Neutron electric dipole moment using lattice QCD simulations at the physical point *[Alexandrou et al., Phys.Rev.D 103, 054501 (2021)]*

STIMULATE
European Joint Doctorates

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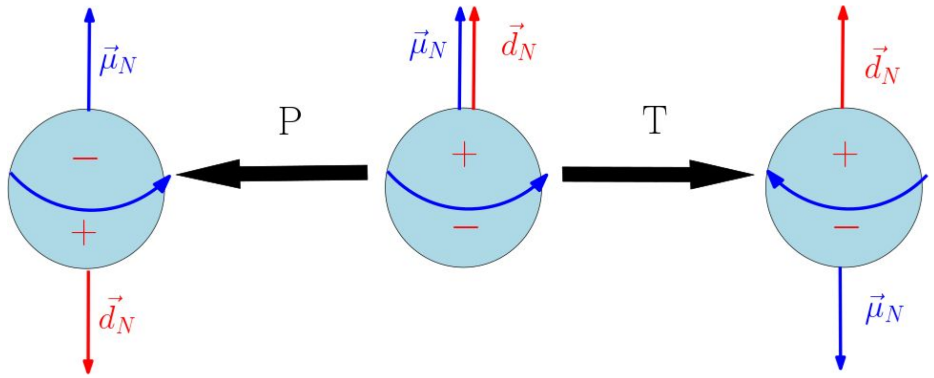
LATTICE 21
JULY 26-30 2021, ZOOM/GATHER@MIT

Introduction



MOTIVATION

The existence of a permanent Neutron Electric Dipole Moment would break CP symmetry, opening to new sources of CP violation within the SM and beyond.

EXPERIMENTAL EVIDENCES

Despite the great effort in the last 70 years

no finite nEDM signal has been measured up to now.

BEST UPPER BOUND

$$|d_n| < 1.8 \times 10^{-13} \text{ e fm}$$

[*Abel et al., Phys. Rev. Lett. 124, 081803 (2020)*]

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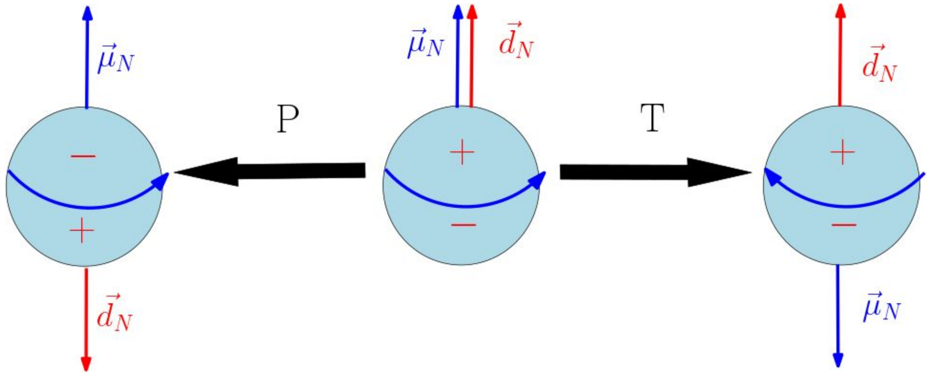

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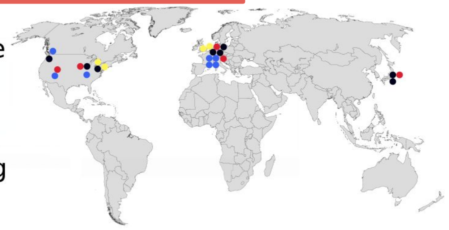
New experiments are coming and plan to further improve current upper bound by 2 order of magnitude

Neutrons: (~ 200 ppl.)
- Beam EDM @ Bern
- LANL nEDM @ LANL
- nEDM @ PSI
- nEDM @ SNS
- PanEDM @ ILL
- PNPI/FTI/ILL @ ILL
- TUCAN @ TRIUMF

Storage rings: (~ 400 ppl.)
- CPEDM/JEDI
- muEDM @ PSI
- g-2 @ FNAL
- g-2 @ JPARC

Atoms: (~ 60 ppl.)
- Cs @ Penn State
- Fr @ Riken
- Hg @ Bonn
- Hg @ Seattle
- Ra @ Argonne
- Xe @ Heidelberg
- Xe @ PTB
- Xe @ Riken



Molecules: (~ 55 ppl.)
- BaF (EDM^3) @ Toronto
- BaF (NLeEDM) @ Groningen/Nikhef
- HfF+ @ JILA
- ThO (ACME) @ Yale
- YBF @ Imperial

source: https://www.psi.ch/en/nedm/edms-world-wide

Experiment	Features	Status
PSI	spallation so-D_2, magnetic fields	analysis/upgrading
PanEDM (ILL/Munich)	reactor He-II, 1st MSR	commissioning
ILL/PNPI/Gatchina	dual cell, 2nd best nEDM meast	upgrading source
LANL	spallation so-D_2 UCN source	2021-
TUCAN (Japan/Canada)	spallation He-II, MSR	upgrading, 2022-
SNS	fully cryogenic source/experiment	2022-
ILL/ESS n-beam	intense pulsed neutron beam	R&D, 2025-
J-PARC crystal	high E in crystal	R&D

source: [*J.W. Martin, J. Phys.: Conf. Ser. 1643 012002 (2020)*]

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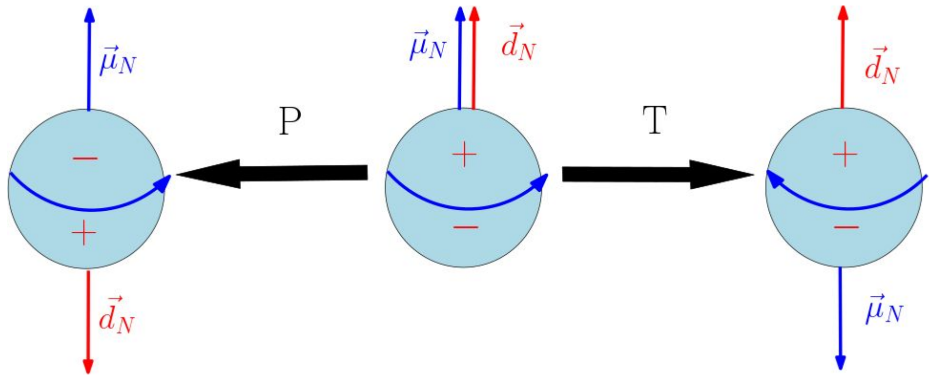

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STANDARD MODEL PREDICTION

- **Electroweak sector:** phase in quark mass from Yukawa couplings with Higgs field;
- **Strong sector:** allowed dimension-four CP-breaking term

$$S_\theta^{\mathcal{E}ucl} = i\theta \underbrace{\boxed{\frac{1}{32\pi^2} \int F^a_{\mu\nu} \tilde{F}^a_{\mu\nu}}}_{\text{Topological charge } \mathcal{Q}}$$

EW CONTRIBUTION

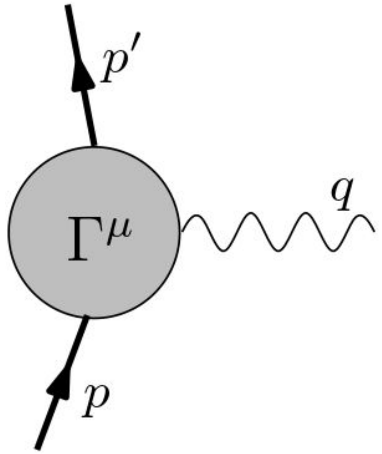
$$|d_n| \sim \mathcal{O}(10^{-19}) \text{ e fm}$$

$$d_n = \boxed{const} \cdot \theta$$

- Translating experimental bound into constraints over couplings
- Can be computed via lattice QCD

θ-nEDM via lattice QCD

Computation of the θ-induced CP-odd electromagnetic form factor [*Shintani et al., Phys. Rev. D 72, 014504 (2005)*]



MATRIX ELEMENT

$$\langle \vec{p}_f, s_f | \mathcal{J}_\mu^{\rm em}(\vec{q}) | \vec{p}_i, s_i \rangle_\theta = \bar{u}_n(\vec{p}_f, s_f)\, \Gamma_\mu(\vec{q})\, u_n(\vec{p}_i, s_i)$$

$$\Gamma_\mu(q) = F_1(q^2)\,\gamma_\mu + \left(F_2(q^2) + i\gamma_5 F_3^\theta(q^2)\right) \frac{\sigma_{\mu\nu} q_\nu}{2m_n}$$

In the limit of zero momentum transfer, we obtain the nEDM from $\boxed{d_n = \lim_{q\to 0} \frac{F_3^\theta(q^2)}{2m_n}}$

LATTICE CORRELATION FUNCTIONS

$$\langle J_N(\vec{p}_f, t_f) | \mathcal{J}_\mu^{\rm em}(\vec{q}, t_{\rm ins}) | \bar{J}_N(\vec{p}_i, t_i) \rangle_\theta \propto e^{i\alpha_n\gamma_5}\, \Lambda_{\frac{1}{2}}(\vec{p}_f)\, \Gamma_\mu(\vec{q})\, \Lambda_{\frac{1}{2}}(\vec{p}_i) e^{i\alpha_n\gamma_5}$$

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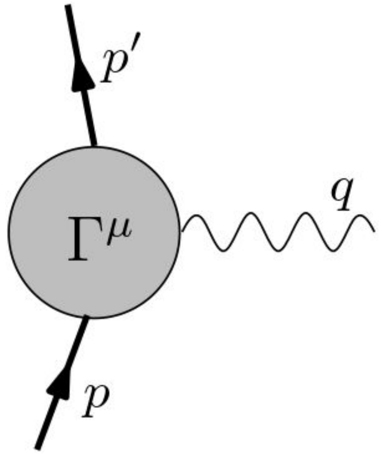

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EXPANSION IN POWERS OF θ

$$\langle \dots \rangle_\theta = \int \dots\; e^{-S + i\theta\mathcal{Q}} \approx \int \dots\; e^{-S} + i\theta \int \dots\; \mathcal{Q} e^{-S}$$

$$\alpha_n^\theta \approx \alpha_n^{(1)}\theta\,,\;\; F_3^\theta \approx \theta F_3^{(1)}$$

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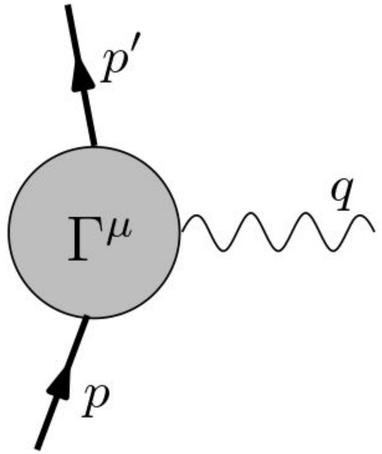

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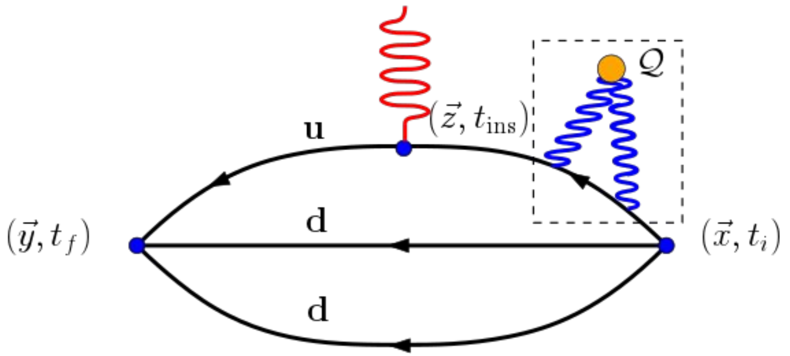
F_3

$$\langle J_N(\vec{y}, t_f) | J^\mu(\vec{z}, t_{\rm ins})(\mathcal{Q}) | \bar{J}_N(\vec{x}, t_i) \rangle$$

connected diagram only!!



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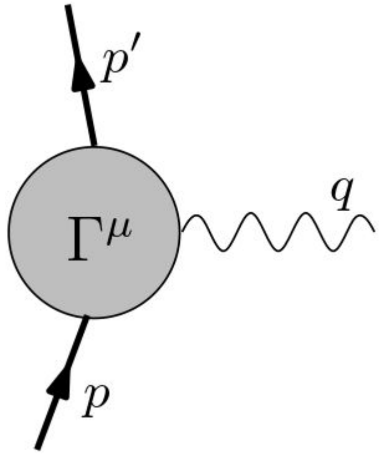

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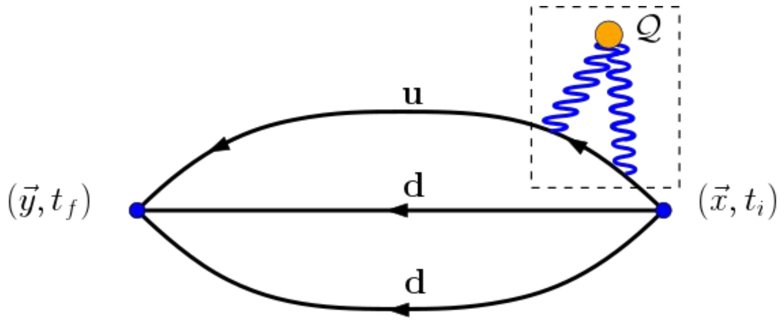
α_n

$$\langle J_N(\vec{y}, t_f) \bar{J}_N(\vec{x}, t_i)(\mathcal{Q}) \rangle$$



Lattice setup

One gauge ensemble of $N_f = 2+1+1$ twisted mass fermions with clover improvement, at physical pion mass.

GAUGE ENSEMBLE

$\beta = 1.778$	$c_{SW} = 1.69$	$a = 0.0801(4)$
$64^3 \times 128$	$m_\pi = 139(1)$ MeV	$m_\pi L = 3.62$
$L = 5.13$ fm	$m_N = 940(2)$ MeV	

STATISTICS EMPLOYED

	Correlation functions	$N_{\rm src}$	$N_{\rm cnfs}$	$N_{\rm tot}$
α_N	G_{2pt}	200	750	150000
F_3	G_{2pt},G_{3pt}	54	750	40500

LATTICE CORRELATION FUNCTIONS

$$G_{2pt}(\vec{p}, t_f, \Gamma_0) = \sum_{\vec{x}} e^{-i\vec{p}\vec{x}} \, \mathrm{Tr}\left[\left\langle J_N(\vec{x}, t_f)\bar{J}_N(\vec{0}, 0)\right\rangle \Gamma_0\right]$$

$$G_{2pt,\mathcal{Q}}(\vec{p}, t_f, \Gamma_5) = \sum_{\vec{x}} e^{-i\vec{p}\vec{x}} \, \mathrm{Tr}\left[\left\langle J_N(\vec{x}, t_f)\bar{J}_N(\vec{0}, 0)\mathcal{Q}\right\rangle \Gamma_5\right]$$

$$G^{\mu}_{3pt}(\vec{p}, \vec{q}, t_f, t_{\rm ins}, \Gamma_k) = \sum_{\vec{y},\vec{z}} e^{-i\vec{p}\vec{y}} e^{+i\vec{q}\vec{z}} \mathrm{Tr}\left[\left\langle J_N(\vec{y}, t_f)|J^{\mu}(\vec{z}, t_{\rm ins})|\bar{J}_N(\vec{0}, 0)\right\rangle \Gamma_k\right]$$

$$G^{\mu}_{3pt,\mathcal{Q}}(\vec{p}, \vec{q}, t_f, t_{\rm ins}, \Gamma_k) = \sum_{\vec{y},\vec{z}} e^{+i\vec{p}\vec{y}} e^{+i\vec{q}\vec{z}} \mathrm{Tr}\left[\left\langle J_N(\vec{y}, t_f)|J^{\mu}(\vec{z}, t_{\rm ins})\mathcal{Q}|\bar{J}_N(\vec{0}, 0)\right\rangle \Gamma_k\right]$$

PROJECTORS

Γ_0	$(1+\gamma_0)/4$
Γ_5	$\gamma_5/4$
Γ_k	$\Gamma_0\gamma_5\gamma_k$

Discretization of topological charge

Field theoretical definition *[Di Vecchia et al, Nucl. Phys. B192, 392 (1981)]*

Discretization of the continuum definition of topological charge based on gluon fields.

$$\mathcal{Q}_{\mathrm{f.t.}} = \frac{1}{32\pi^2} \sum_{\hat{n}} \epsilon_{\mu\nu\rho\sigma} \, \mathrm{Tr} \left[Q^{(\mathrm{cl})}_{\mu\nu}(\hat{n},\tau) \, Q^{(\mathrm{cl})}_{\rho\sigma}(\hat{n},\tau) \right]$$

- Gradient flow is applied to gauge links entering the clover definition to suppress UV fluctuations.

Spectral projector definition *[Giusti et al, J. High Energy Phys. 03 (2009)]*

Topological charge can be related to the number of left and right-handed zero modes of the Dirac operator

$$\mathcal{Q}_{\mathrm{s.p.}} = \frac{Z_S}{Z_P} \sum_{i}^{\lambda < M_{\mathrm{thr}}^2} u_i^\dagger \gamma_5 u_i \,, \qquad (D_W^\dagger D_W) u_i = \lambda_i u_i$$

- Multiplicative renormalization and threshold >0 due to explicit chiral symmetry breaking of Wilson fermions.

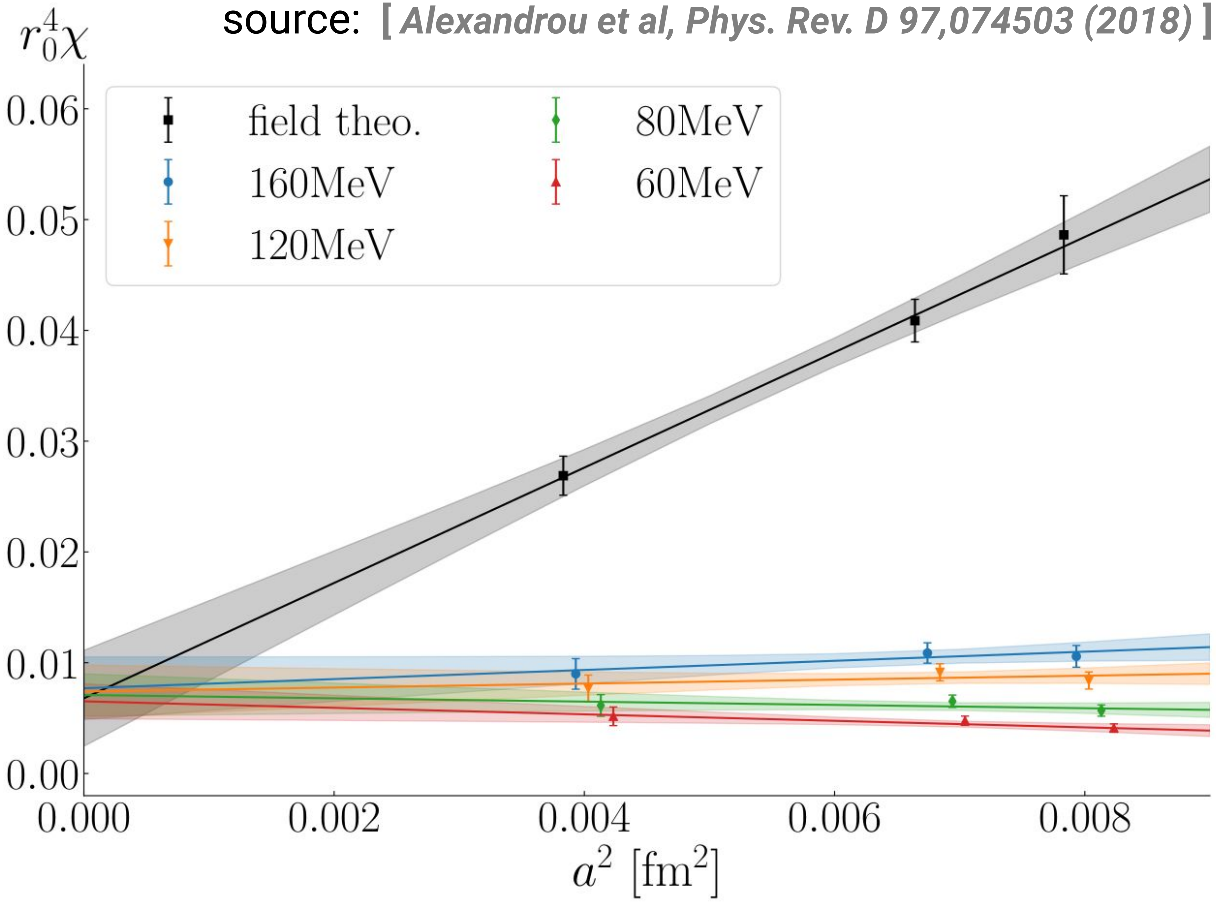
In the continuum definitions are equivalent, but on lattice different discretization effects !!

Topological susceptibility with spectral projectors

TOPOLOGICAL SUSCEPTIBILITY

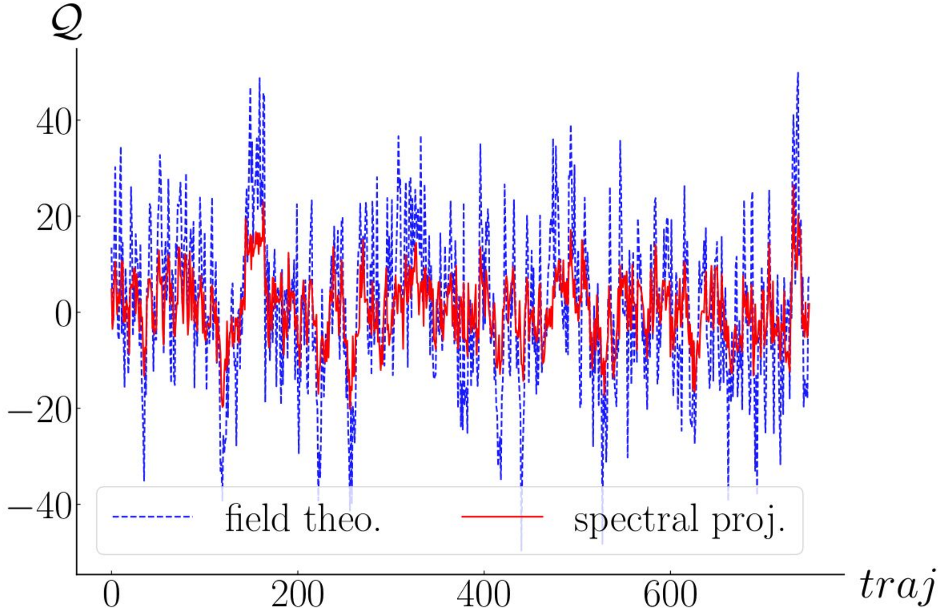
$$\chi_{\text{top}} = \left\langle Q^2 \right\rangle / V_4$$

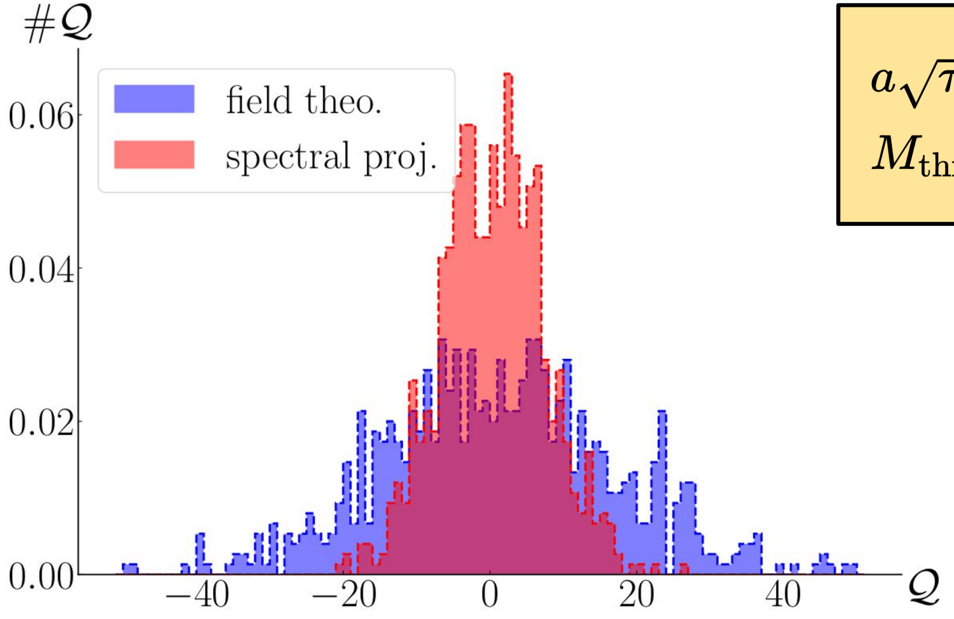
source: [*Alexandrou et al, Phys. Rev. D 97,074503 (2018)*]



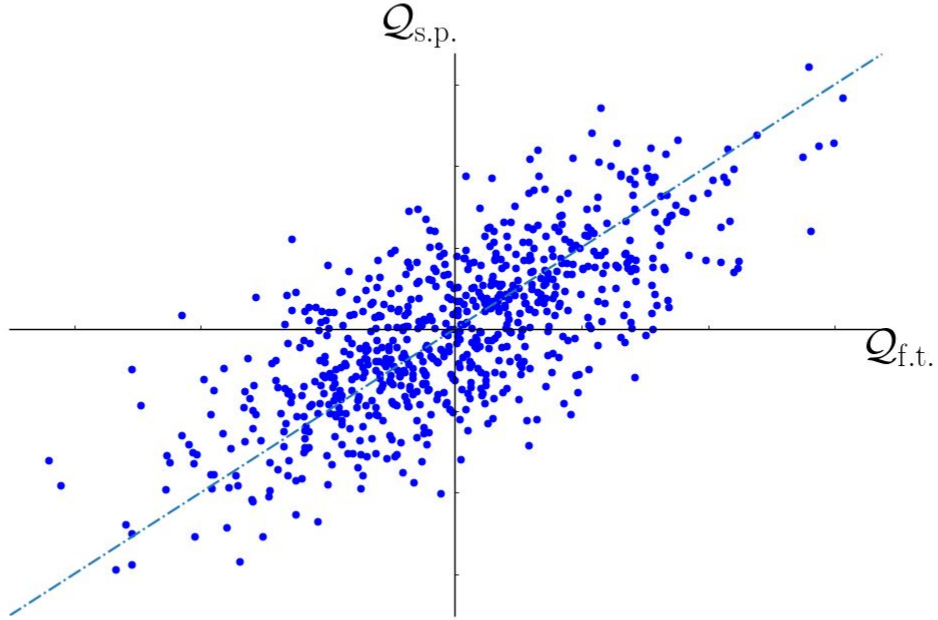
1. The topological susceptibility via spectral projectors shows milder cutoff effects w.r.t. the field theoretical definition;
2. The choice of the threshold doesn't affect the continuum extrapolation;

Field theoretical vs Spectral projectors





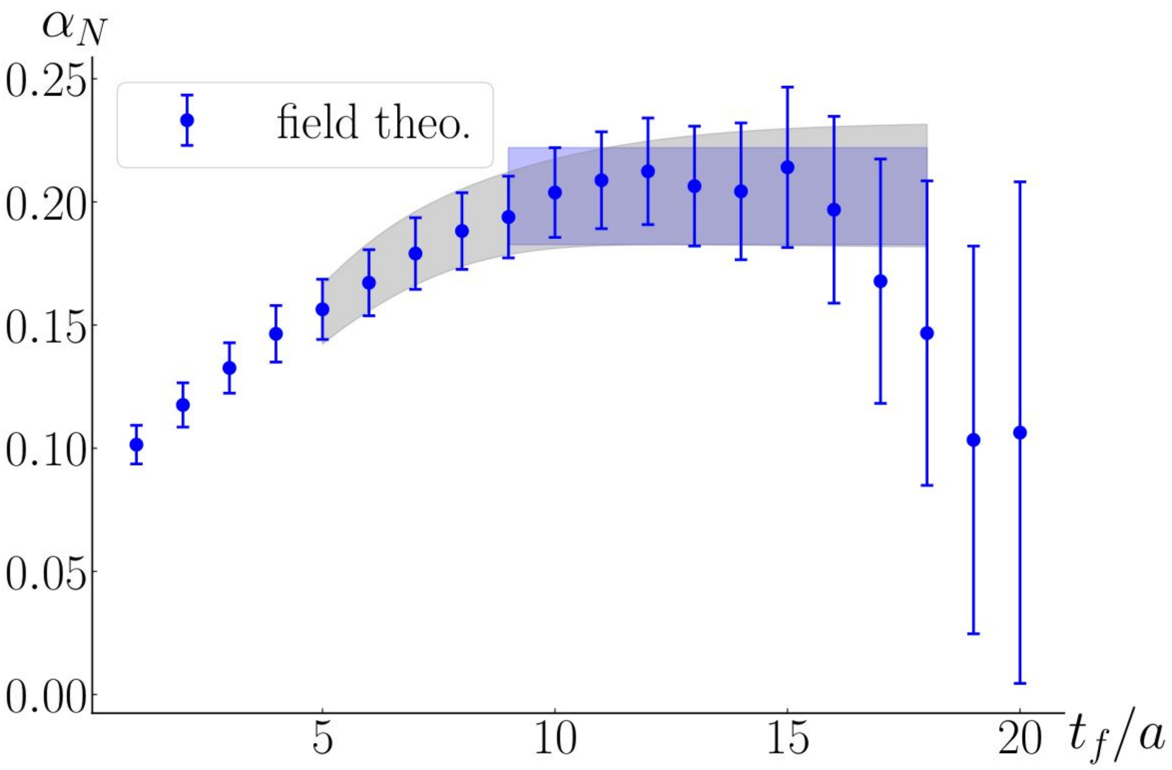
$$a\sqrt{\tau_{\text{flow}}} = 0.15 \text{ fm}$$
$$M_{\text{thr}} = 65 \text{ MeV}$$

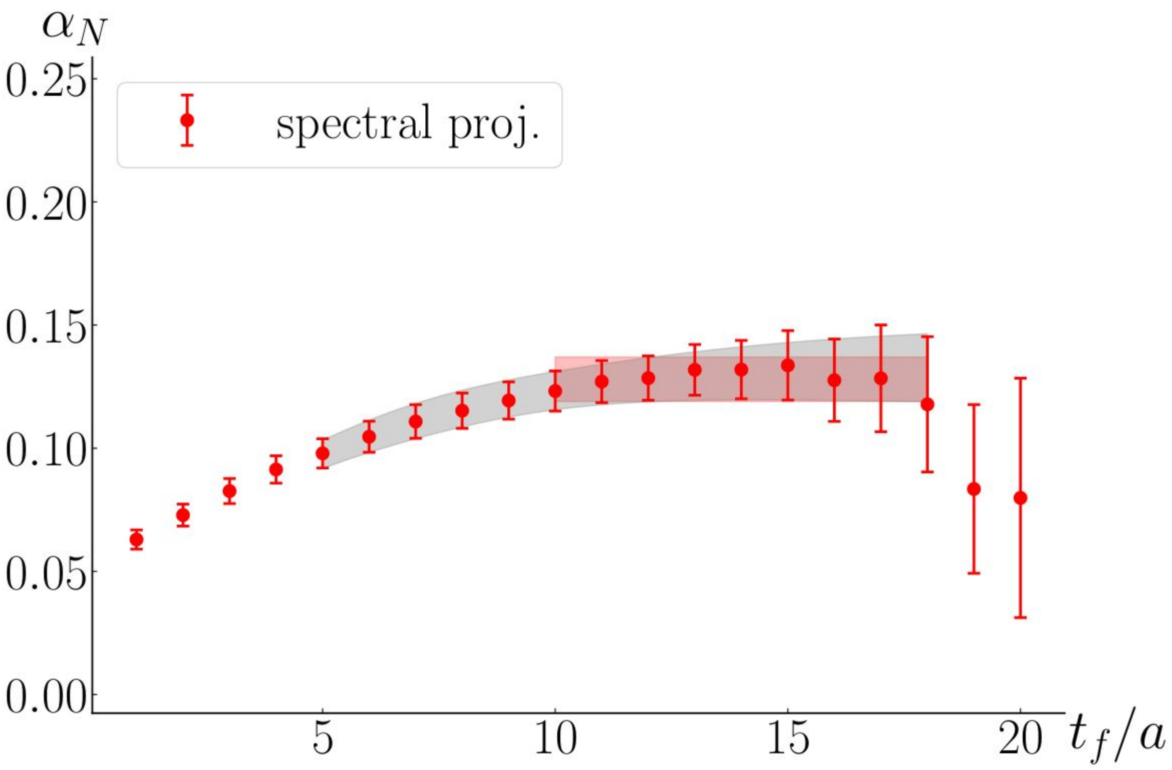


	Field Theoretical	Spectral Projectors
$< \mathcal{Q} >$	1.2(1.4)	0.46(54)
$< \mathcal{Q}^2 >$	254(22)	49.1(4.6)
Pearson Correlation Coefficient		72(2) %

Nucleon mixing angle

$$\alpha_n = \lim_{t_f \to +\infty} \frac{G_{2pt,\mathcal{Q}}(\vec{0}, t_f, \Gamma_5)}{G_{2pt}(\vec{0}, t_f, \Gamma_0)}$$





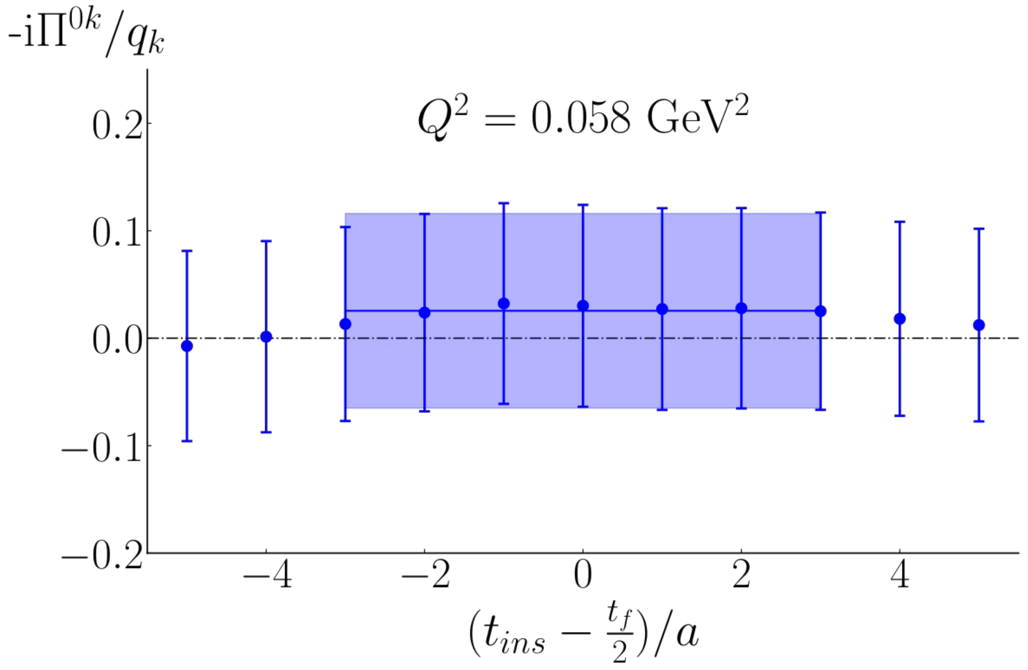
CP-odd form factor

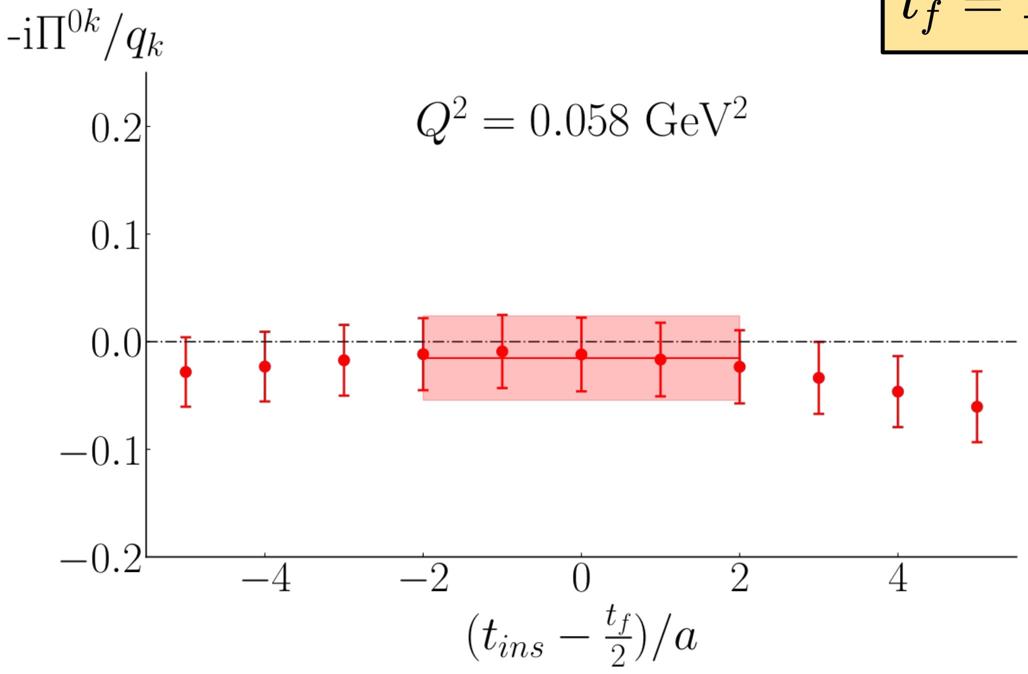
3-POINT FUNCTION RATIO

$$\Pi^{\mu\nu}_{3pt,\mathcal{Q}}(\vec{q}) = \lim_{t_f,t_{\text{ins}}\to+\infty}\left(\frac{G^{\mu}_{3pt,\mathcal{Q}}(\vec{0},\vec{q},t_f,t_{\text{ins}}\Gamma_\nu)}{G_{2pt}(\vec{0},t_f,\Gamma_0)}\right) R$$

$$\Pi^{0k}_{3pt,\mathcal{Q}}(\vec{q}) = \frac{iq_k}{2m_n}\left(\alpha_N \Pi^{00}_{3pt}(q^2)\frac{2m_n}{E_n+m_n} - F_3(q^2)\frac{\mathcal{K}(E_n+m_n)}{2m_n}\right)$$

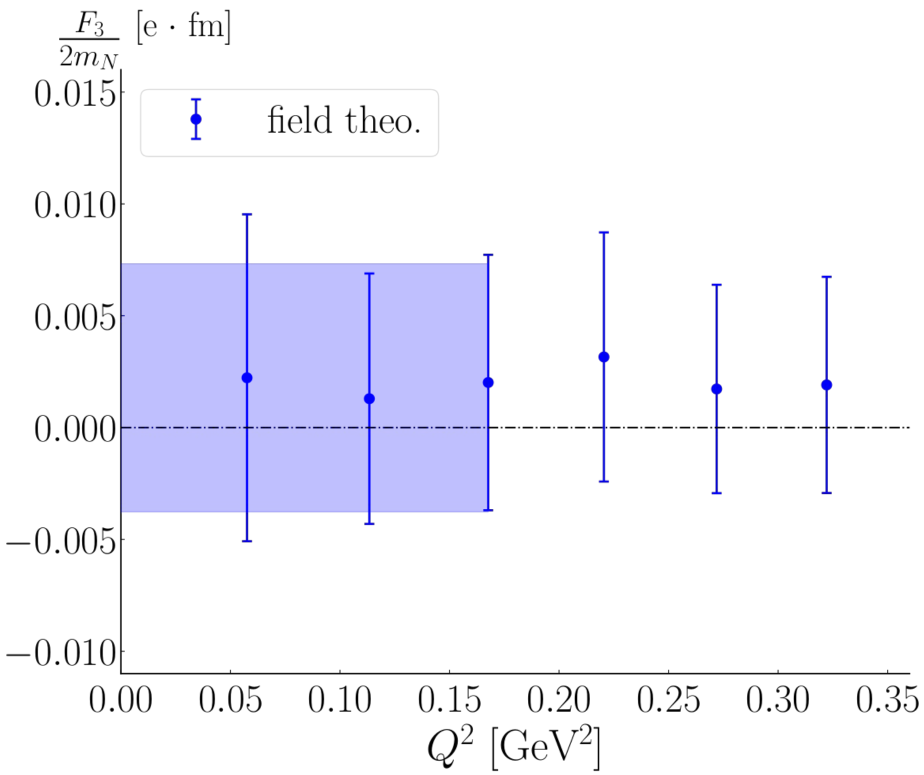
$t_f = 12a$

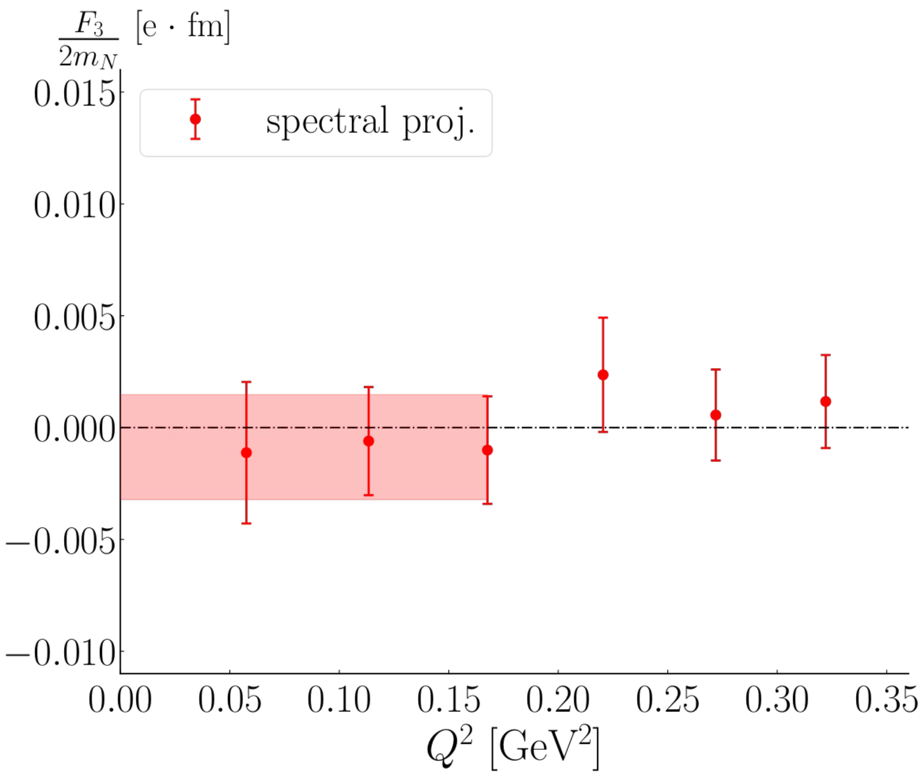




CP-odd form factor

Weighted average of values at three smallest momentum transfer, no momentum dependence with this level of uncertainty.

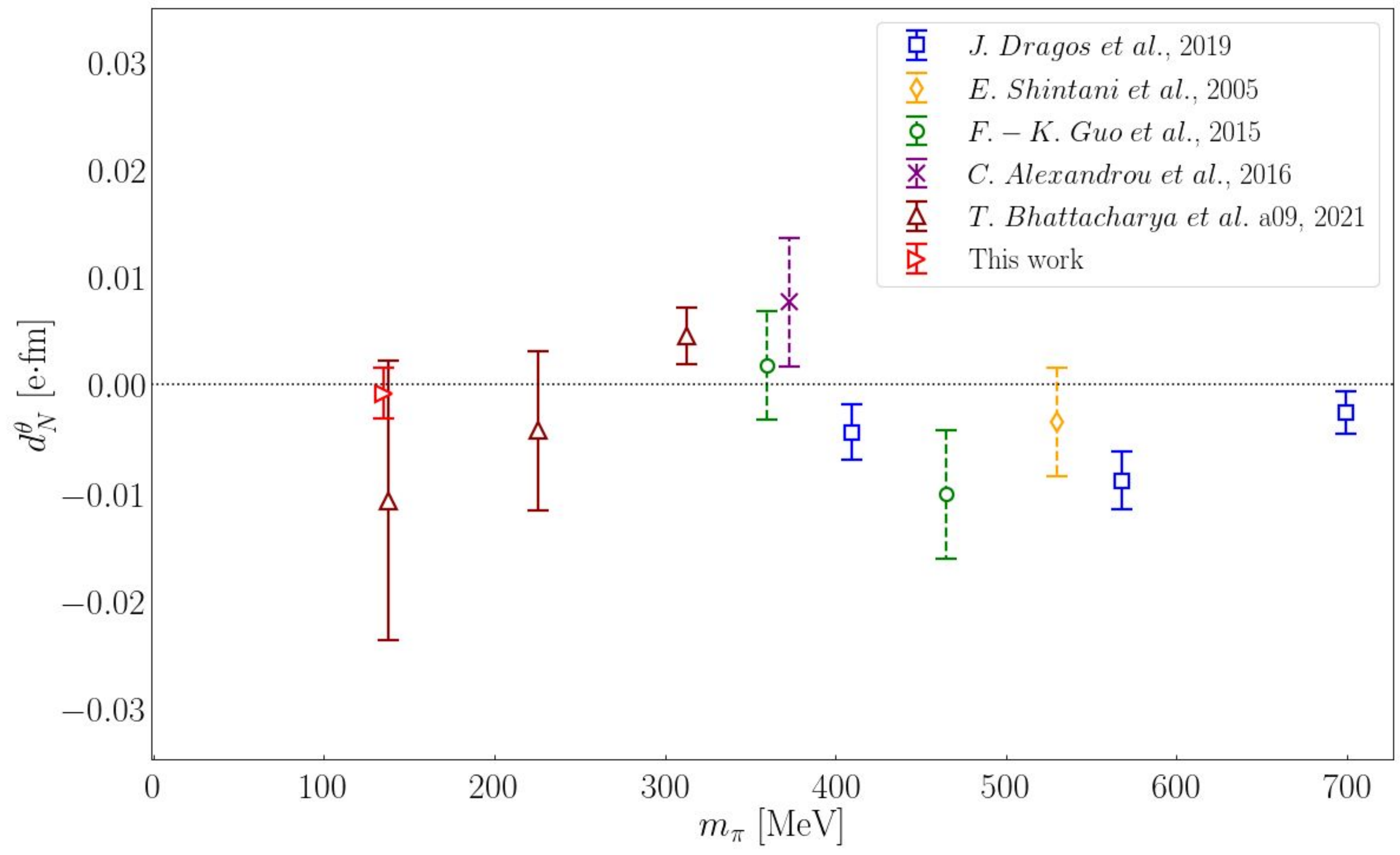




FIELD THEORETICAL	$d_n^\theta = 0.0018(56)\theta$ e fm
SPECTRAL PROJECTORS	$d_n^\theta = -0.0009(24)\theta$ e fm

Status of nEDM lattice calculations

- Dashed lines: results with spurious contribution subtracted, from [*Abramczyk et al., Phys. Rev. D 96, 014501 (2017)*]



CHIRAL-CONTINUUM EXTR.

Paper	Neutron EDM
J.Dragos et al. (2019)	-0.00152(71)
T.Bhattacharya et al. (2021)	-0.003(7)(20)
T.Bhattacharya et al. (2021) with Nπ	-0.028(18)(54)
This work	-0.0009(24)

Conclusions

ACHIEVEMENTS

- Impact of fermion topological charge definition: **x2 improvement** in the final statistical error;
- Lattice determination of the nEDM at the physical point with unmatched precision;

$$d_n^\theta = -0.0009(24)\theta \text{ e fm}$$

OPEN ISSUES

- Result compatible with zero: large uncertainty or no CP violation?
  - ruling out a zero value would require at least 2-order-of-magnitude increase in statistics;
- Different lattice spacings for continuum extrapolation (**large computational effort, exascale problem);**

Alternative approaches of extracting θ-nEDM highly desirable:

- We are currently carrying out exploratory studies with ensembles at imaginary θ;

THANK YOU

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No 765048

BACKUP

Setup Spectral Projectors

$$\mathcal{Q}_{\text{s.p.}} = \frac{Z_S}{Z_P} \sum_i^{\lambda < M_{\text{thr}}^2} u_i^\dagger \gamma_5 u_i \,, \quad (D_W^\dagger D_W) u_i = \lambda_i u_i$$

- Lowest 200 eigenvalues of the squared twisted mass Dirac operator using the implicitly restarted Lanczos method (IRLM) with polynomial accelleration;
- The renormalization constants are computed in the so-called RI' scheme using five ensembles at different pion masses and performing a chiral extrapolation to extract the Z-factors at the chiral limit [*Alexandrou et al., Phys. Rev. D 102, 054517 (2020)*].
- Conversion to $\overline{\text{MS}}$-scheme via RGI, that provides us the values of the renormalization constants employed in this work

$$Z_P = 0.462(4) \qquad\qquad Z_S = 0.620(4)$$

Dependence of F3 from smoothing scale