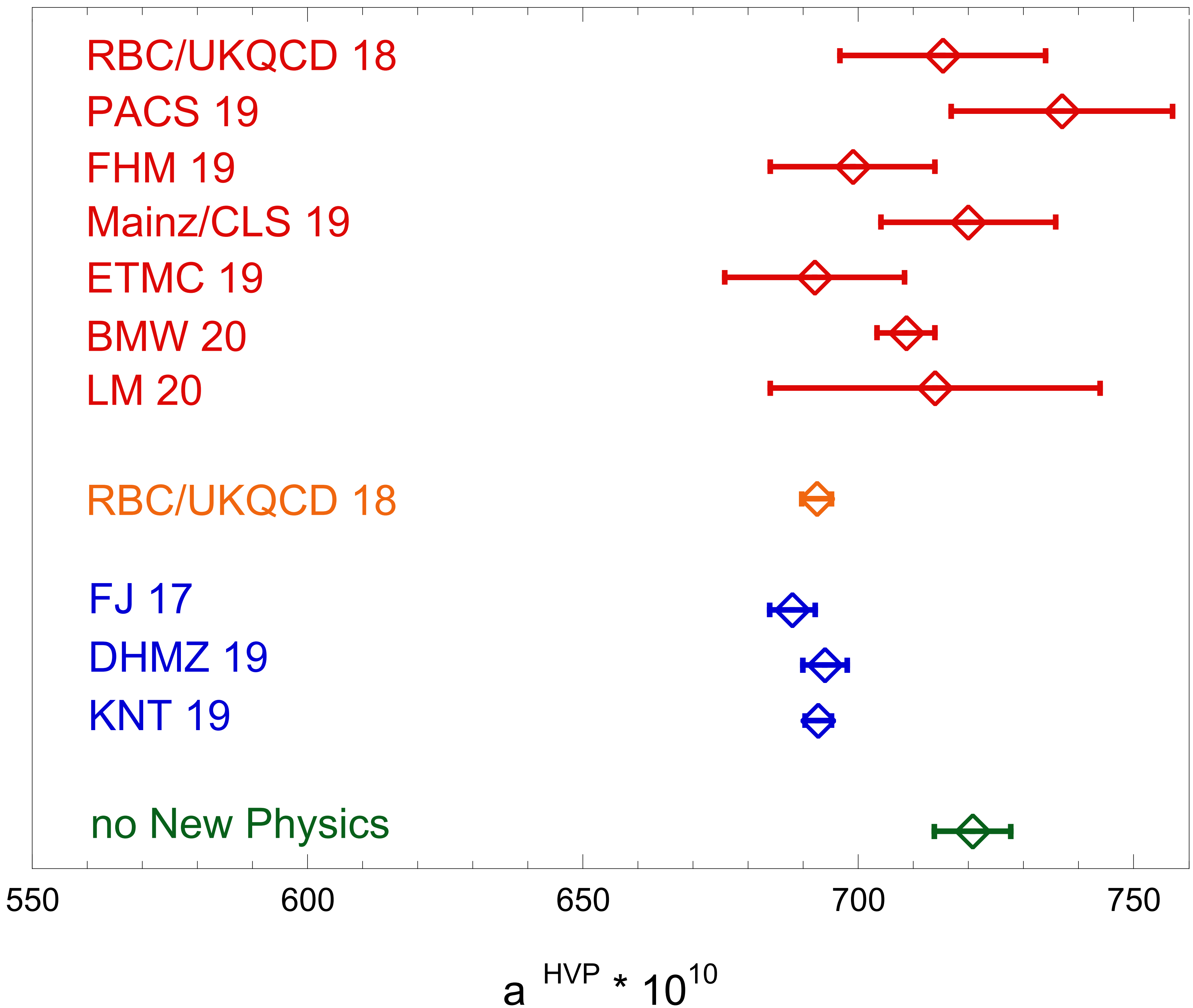
Hadronic Vacuum Polarization

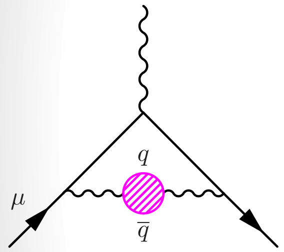


lattice data
100%

lattice + e^+e^-
~ 30% + 70%

e^+e^- data
100%



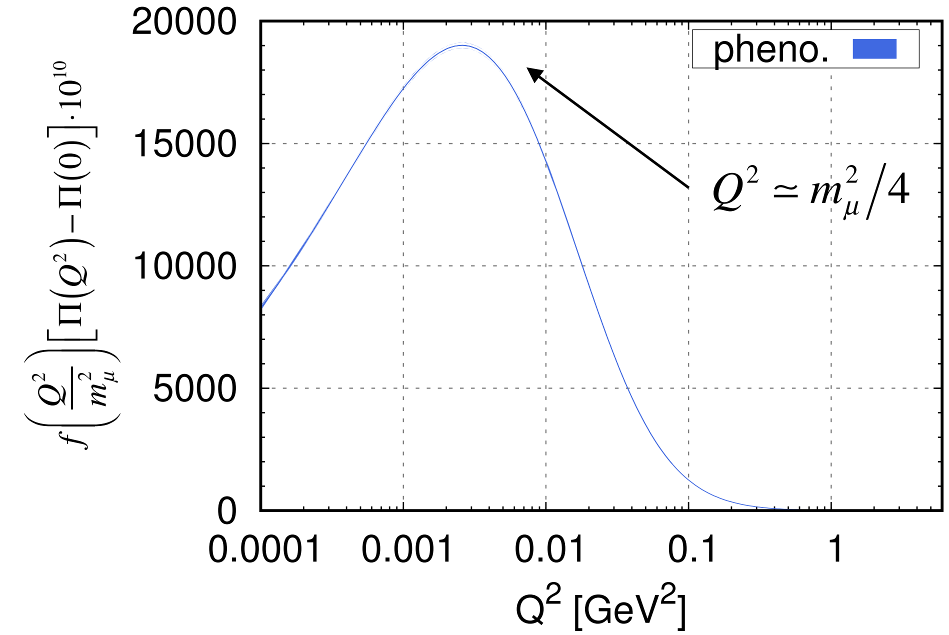


HVP from LQCD

$$\Pi_{\mu\nu}(Q) = \int d^4x\, e^{iQ\cdot x} \langle J_\mu(x) J_\nu(0)\rangle = \left[\delta_{\mu\nu}Q^2 - Q_\mu Q_\nu\right]\Pi(Q^2)$$

$$a_\ell^{\mathrm{HVP}} = 4\alpha_{em}^2 \int_0^\infty dQ^2 \frac{1}{m_\ell^2} f\left(\frac{Q^2}{m_\ell^2}\right)\left[\Pi(Q^2) - \Pi(0)\right]$$

B. E. Lautrup *et al.*, 1972; T. Blum, 2002



F. Jegerlehner, "alphaQEDc17"

Time-Momentum Representation

$$a_\ell^{\mathrm{HVP}} = 4\alpha_{em}^2 \int_0^\infty dt\; K_\ell(t)\, V(t)$$

D. Bernecker and H. B. Meyer, 2011

$$V(t) \equiv \frac{1}{3}\sum_{i=1,2,3}\int d\vec{x}\,\langle J_i(\vec{x},t) J_i(0)\rangle$$

$$\boxed{a_\ell^{\mathrm{HVP}} = \sum_{f=u,d,s,c} 4\alpha_{em}^2 \left\{ \sum_{t=0}^{T_{data}} K_\ell(t) V^f(t) + \sum_{t=T_{data}+a}^{\infty} K_\ell(t) \frac{G_V^f}{2M_V^f} e^{-M_V^f t} \right\}}$$

$t \le T_{data} < T/2$ (avoid bw signals)

$t > T_{data} > t_{min}$ (ground-state dom.)

quark-connected terms only

lattice data

local vector currents

analytic representation

Window contributions to the muon HVP

Windows

$$a_\mu^{\rm HVP} = a_\mu^{SD} + a_\mu^{W} + a_\mu^{LD}$$

$$a_\mu^{SD}(f; t_0, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, K_\mu(t) V^f(t) \left[1 - \Theta\left(t, t_0, \Delta\right)\right]$$

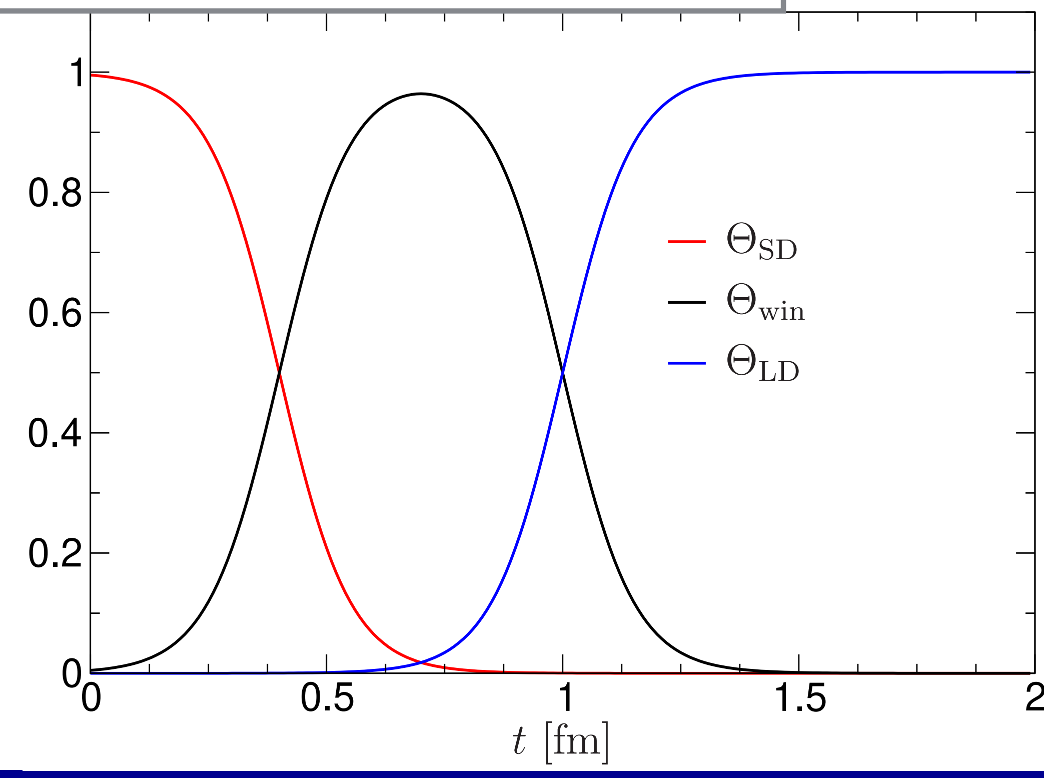
$$a_\mu^{W}(f; t_0, t_1, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, K_\mu(t) V^f(t) \left[\Theta\left(t, t_0, \Delta\right) - \Theta\left(t, t_1, \Delta\right)\right]$$

$$a_\mu^{LD}(f; t_1, \Delta) \equiv 4\alpha_{em}^2 \int_0^\infty dt\, K_\mu(t) V^f(t)\, \Theta\left(t, t_1, \Delta\right)$$

$$\Theta\left(t, t', \Delta\right) = \frac{1}{1 + e^{-2(t-t')/\Delta}}$$

$$t_0 = 0.4 \text{ fm} \quad t_1 = 1.0 \text{ fm}$$

$$\Delta = 0.15 \text{ fm}$$



Details of the lattice simulation

We have used the gauge field configurations generated by ETMC, European Twisted Mass Collaboration, in the pure isosymmetric QCD theory with Nf=2+1+1 dynamical quarks

ensemble	β	V/a^4	$a\mu_{ud}$	$a\mu_\sigma$	$a\mu_\delta$	N_{cf}	$a\mu_s$	M_π (MeV)	M_K (MeV)
$A40.40$	1.90	$40^3 \cdot 80$	0.0040	0.15	0.19	100	0.02363	317(12)	576(22)
$A30.32$		$32^3 \cdot 64$	0.0030			150		275(10)	568(22)
$A40.32$			0.0040			100		316(12)	578(22)
$A50.32$			0.0050			150		350(13)	586(22)
$A40.24$		$24^3 \cdot 48$	0.0040			150		322(13)	582(23)
$A60.24$			0.0060			150		386(15)	599(23)
$A80.24$			0.0080			150		442(17)	618(14)
$A100.24$			0.0100			150		495(19)	639(24)
$A40.20$		$20^3 \cdot 48$	0.0040			150		330(13)	586(23)
$B25.32$	1.95	$32^3 \cdot 64$	0.0025	0.135	0.170	150	0.02094	259 (9)	546(19)
$B35.32$			0.0035			150		302(10)	555(19)
$B55.32$			0.0055			150		375(13)	578(20)
$B75.32$			0.0075			80		436(15)	599(21)
$B85.24$		$24^3 \cdot 48$	0.0085			150		468(16)	613(21)
$D15.48$	2.10	$48^3 \cdot 96$	0.0015	0.1200	0.1385	100	0.01612	223 (6)	529(14)
$D20.48$			0.0020			100		256 (7)	535(14)
$D30.48$			0.0030			100		312 (8)	550(14)

- Gluon action: Iwasaki
- Quark action: twisted mass at maximal twist (automatically O(a) improved)

OS for *s* and *c* valence quarks

Pion masses in the range 220 - 490 MeV

4 volumes @ $M_\pi \simeq 320$ MeV and $a \simeq 0.09$ fm

$M_\pi L \simeq 3.0 \div 5.8$

Effective lepton mass and effective windows

$$m_\mu^{eff} \equiv \left(m_\mu / X^{phys}\right) X$$

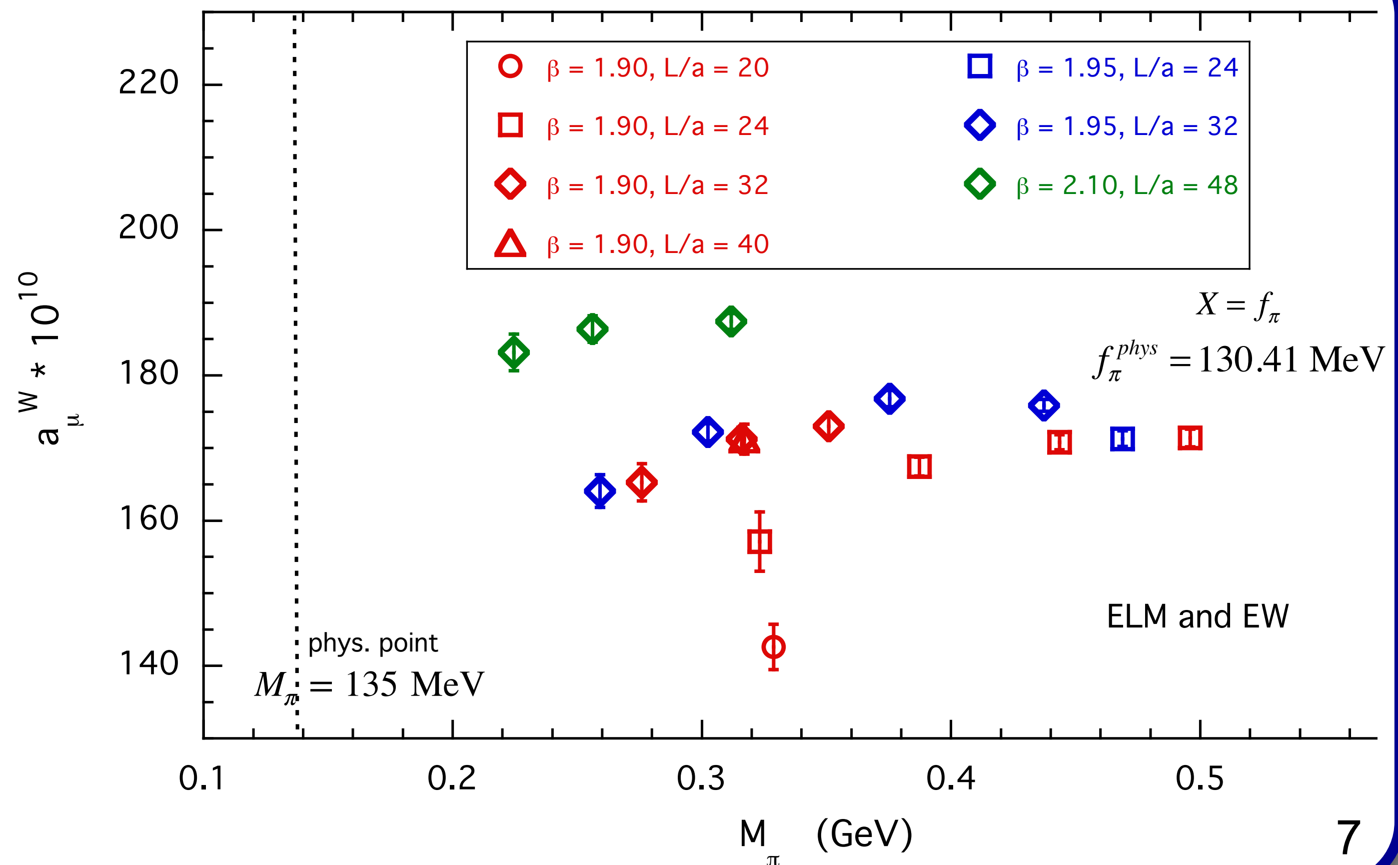
$$\Delta^{eff} \equiv \Delta X^{phys} / X$$

$$t_0^{eff} \equiv t_0 X^{phys} / X \qquad t_1^{eff} \equiv t_1 X^{phys} / X$$

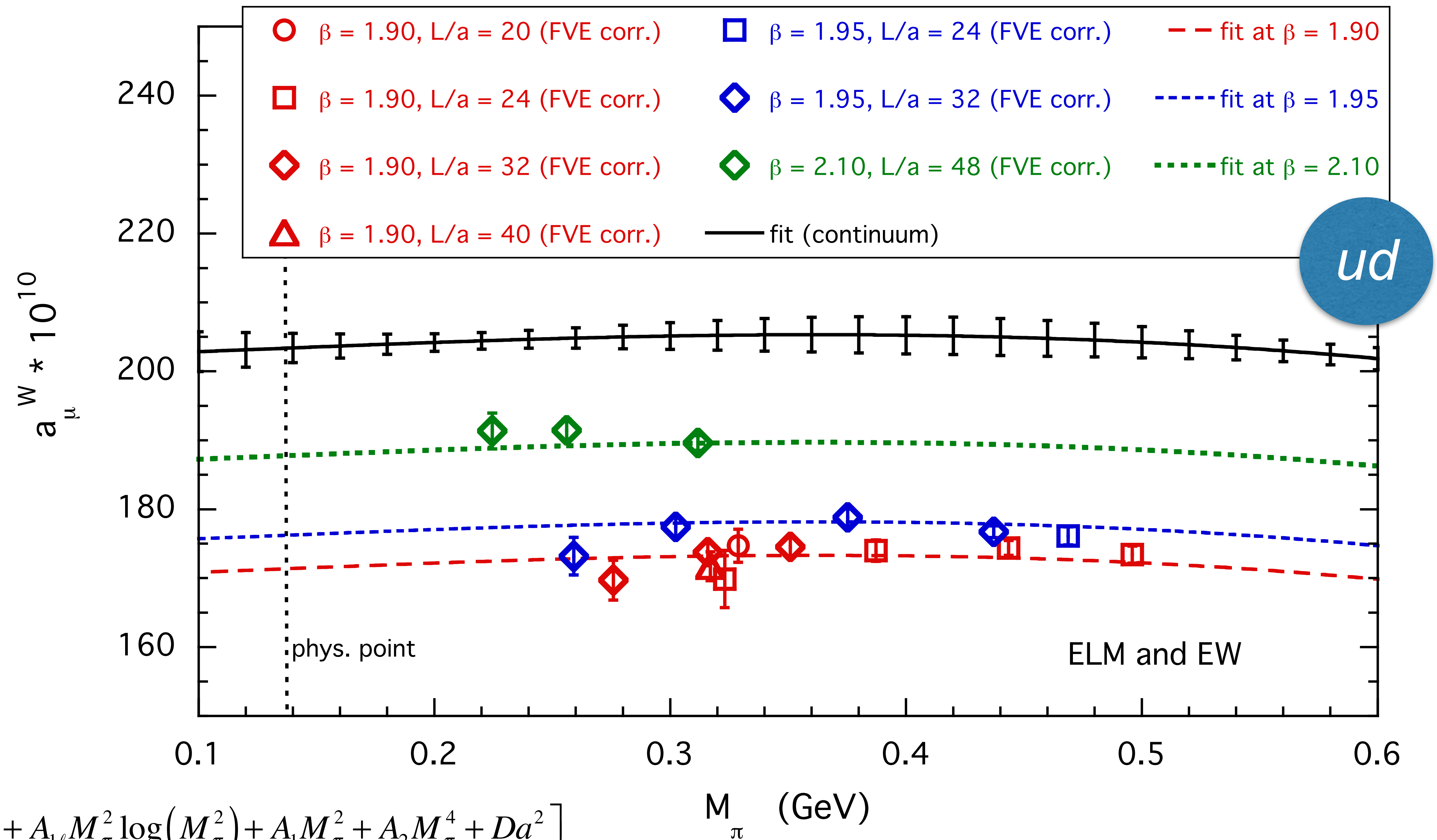
$$a_\mu^W\left(f; t_0^{eff}, t_1^{eff}, \Delta^{eff}\right) = 4\alpha_{em}^2 \frac{1}{m_\mu^2}\left(\frac{X^{phys}}{aX}\right)^2 \sum_{n=1}^{N_T} \widetilde{K}_\mu\left(m_\mu \frac{aX}{X^{phys}} n\right) a^3 V^f(an)$$
$$\cdot\left[\Theta\left(aXn, t_0 X^{phys}; \Delta X^{phys}\right) - \Theta\left(aXn, t_1 X^{phys}; \Delta X^{phys}\right)\right]$$

$a_\mu^W(ud)$

- **Advantage**: uncertainty of the scale setting does not play any role
- For $X = f_\pi$ the pion mass dependence is mild
- Visible FVEs and large discretization effects



Intermediate window



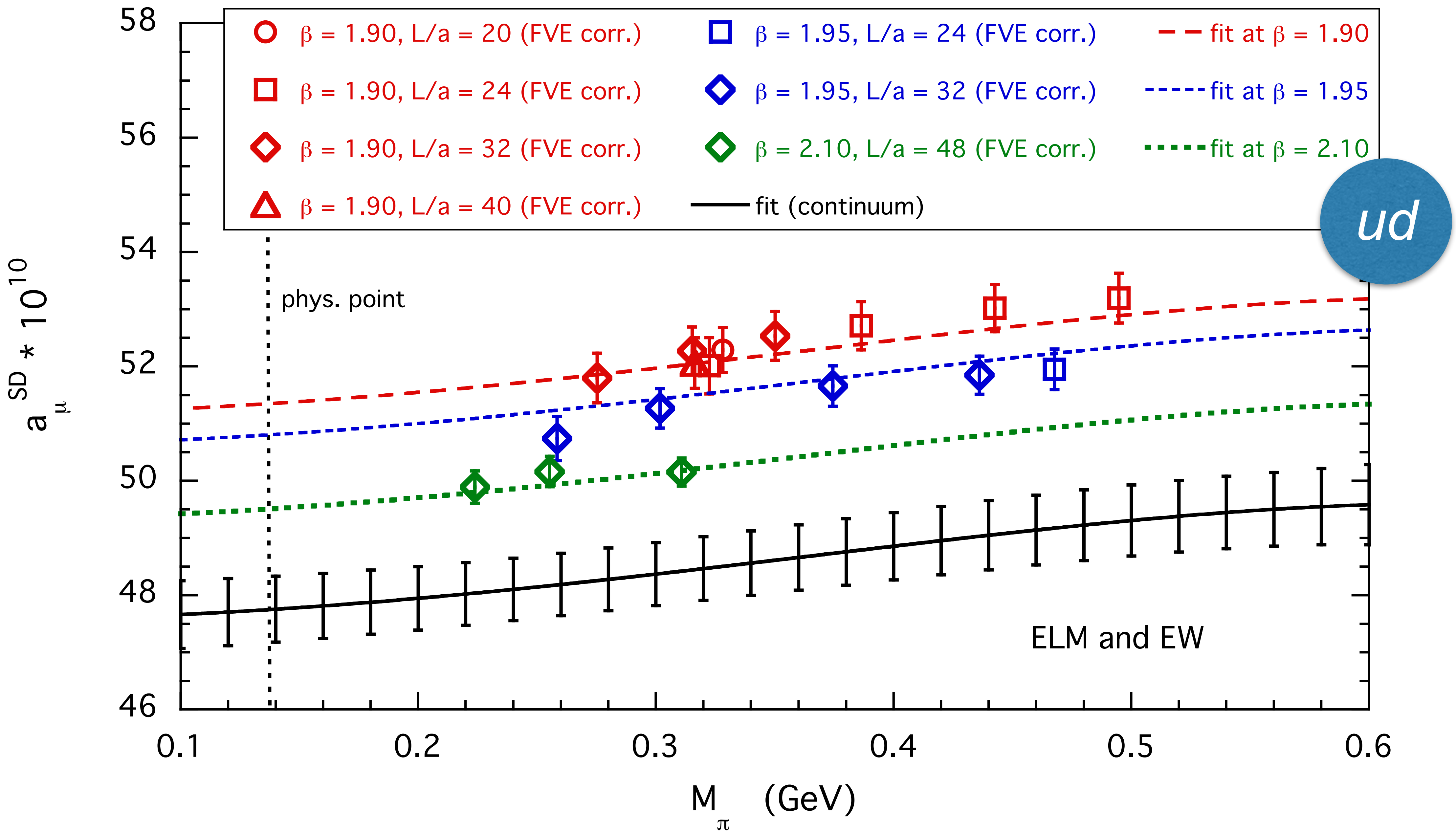
$$a_\mu^W = A_0\left[1 + A_{1\ell} M_\pi^2 \log\left(M_\pi^2\right) + A_1 M_\pi^2 + A_2 M_\pi^4 + Da^2\right]$$

$$\cdot\left[1 + F_0 M_\pi^2 e^{-M_\pi L}/(M_\pi L)^p\right]$$ optimal choice $p \simeq 2$

$a^4, a^2\alpha_s^n(1/a)$ terms

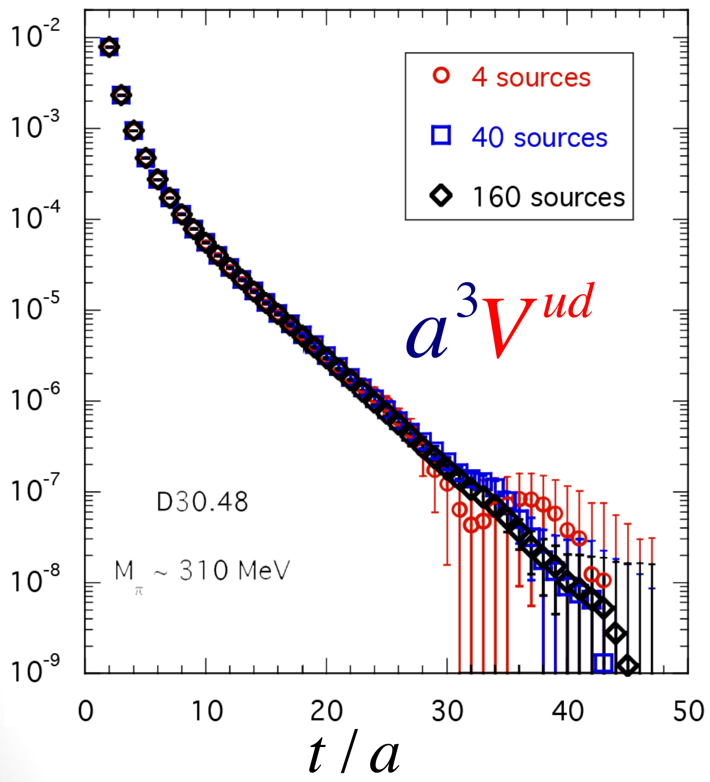
$$a_\mu^W(ud) = 202.2(2.0)_{stat}(0.4)_{chir}(1.5)_{disc}(0.7)_{FVE}[2.6] \cdot 10^{-10}$$

SD window



$$a_\mu^{SD}(ud) = 48.21(0.56)_{stat}(0.10)_{chir}(0.50)_{disc}(0.25)_{FVE}[0.80] \cdot 10^{-10}$$

LD window?



- FVEs are large
- Strong pion mass dependence

analytic representation for $V^{ud}(t)$

ArXiv:1808.00887

Correlator representation

$$\boxed{V^{ud}(t) = V_{dual}(t) + V_{\pi\pi}(t)}$$

low and intermediate time distances

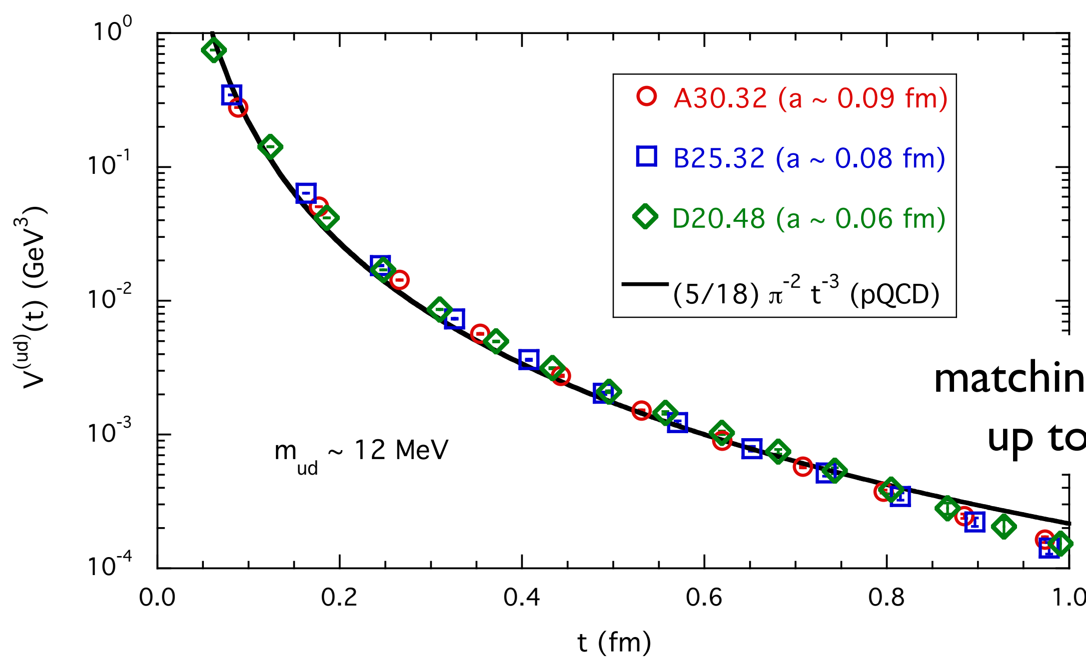
$$V_{dual}(t) \equiv \frac{1}{24\pi^2} \int_{s_{dual}}^{\infty} ds \sqrt{s}\; e^{-\sqrt{s}t} R^{pQCD}(s)$$

$$s_{dual} = \left(M_\rho + E_{dual}\right)^2 \qquad R_{dual} = 1 + O\left(\frac{m_{ud}^4}{s_{dual}^2}\right) + O(\alpha_s) + O(a^2)$$

$$V_{dual}(t) = \frac{5}{18\pi^2} \frac{R_{dual}}{t^3} e^{-(M_\rho + E_{dual})t} \left[1 + \left(M_\rho + E_{dual}\right)t + \frac{1}{2}\left(M_\rho + E_{dual}\right)^2 t^2\right]$$

quark-hadron duality *à la SVZ*

SVZ, 1979



matching with pQCD
up to t ~ 0.8 fm

long time distances

$$V_{\pi\pi}(t) = \sum_n \nu_n \left|A_n\right|^2 e^{-\omega_n t}$$

M. Lüscher
1991

$$\omega_n = 2\sqrt{M_\pi^2 + k_n^2}$$

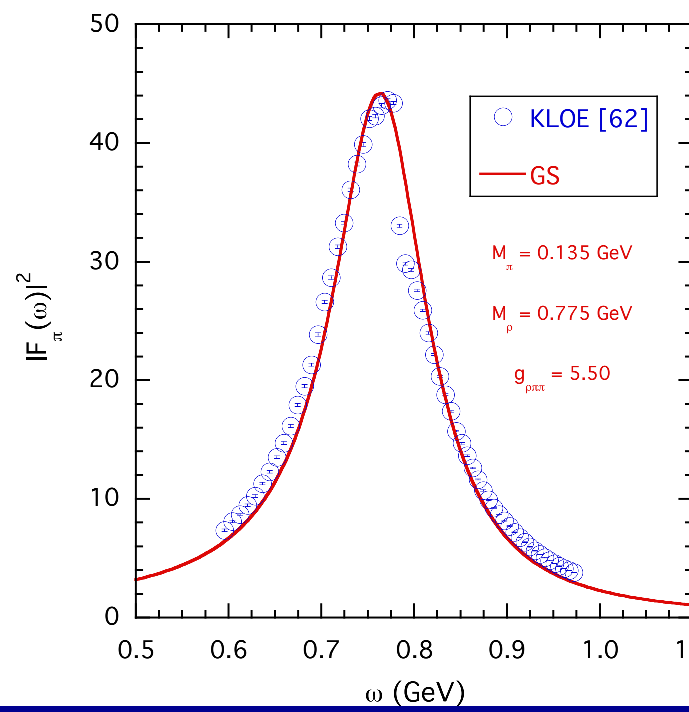
Lüscher condition

L. Lellouch and M. Lüscher, 2001
H.B. Meyer, 2011

$$\left|A_n\right|^2 \rightarrow \left|F_\pi(\omega_n)\right|^2$$

Gounaris-Sakurai parameterization

M_ρ, $g_{\rho\pi\pi}$ GS, 1968



LD window

$$a_\mu^{LD}(ud) = 382.5(10.5)_{stat}(5.2)_{syst}[11.7] \cdot 10^{-10}$$

analytic representation

analytic representation ⟷ data driven

analytic representation

$$a_\mu^{W}(ud) = 198.0(3.4)_{stat}(4.7)_{syst}[5.8] \cdot 10^{-10}$$

$$a_\mu^{SD}(ud) = 48.6(1.8)_{stat}(1.0)_{syst}[2.0] \cdot 10^{-10}$$

data driven

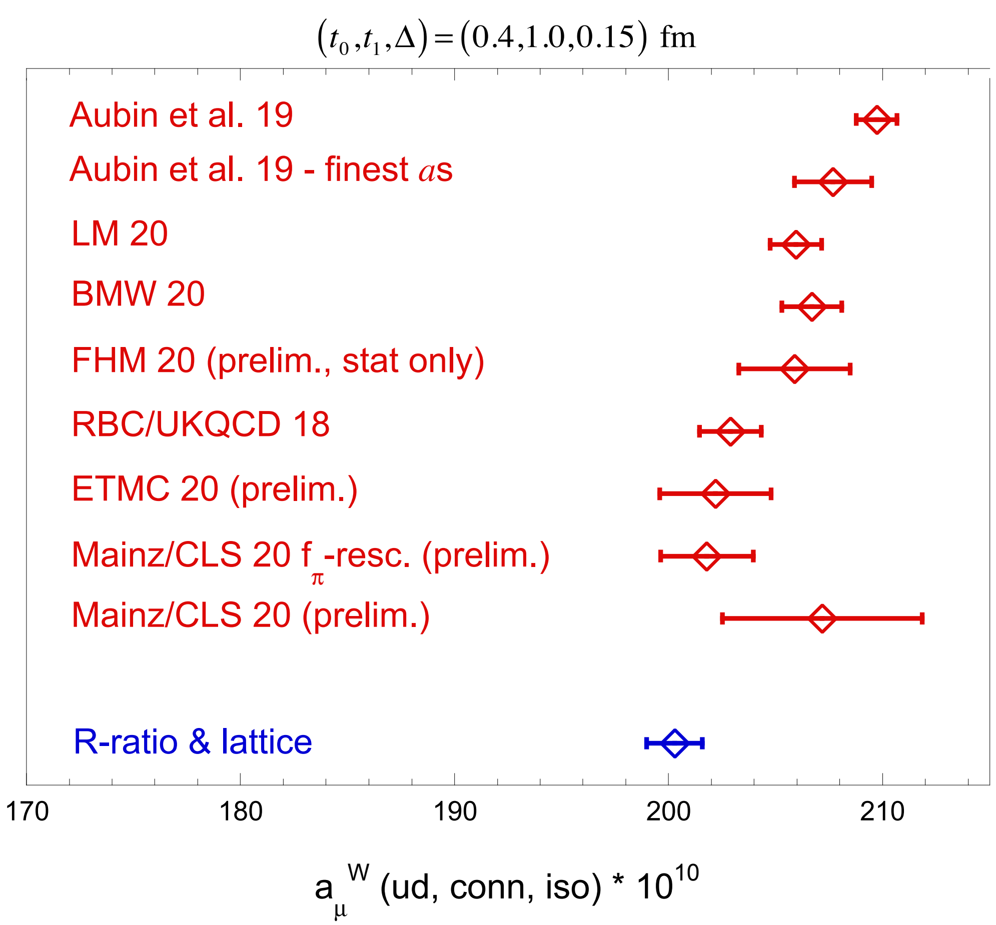
$$a_\mu^{W}(ud) = 202.2(2.0)_{stat}(1.7)_{syst}[2.6] \cdot 10^{-10}$$

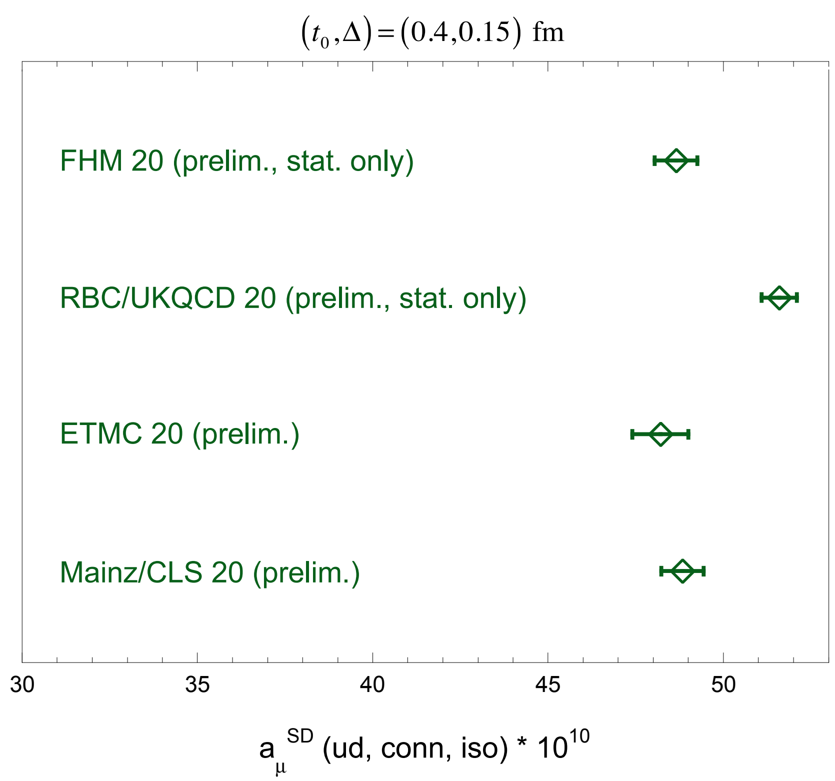
$$a_\mu^{SD}(ud) = 48.21(0.56)_{stat}(0.57)_{syst}[0.80] \cdot 10^{-10}$$

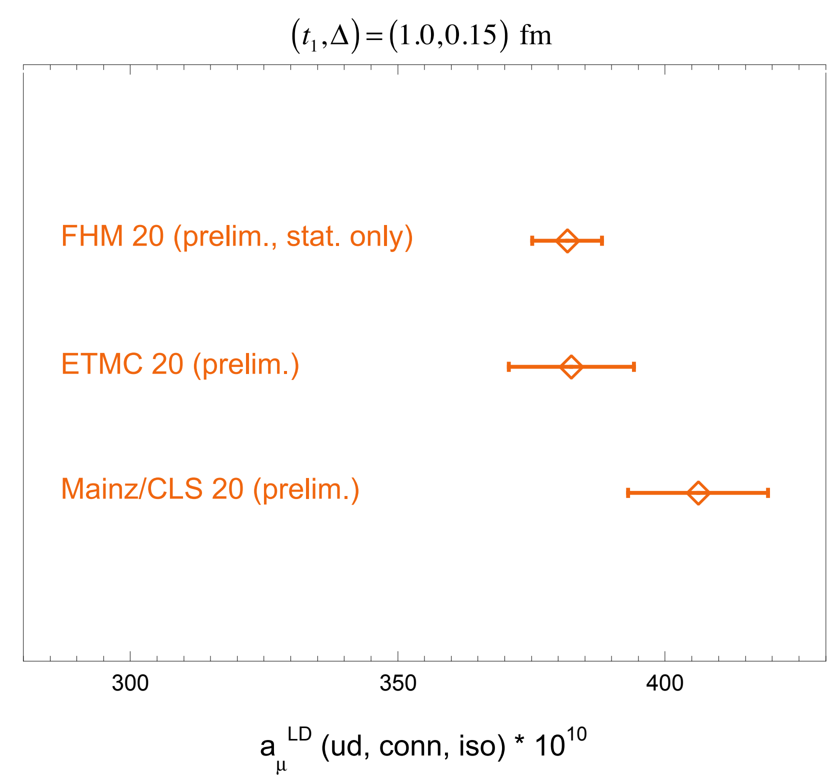
good consistency

Summary: *ud* contribution

f	$a_\mu^{SD}(f)\cdot 10^{10}$	$a_\mu^{W}(f)\cdot 10^{10}$	$a_\mu^{LD}(f)\cdot 10^{10}$
ud	48.2 (0.8)	202.2 (2.6)	382.5 (11.7)



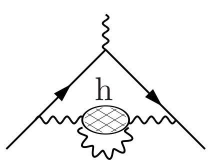




Intermediate window other contributions

preliminary

f	$a_\mu^W(f) \cdot 10^{10}$
ud	202.2 (2.6)
s	26.9 (1.0)
c	2.81 (0.11)
IB	0.7 (0.4)
disc	-0.9 (0.2)



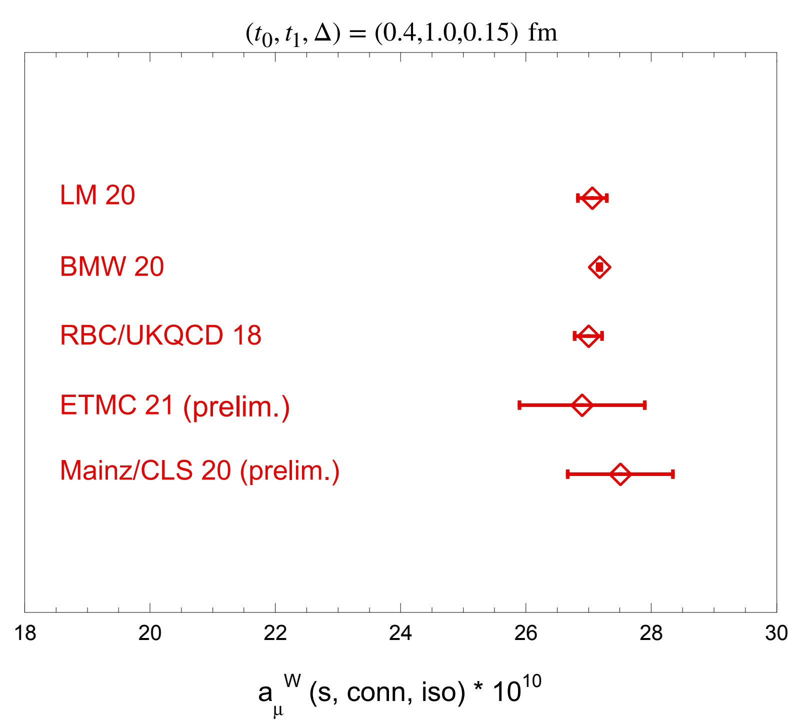
RBC/UKQCD 18 & BMW 20

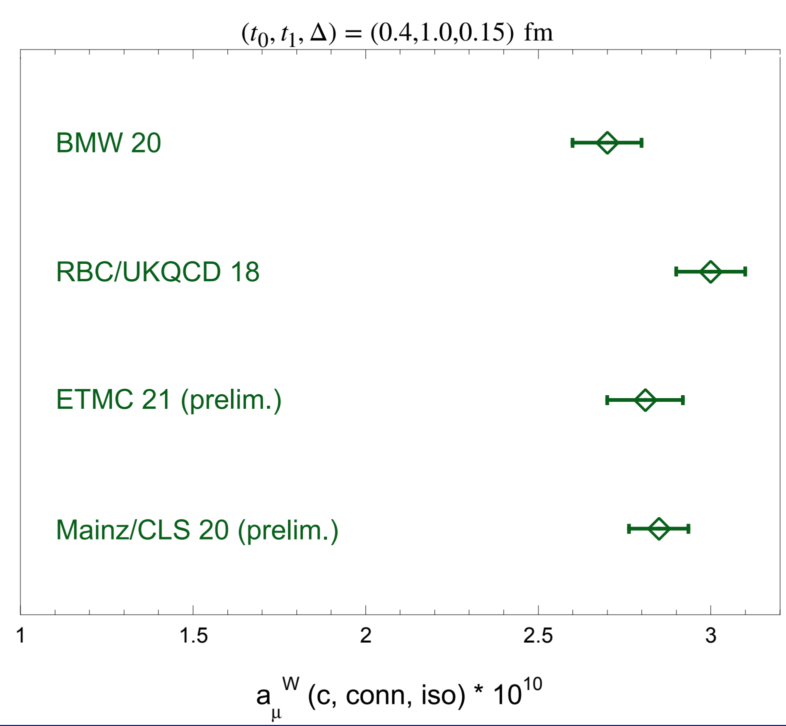
Lattice

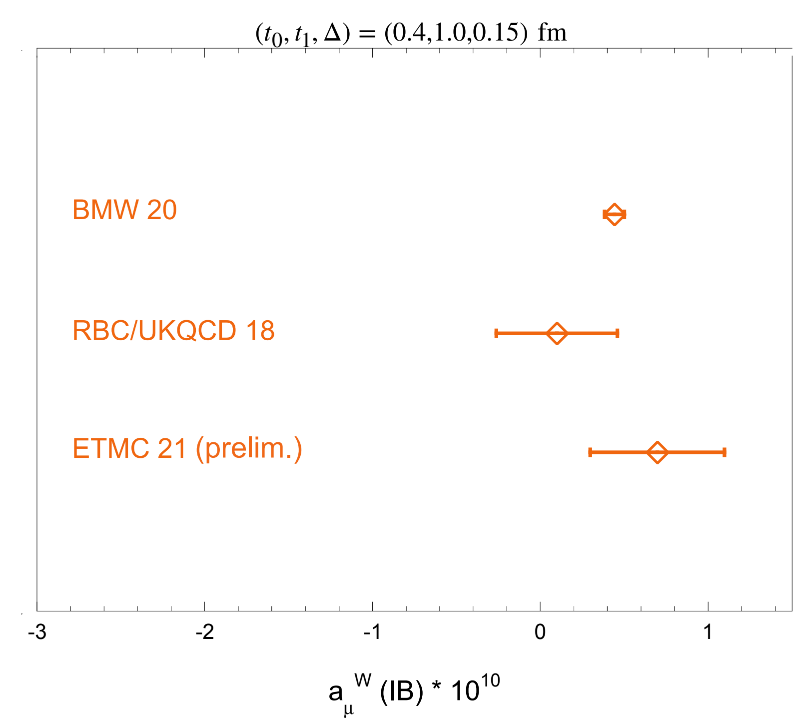
$$a_\mu^W = 231.7(2.8) \cdot 10^{-10}$$

$$a_\mu^W = 229.7(1.3) \cdot 10^{-10}$$

R-ratio







Conclusions

- The HVP contribution is currently one of the most important sources of the theoretical uncertainty to the muon (g-2) → LQCD

- We have performed a first-principles lattice QCD+QED calculation of $a_\ell^{\rm HVP}$. Our results agree with recent determinations based on dispersive analyses.

$$a_e^{\rm HVP} = 185.8(4.2) \cdot 10^{-14} \qquad a_\mu^{\rm HVP} = 692.1(16.3) \cdot 10^{-10} \qquad a_\tau^{\rm HVP} = 335.9(6.9) \cdot 10^{-8}$$

DG and S. Simula, 2019

- Window contributions are sharp benchmark quantities. Our result for the intermediate window is in agreement with the R-ratio prediction.

preliminary

$$a_\mu^W = 231.7(2.8) \cdot 10^{-10}$$

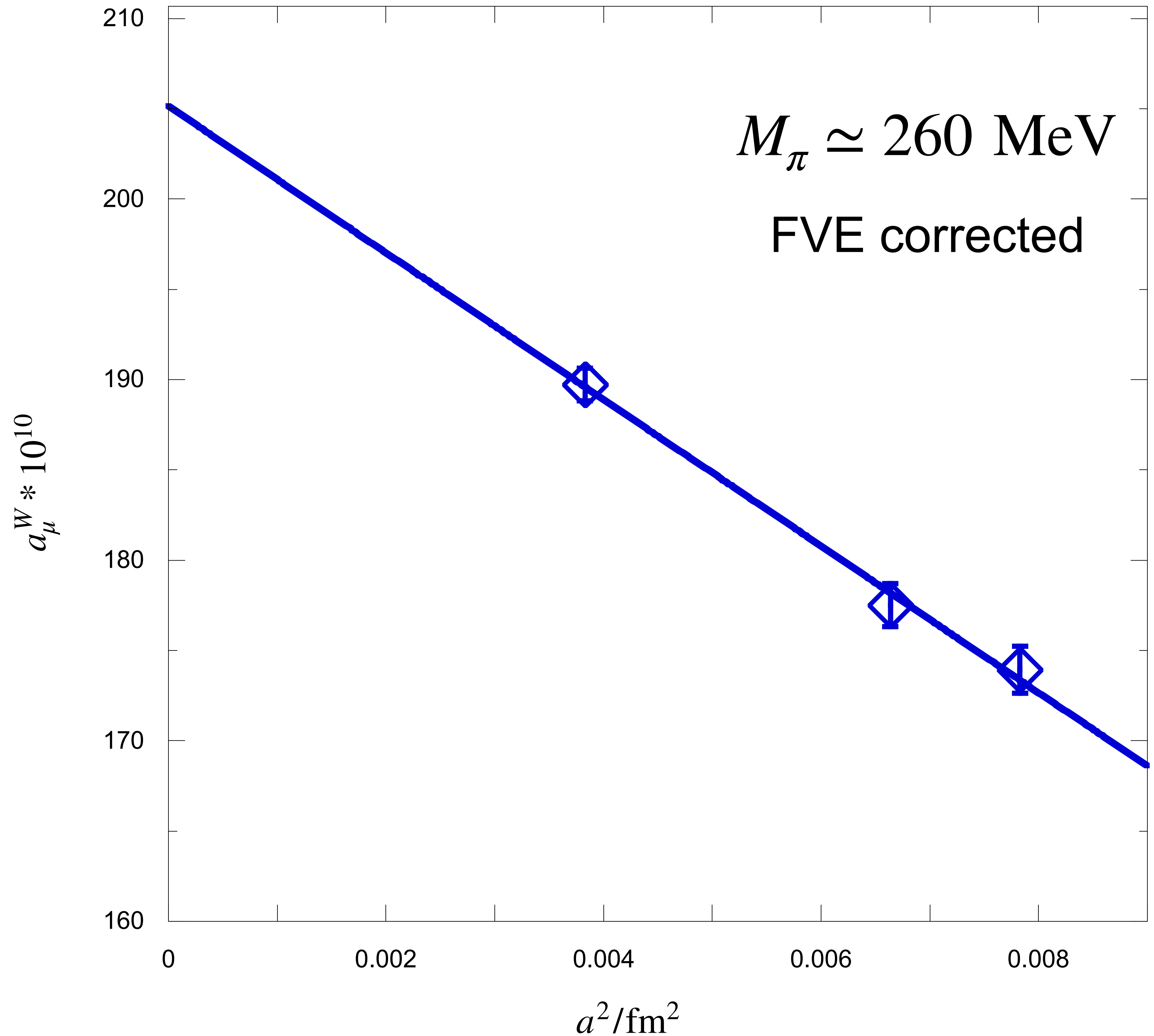
In progress…

- evaluation of the quark-disconnected terms and relaxation of the *qQED* approximation

- use of the new ETMC lattice setup @ the physical pion point

Backup slides

Intermediate window

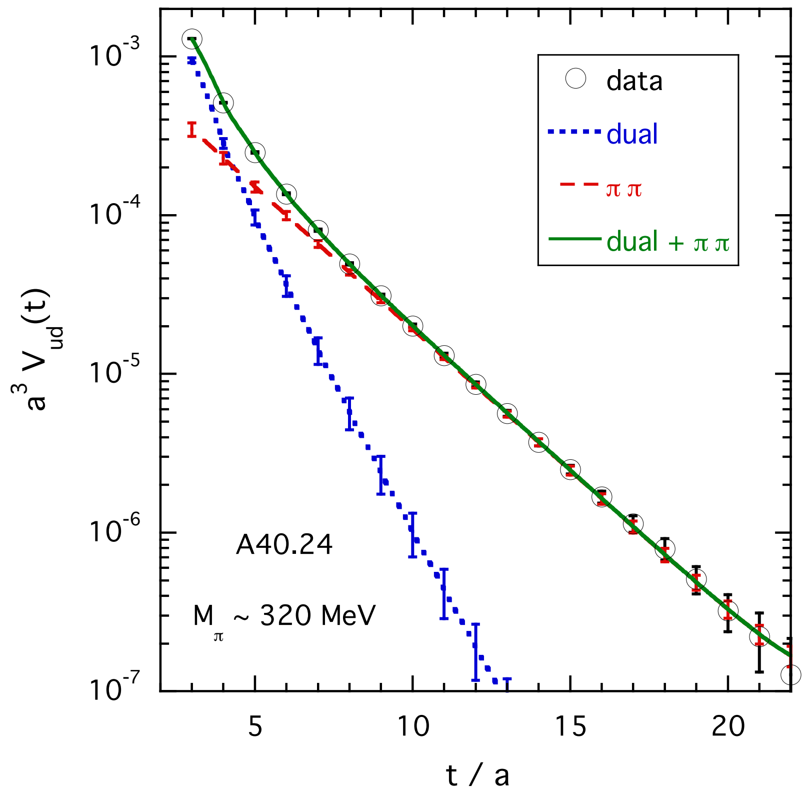


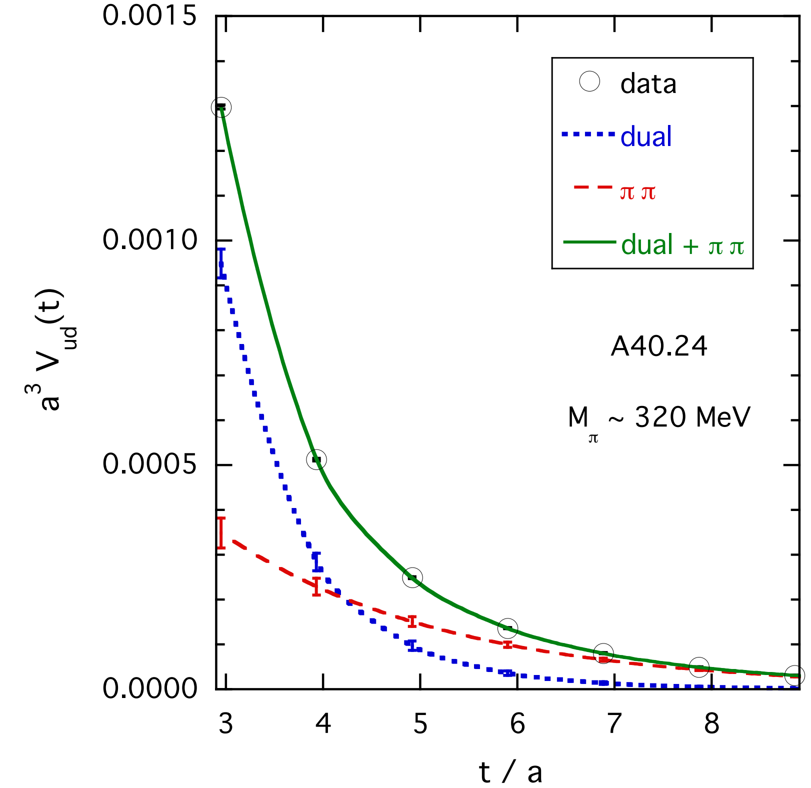
Subtraction of FVEs

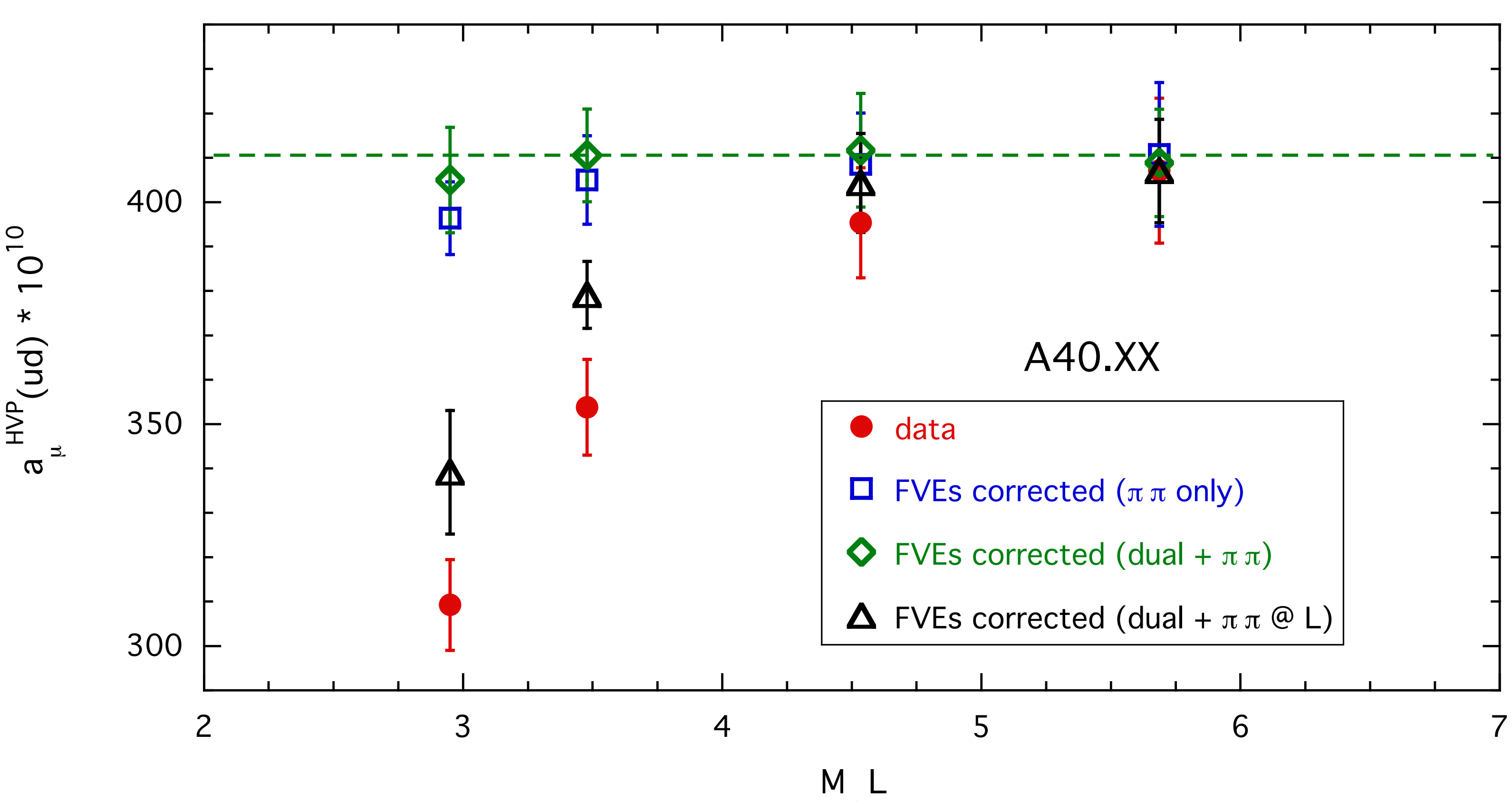
Accurate reproduction
for all the ETMC ensembles
$t \geq 0.2\ \mathrm{fm}$

$$R_{dual},\ \frac{E_{dual}}{M_\pi},\ g_{\rho\pi\pi},\ \frac{M_\rho}{M_\pi}$$

$\boxed{\pi\text{-}\pi:\ \text{4 energy levels}}$







$$a_\mu^{\mathrm{HVP}}(\infty) = a_\mu^{\mathrm{HVP}}(L) + \Delta_{\mathrm{FVE}} a_\mu^{\mathrm{HVP}}$$

infinite-volume limit

$$V_{dual}^\infty(t):\ R_{dual}^\infty\ \ M_\rho^\infty\ \ E_{dual}^\infty$$

$$V_{\pi\pi}^\infty(t) = \frac{1}{48\pi^2}\int_{2M_\pi^\infty}^\infty d\omega\ \omega^2\left[1-\frac{(2M_\pi^\infty)^2}{\omega^2}\right]^{3/2}\left|F_\pi^\infty(\omega)\right|^2 e^{-\omega t}$$

H. B. Meyer, 2011