HVP contribution to Running Coupling and Electroweak Precision Science

Kohtaroh Miura
GSI, Helmholtz-Institut Mainz

Lattice 2021 at MIT, July 27, 2021

Mainz $\Delta\alpha_{\rm had}(-Q^2)$ Working Group

M. Cè, A. Géradin, G. Hippel, H.B. Meyer, K. Miura, D. Mohler, K. Ottnad, T. San José, A. Risch, J. Wilhelm, and H. Wittig.

Reference: M. Cè et.al. PoS**Lattice2019** (2020), arXiv:1910.09525.
$\Delta\alpha_{\rm had}(-Q^2)$ and $\Delta^{\rm had}\sin^2\theta_{\rm W}(-Q^2)$.

Motivation

QED Running Coupling

- $\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}$, $\alpha = \frac{1}{137.03\cdots}$, $\Delta\alpha(-Q^2) \ni \Delta\alpha^{(5)}_{\rm had}(-Q^2) = 4\pi\alpha\hat{\Pi}(Q^2)$.
- Soft-Tension Problem in Pheno $\Delta\alpha^{(5)}_{\rm had}(M_z^2)$:
  - 0.02761(11) [Keshavarzi et al. PRD2020].
  - 0.02760(10) [Davier et al. EPJC20].
  - 0.02776(16) [Jegerlehner-19].
  - 0.02722(41) EW-fit, $M_H = 94^{+20}_{-18} GeV$ [Keshavarzi et.al. PRD2020]
  - 0.02716(33) EW-fit, $M_H = fit - prm$ [Malaescu et al. 2008.08107].
  - 0.02716(39) EW-fit, $M_H = 90^{+21}_{-18} GeV$ [Gfitter. EPCJ18].
- [Crivellin et al. PRL2020]: If $a_\mu^{\text{LO-HVP}}$ gets closer to NoNewPhys, the tension increases at EW-Global fit. c.f. [M.Passera et al. PRD08]
- [BMW-2020, 2002.12347]: The tension is not necessarily suggested by naive looking at $\Delta\alpha_{\rm had}(-10GeV^2) - \Delta\alpha_{\rm had}(-1GeV^2)$.

Motivation

QED Running Coupling

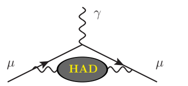
- $\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}$, $\alpha = \frac{1}{137.03\cdots}$, $\Delta\alpha(-Q^2) \ni \Delta\alpha^{(5)}_{\text{had}}(-Q^2) = 4\pi\alpha\hat{\Pi}(Q^2)$.
- Soft-Tension Problem in Pheno $\Delta\alpha^{(5)}_{\text{had}}(M_z^2)$:
  - 0.02761(11) [Keshavarzi et al. PRD2020].
  - 0.02760(10) [Davier et al. EPJC20].
  - 0.02776(16) [Jegerlehner-19].
  - 0.02722(41) EW-fit, $M_H = 94^{+20}_{-18} GeV$ [Keshavarzi et.al. PRD2020]
  - 0.02716(33) EW-fit, $M_H = fit - prm$ [Malaescu et al. 2008.08107].
  - 0.02716(39) EW-fit, $M_H = 90^{+21}_{-18} GeV$ [Gfitter. EPCJ18].
- [Crivellin et al. PRL2020]: If $a_\mu^{\text{LO-HVP}}$ gets closer to NoNewPhys, the tension increases at EW-Global fit. c.f. [M.Passera et al. PRD08]
- [BMW-2020, 2002.12347]: The tension is not necessarily suggested by naive looking at $\Delta\alpha_{\text{had}}(-10GeV^2) - \Delta\alpha_{\text{had}}(-1GeV^2)$.

This Talk

$a_\mu^{\text{LO-HVP}}$: Tension to NoNewPhys.?

$\Updownarrow$

$\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$: (Soft) Tension in EW scale?



Mainz/CLS Efforts for HVP

- $a_\mu^{\text{LO-HVP}}$: Talk by H. Wittig.
- $\Delta\alpha_{\rm had}(-Q^2)$: Previous Talk by T. S. José.
- $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$: This Talk: first lattice study for the Z-pole value.

From LQCD to EW Precision Science

- Euclidean Split Method [Jegerlehner hep-ph/0807.4206]:

$$\begin{aligned}\Delta\alpha^{(5)}_{\rm had}(M_z^2) &= \Delta\alpha^{(5)}_{\rm had}(-Q_0^2) \longleftarrow \text{LQCD/R-ratio} \\ &+ \left[\Delta\alpha^{(5)}_{\rm had}(-M_z^2) - \Delta\alpha^{(5)}_{\rm had}(-Q_0^2)\right] \longleftarrow \text{pQCD}'\text{/R-ratio} \\ &+ \left[\Delta\alpha^{(5)}_{\rm had}(M_z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_z^2)\right] \longleftarrow 0.000045(\text{pQCD})\ . \quad (1)\end{aligned}$$

- Q_0^2: Threshold Energy. Typically $\sim 5\ GeV^2$.

- $\Delta\alpha^{(5)}_{\rm had}(-Q_0^2)$: $\Delta\alpha_{\rm had}(-Q_0^2)|_{2+1}$(Prev. Talk) combined with heavy quarks contributions (First Topic of This Talk).

Charm Sea-Quarks

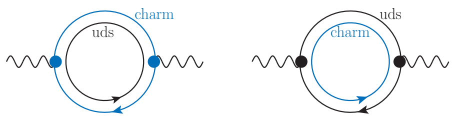
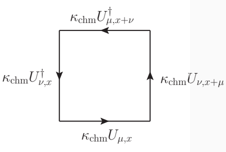


Figure: D-Meson contributions in HVP.

- Charm valence-quark contributions (e.g. Left Fig.) are measured with $N_f = 2+1$ ensembles and already included in Mainz/CLS $\Delta\alpha_{\rm had}(-Q_0^2)$.
- Charm Sea-Quark contributions (e.g. Right Fig.) are missing and investigated by Hopping Parameter Expansion (HPE).

Charm Sea-Quarks in HPE



- The HPE leading-order is a shift of lattice coupling as $c_0\beta_{\rm L} \to c_0\beta_{\rm L} + \delta\beta_{\rm HPE}$, where

$$\delta\beta_{\rm HPE} = 24N(1+2r^2-r^4)\kappa^4_{\rm chm} \xrightarrow[\kappa_{\rm chm}\sim 0.12]{N=1,r=1} \mathcal{O}(0.01)\,. \quad (2)$$

- 4-flavor ($uds + c$) system can be approximated with 3-flavor (uds) system with shifted coupling,

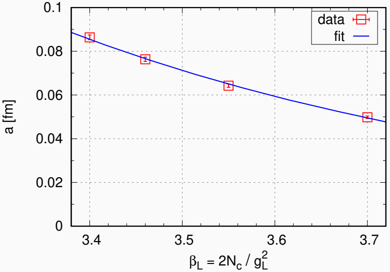
$$\left\langle O(c_0\beta_{\rm L})\right\rangle_4 = \left\langle O(c_0\beta_{\rm L}+\delta\beta_{\rm HPE})\right\rangle_3 + \mathcal{O}(\kappa^6_{\rm chm})\,. \quad (3)$$

- Expanding in terms of $\delta\beta_{\rm HPE}$, the charm sea-quark effects are evaluated as

$$\delta_{\rm chm}O \equiv \left\langle O(c_0\beta_{\rm L})\right\rangle_4 - \left\langle O(c_0\beta_{\rm L})\right\rangle_3 \quad (4)$$

$$= \delta\beta_{\rm HPE}\underbrace{\left[\frac{1}{a(\beta_{\rm L})}\frac{\partial a(\beta_{\rm L})}{\partial\beta_{\rm L}}\right]}_{\text{2-loop scaling}}\underbrace{\left[a(\beta_{\rm L})\frac{\partial\langle O\rangle_3}{\partial a(\beta_{\rm L})}\right]_{a\to a_{\rm simu}}}_{\text{utilize continuum extrap. results}}\,. \quad (5)$$

Asymptotic Scaling



$$c_{\rm ren} = 1.323(36)\ ,$$

$$\Lambda = 8.89(1.56)\ \mathrm{MeV}\ ,$$

$$\chi^2/\mathrm{dof} = 2.2/2\ .$$

- Assume a 2-loop asymptotic scaling by approximating uds-quarks to be massless,

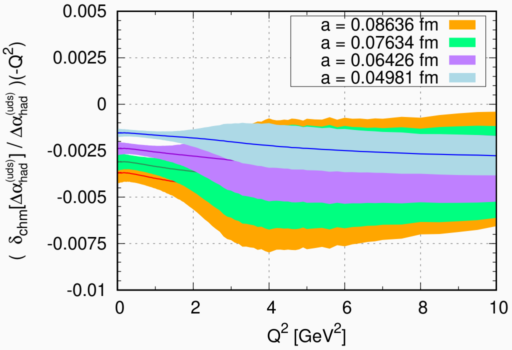
$$a(\beta_{\rm L})\ [\mathrm{fm}] = \frac{\hbar c[\mathrm{MeV\ fm}]}{\Lambda[\mathrm{MeV}]}\left(\frac{c_{\rm ren}\beta_{\rm L}}{2N_c b_0}\right)^{b_1/(2b_0^2)} \exp\left[-\frac{c_{\rm ren}\beta_{\rm L}}{4N_c b_0}\right]\ , \quad (6)$$

$$b_0(N_c, N_f)|_{N_c=N_f=3} \simeq 0.0570\ , \quad b_1(N_c, N_f)|_{N_c=N_f=3} \simeq 0.0026\ . \quad (7)$$

- Here, $c_{\rm ren}$ and Λ are treated as fit parameters. Then, we can analytically evaluate the derivative as:

$$\frac{1}{a(\beta_{\rm L})}\frac{\partial a(\beta_{\rm L})}{\partial \beta_{\rm L}} = \left(\frac{b_1}{2b_0^2\beta_{\rm L}} - \frac{c_{\rm ren}}{4N_c b_0}\right)\ . \quad (8)$$

Charm Sea-Quarks in $\Delta\alpha^{\rm uds}_{\rm had}(-Q_0^2)$



- The charm sea-quark effects $\delta_{chm}[\Delta\alpha^{\rm uds}_{\rm had}(-Q_0^2)]$ are a few permil contributions.
- The effects become smaller at finer lattice ensembles. We adopt the finest lattice result as our estimate at the continuum limit.

$\Delta\alpha^{(5)}_{\rm had}(-Q_0^2)$ Summary

Error Type	%	Comments
statistical	0.7	simulation based
chiral/continuum extrap.	0.1	simulation based
scale setting	0.7	simulation based
isospion breaking	0.4	simulation based, talk by A. Risch
charm sea-quark	0.2 - 0.3	hopping param. expn.
charm disconnected	~ 0.01	1% of uds-disc. c.f. [BMW-PRL18]
bottom	0.2	w. time-moments by [HPQCD-PRD2015]

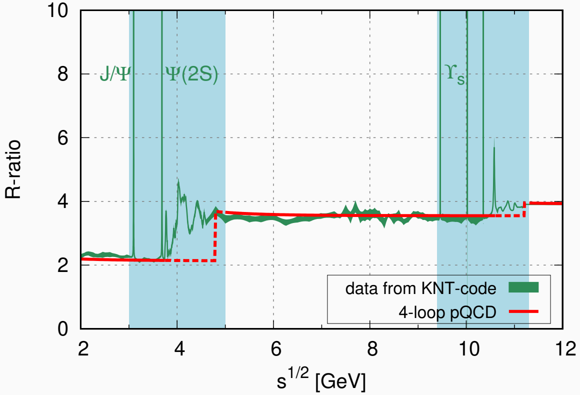
Table: Error Budget in $\Delta\alpha^{(5)}_{\rm had}(-5GeV^2)$

- For the central value of $\Delta\alpha^{(5)}_{\rm had}(-Q_0^2)$, we adopt Mainz/CLS $\Delta\alpha_{\rm had}(-Q_0^2)$ obtained from $N_f = 2+1$ ensembles.
- Charm sea-quark effects are treated as an additional uncertainty on the central value.
- Similarly, charm disconnected, bottom quark effects are considered as additional uncertainty.

Next Step: pQCD′

$$\begin{aligned}\Delta\alpha^{(5)}_{\rm had}(M_z^2) &= \Delta\alpha^{(5)}_{\rm had}(-Q_0^2) \longleftarrow \text{LQCD: DONE!}\\ &+\left[\Delta\alpha^{(5)}_{\rm had}(-M_z^2) - \Delta\alpha^{(5)}_{\rm had}(-Q_0^2)\right] \longleftarrow \text{pQCD}' : \text{Next Step}\\ &+\left[\Delta\alpha^{(5)}_{\rm had}(M_z^2) - \Delta\alpha^{(5)}_{\rm had}(-M_z^2)\right] \longleftarrow 0.000045(\text{pQCD})\,. \quad (9)\end{aligned}$$

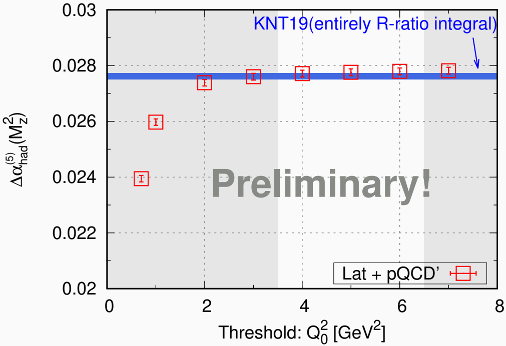
pQCD′= pQCD + J/Ψ + Υ



$$\left[\Delta\alpha^{(5)}_{\rm had}(-M_Z^2) - \Delta\alpha^{(5)}_{\rm had}(-Q_0^2)\right] \simeq \frac{(M_Z^2-Q_0^2)\alpha_0}{3\pi}\int_{4M_\pi^2}^{\infty} ds \frac{R_{4loop}(s)+Reso(J/\Psi,\Upsilon)(s)}{(s+M_Z^2)(s+Q_0^2)} \,.$$

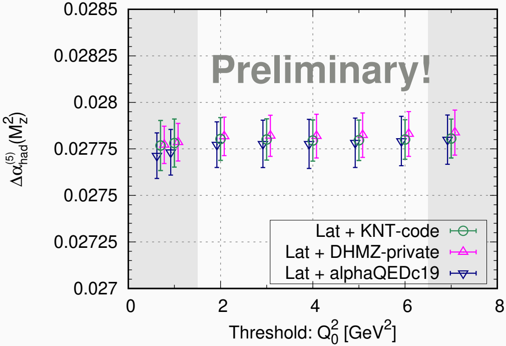
pQCD: rhad-1.0.1, Resonances: KNT-Code [KNT-PRD18].

$\Delta\alpha_{\rm had}^{(5)}(M_Z^2)$ Lattice + pQCD′



- **Preliminary:** $\Delta\alpha_{\rm had}^{(5)}(M_Z^2) = 0.0278[0.4\%]_{Q_0^2}[1\%]_{\rm tot}$.
- Central value is determined at $Q_0^2 = 5\ GeV^2$.
  Total error = max. deviation in $Q_0^2 = 4 - 6\ GeV^2$.

$\Delta\alpha_{had}^{(5)}(M_Z^2)$ Lattice + Pheno



- **Preliminary:** pQCD′is replaced with direct R-ratio integrals. The result is unchanged in this precision, $\Delta\alpha_{had}^{(5)}(M_Z^2) = 0.0278[0.4\%]_{Q_0^2}[1\%]_{tot}$.
- Central value is the average of three data at $Q_0^2 = 5\ GeV^2$.
  Total error = max. deviation over 15 data in $Q_0^2 = 2 - 6\ GeV^2$.

Comparison of $\Delta\alpha^{(5)}_{had}(M_Z^2)$

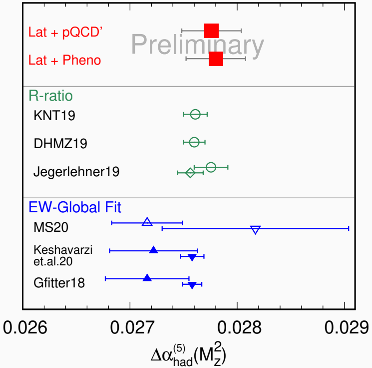


Fig:
○: R-ratio integral over entire energy range.
◊: Adler function in Euclidean split method.
△ ▲: EW-Global fit with M_{higgs} = fit param.
▽▼: EW-Global fit with $M_{higgs} = 125\ GeV^2$.
In ▲▼, the R-ratio results are used as an input while not in △ ▽.

- Our results (red-squares, preliminary) are consistent with the dispersive ones (green-circles/diamond) and the EW-Global fits (blue-triangles).
- We have assigned 1% error as a preliminary estimate. We will further elaborate the error reduction of the high energy part.

Summary

- We have investigated $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ by using our $N_f = 2 + 1$ results for $\Delta\alpha_{\rm had}(-Q_0^2)$ at low-energy Q_0^2 in the Euclidean split method. The higher energy part was evaluated by the pQCD′or the direct R-ratio integral.
- We have developed the HPE-based method for charm sea-quark contributions, giving a few permil effects over total.
- Our estimates for $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ with preliminary conservative error assignments are consistent with known results by pure dispersive approach and EW Global fits.
- To obtain reasonablly reduced errors for high energy contributions, we need further discussion with phenomenologists.
- It is interesting to perform EW-Global fits by using our $\Delta\alpha^{(5)}_{\rm had}(M_Z^2)$ as an input or prior.

Summary

- We have investigated $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ by using our $N_f = 2+1$ results for $\Delta\alpha_{\text{had}}(-Q_0^2)$ at low-energy Q_0^2 in the Euclidean split method. The higher energy part was evaluated by the pQCD′or the direct R-ratio integral.
- We have developed the HPE-based method for charm sea-quark contributions, giving a few permil effects over total.
- Our estimates for $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ with preliminary conservative error assignments are consistent with known results by pure dispersive approach and EW Global fits.
- To obtain reasonablly reduced errors for high energy contributions, we need further discussion with phenomenologists.
- It is interesting to perform EW-Global fits by using our $\Delta\alpha^{(5)}_{\text{had}}(M_Z^2)$ as an input or prior.