Flavour singlet scattering of Goldstone bosons in SU(2) with N_f=2 fundamental fermions

based on 2107.09974

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Outline

- Introduction and context
- Scattering in SU(2)
- Lattice results
- Conclusion

Scattering observables as a probe of New Physics

- LHC Run II :
  - Study of the properties of the Higgs boson
  - Measure Vector Boson Scattering

- Vector Boson Scattering :
  - Probe the structure of electroweak interactions in the SM
  - Very sensitive to new physics.

[Covarelli *et al*, *Int.J.Mod.Phys.A* 36 (2021) 16, 2130009, 2102.10991]

- New strong dynamics Beyond the Standard Model
  - Dynamical origin to the EWSB
  - Scattering processes of the new strong sector contribute to SM processes investigated at the LHC.

Why is the flavour singlet channel interesting?

- Theoretical perspective:
  - Plays a central role in building effective theories of composite scenario at the EW scale: need to accumulate reliable results on the 0^{++}.

- Phenomenological perspectives in the context of Composite Higgs models:
  - 0^{++} state mixes with the Higgs boson: alter its physical properties
  - 0^{++} is expected to show up at the LHC as a heavy resonance
  - Phenomenological implication for theories based on SU(4) ⟶ Sp(4) breaking considered

[Buarque Franzosi, Cacciapaglia et al *Eur.Phys.J.C* 80 (2020) 1, 28, 1809.09146]

$SU(2)_c$ with N_f=2 fundamental Dirac flavours

- Fundamental representation of SU(2) is pseudo-real
- Chiral symmetry breaking pattern : SU(4) → Sp(4) (~ SO(6) → SO(5))

→ **5 Goldtone bosons**

- UV completion of a Minimal composite Higgs model

[G. Cacciapaglia & F. Sannino, *JHEP* 04 (2014) 111 1402.0233]

- The Higgs is a linear combination of GBs and of the 0^+ state
- Not excluded by experimental data

[Arbey *et al*, *Phys.Rev.D* 95 (2017) 1, 015028 1502.04718]

Scattering of GBs in $SU(2)_c$ with $N_f=2$

- Goal: build a correlation of operator which flavour singlet quantum numbers
- Defining the following multiplet $\Pi^i = \frac{1}{2}\left[Q^T(-i\sigma_2)C\gamma_5 X^i E Q + \text{h.c}\right]$

where $Q \equiv \begin{pmatrix} U_L \\ D_L \\ \widetilde{U}_L \\ \widetilde{D}_L \end{pmatrix} \equiv \begin{pmatrix} U_L \\ D_L \\ -i\sigma_2 C\bar{u}_R^T \\ -i\sigma_2 C\bar{d}_R^T \end{pmatrix}, \quad E = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}$

and $X^{1,\dots,5}$ are the generators of SU(4)/Sp(4) .

- Π transforms as 5 dimensional irrep of Sp(4) [proof by explicit calculation]
- Any operator proportional to $\text{tr}\{\Pi \otimes \Pi\}$ is a flavour singlet.

Scattering of GBs in $SU(2)_c$ with $N_f=2$ (cont.)

- Defining

$$\begin{aligned}
\Pi_{ud}(x) &= u^T(x)(-i\sigma_2)C\gamma_5 d(x),\\
\Pi_{\bar{u}\bar{d}}(x) &= \bar{u}(x)(-i\sigma_2)C\gamma_5 \bar{d}(x)^T,\\
\pi^-(x) &= \bar{u}(x)\gamma_5 d(x),\\
\pi^+(x) &= -\bar{d}(x)\gamma_5 u(x),\\
\pi^0(x) &= \frac{1}{\sqrt{2}}\left[\bar{u}(x)\gamma_5 u(x) - \bar{d}(x)\gamma_5 d(x)\right]
\end{aligned}$$

- We have shown that a two GBs flavour singlet operator is

$$\mathcal{O}_{\pi\pi} = -\frac{4}{\sqrt{5}}\sum_i \Pi^i\Pi^i = \frac{1}{\sqrt{5}}\Big[+\pi^+\pi^- + \pi^-\pi^+ - \pi^0\pi^0 \quad + \Pi_{ud}\Pi_{\bar{u}\bar{d}} + \Pi_{\bar{u}\bar{d}}\Pi_{ud}\Big]$$

- The fermion bilinear operator with the flavour singlet quantum number reads:

$$\mathcal{O}_\sigma = \frac{1}{\sqrt{2}}\left[Q^T(-i\sigma_2)CEQ + \text{h.c}\right] = \frac{1}{\sqrt{2}}\left[\bar{u}(x)u(x) + \bar{d}(x)d(x)\right].$$

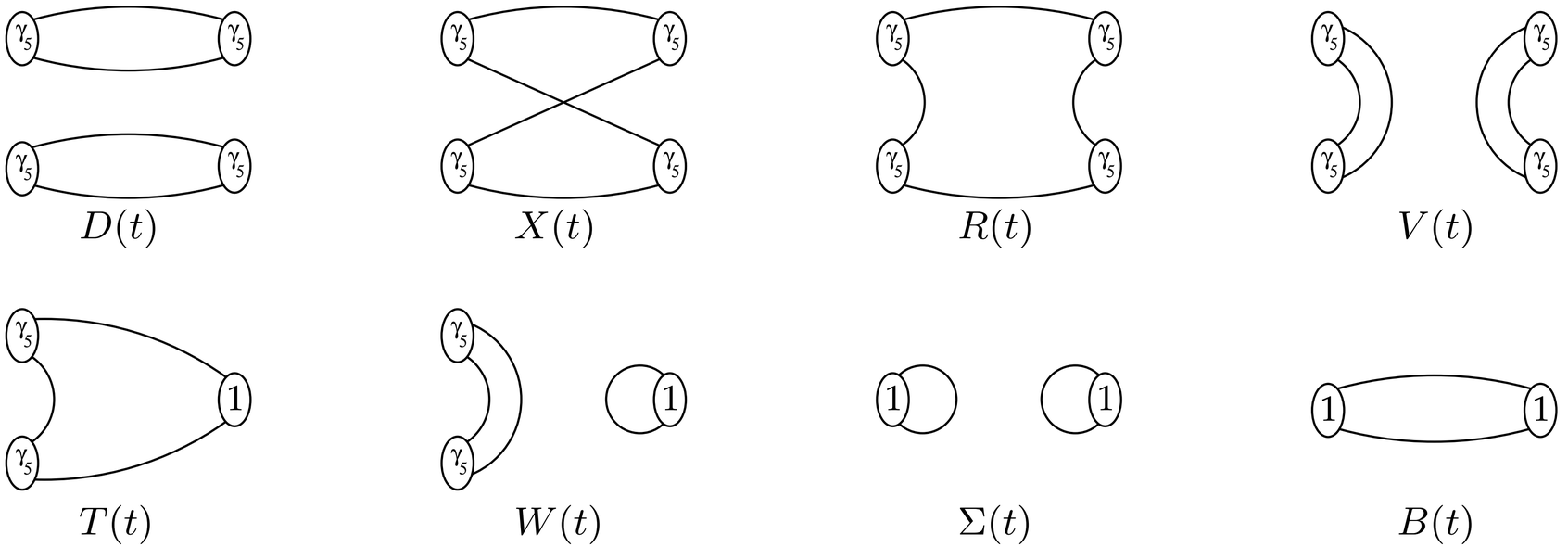
→ $\{\mathcal{O}_{\pi\pi}, \mathcal{O}_\sigma\}$ are the two operators used to build our correlation matrix

GEVP - Contractions

- Correlation matrix : $$C_{X\to Y}(\delta t) = \frac{1}{T}\sum_t \langle O_X(t+\delta t) O_Y(t)^\dagger \rangle.$$

- Wick contractions read :

$$C_{\sigma\to\sigma}(t) = -B(t) + 2\Sigma(t),$$
$$C_{\pi\pi\to\pi\pi}(t) = 2D(t) + 3X(t) - 10R(t) + 5V(t),$$
$$C_{\pi\pi\to\sigma}(t) = \sqrt{10}\,(T(t) - W(t))\,.$$



Note: the combinatoric is different from the QCD case !

Lüscher's method

- Quantisation condition for s-wave scattering:

$$k \cot \delta_0 (k) = \frac{2}{\sqrt{\pi} L} \mathcal{Z}_{00}(\eta^2), \quad \eta = \frac{Lk}{2\pi}$$

k: relative momentum of the two GBs in the c.m.f obtained from the energy levels.

$\mathcal{Z}_{00}$: Lüscher zeta function

- Bound-state condition: ($k \cot \delta_0$ must cross the bound state condition from below for decreasing k^2)

$$k \cot \delta_0 (k) = -\sqrt{-k^2},$$

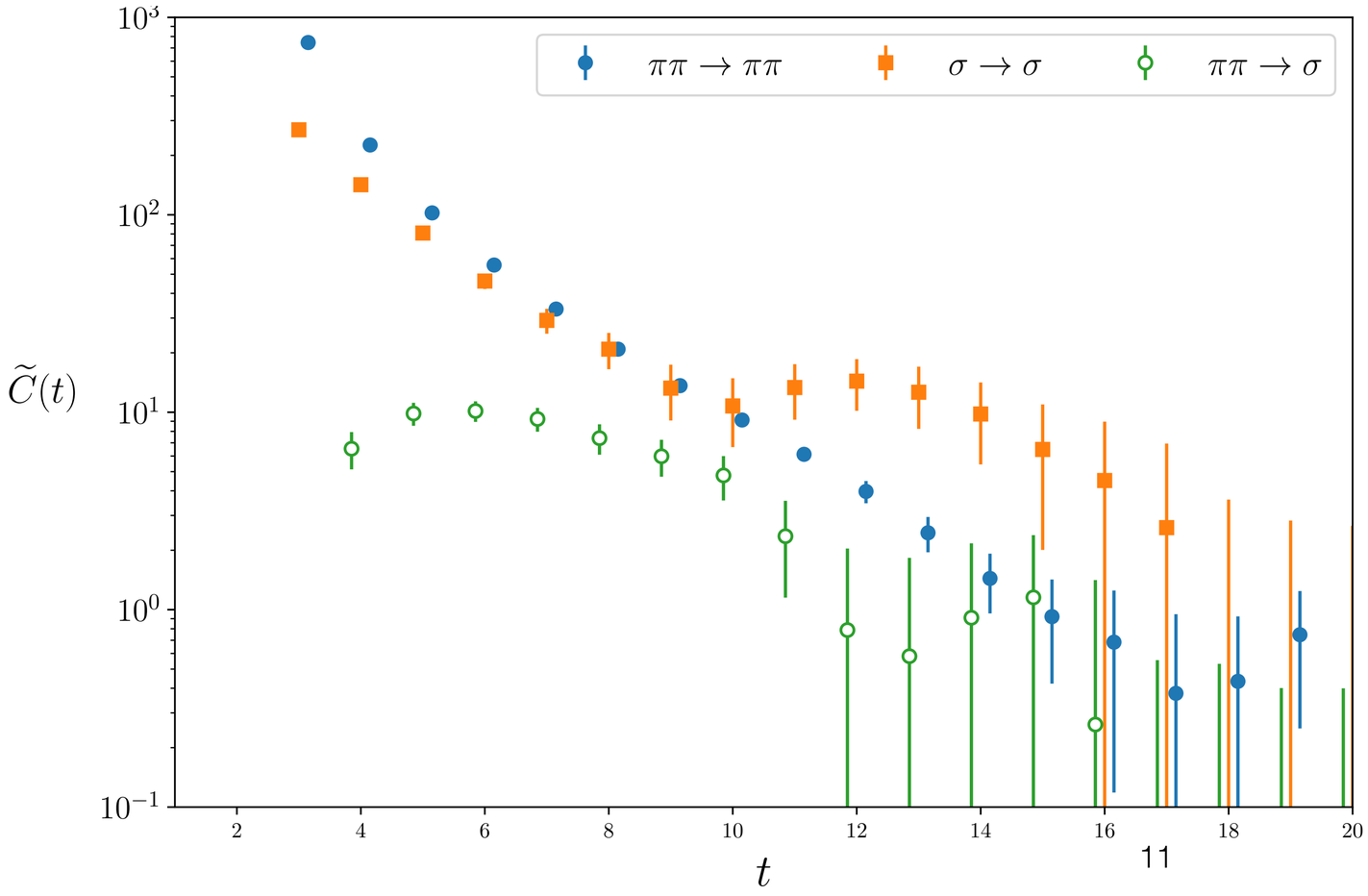
Results

Analysis of the correlation matrix

- Constant contribution to the correlation functions removed by defining shifted-correlators:

$$\widetilde{C}(t) = \frac{1}{2}\left[C(t-1) - C(t+1)\right]$$

- GEVP from the shifted correlators



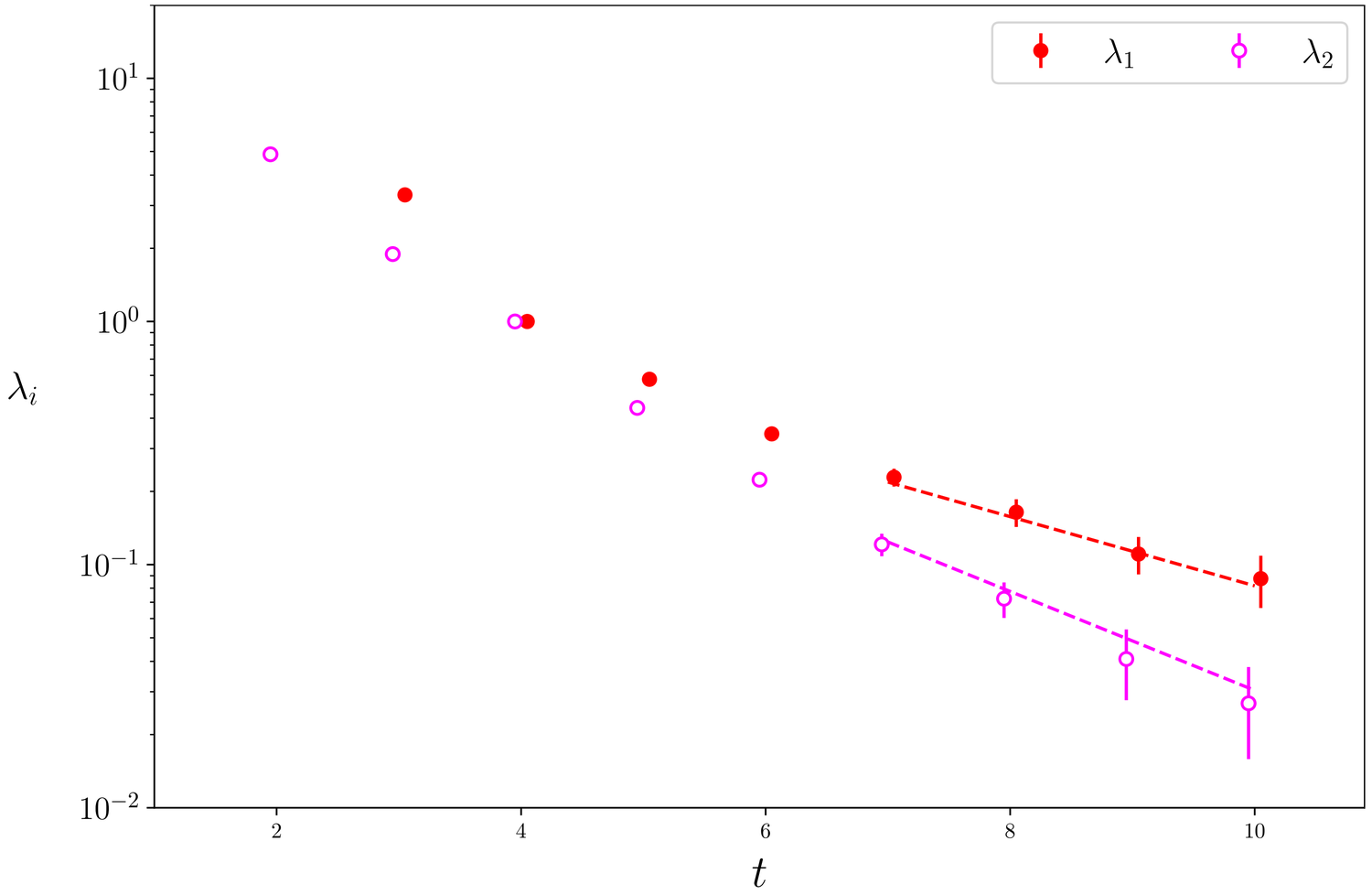
L=24a, T=48a
Vector resonance unstable

Correlators involving $\mathcal{O}_\sigma$
have larger statistical noise

Signal dominated by noise for t > 12

Eigenvalues

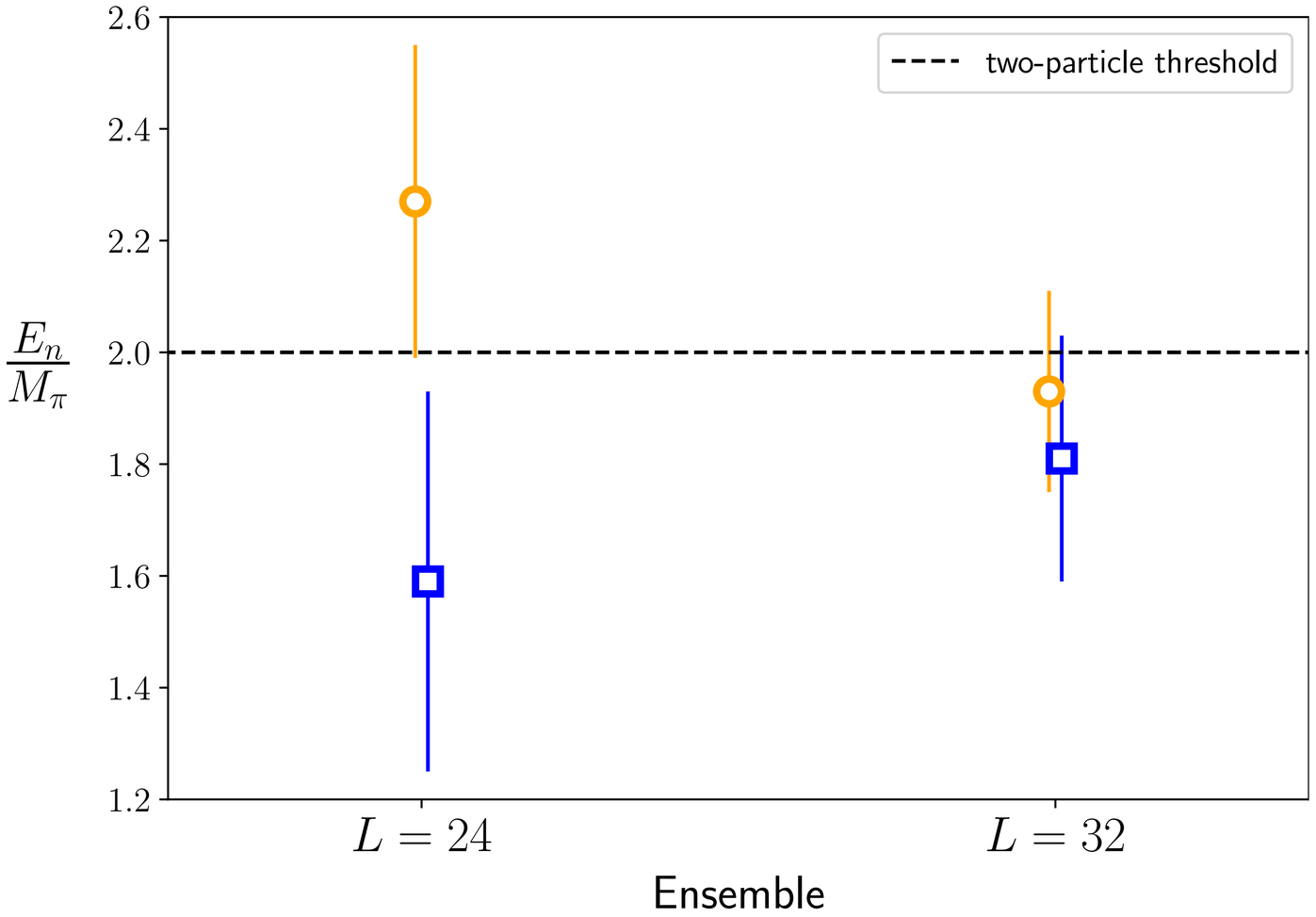
L=24a, T=48a, Vector resonance unstable



Best fit : dashed line

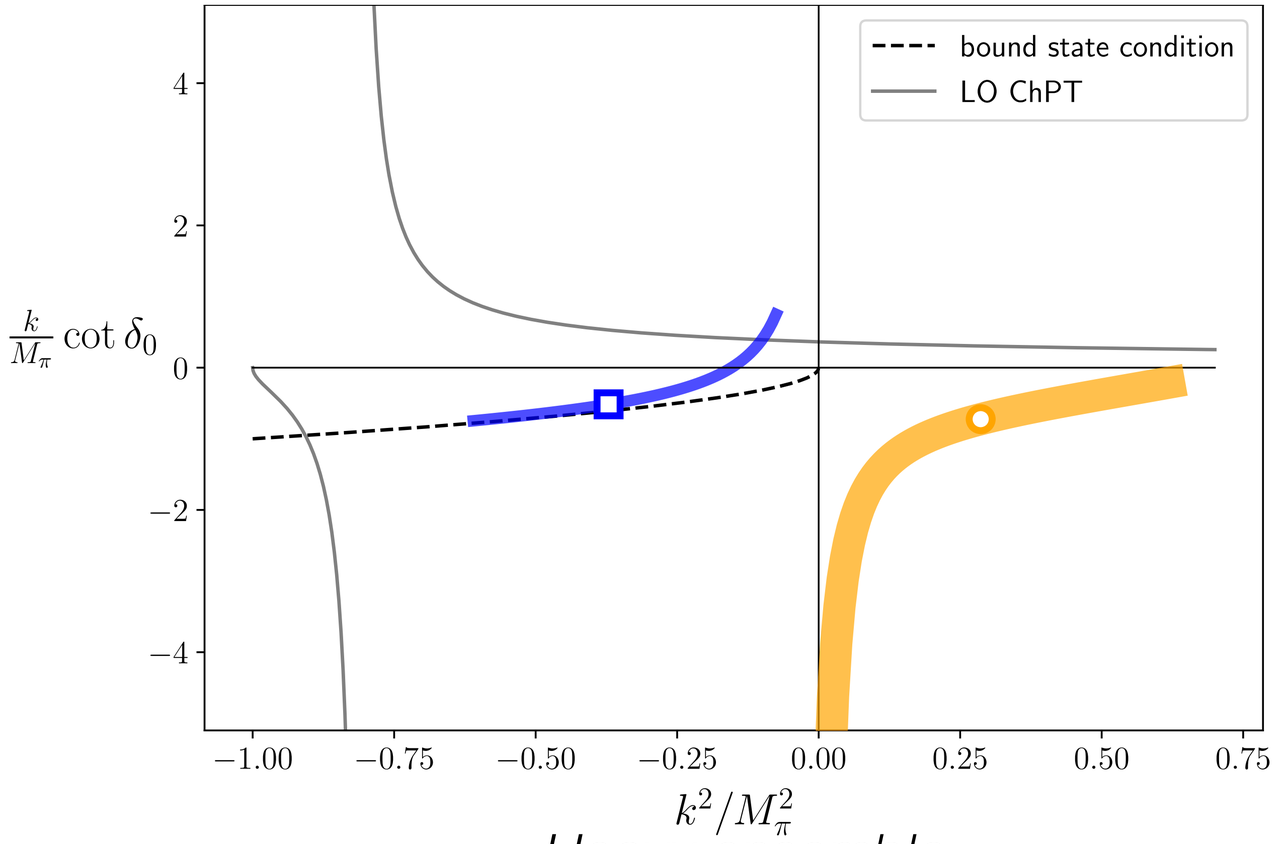
Noise dominated region excluded from fit

Eigenvalues summary

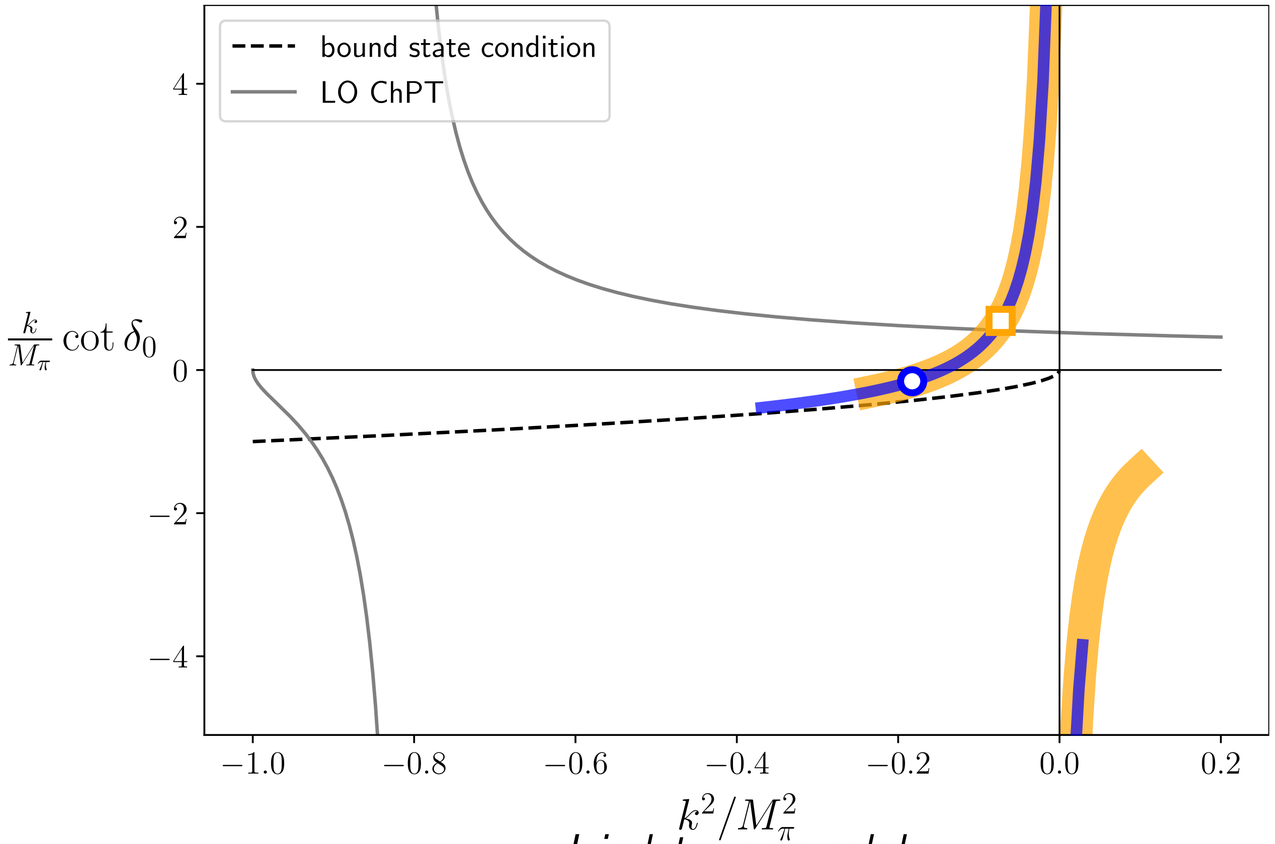


The ensemble with the lightest fermion mass is close to threshold

Scattering amplitude



Heavy ensemble



Light ensemble

Empty marker: central value
shaded area: represents the 1sigma error
LO chiral perturbation theory prediction

[Bijnens & Lu, J *JHEP* 03 (2011) 028 1102.0172]

→ Results suggest the presence of a bound state.

Conclusion

- First study of the singlet channel in four-dimensional gauge theories beyond QCD
- Study of scattering processes is crucial to constrain underlying dynamics of Pseudo-Nambu Goldstone Bosons Composite Higgs models

- We use :
  - Two ensembles below vector channel threshold at fixed lattice spacing
  - 2x2 GEVP (including all disconnected contributions)
  - Lüscher's method

- Results are compatible with a bound state in the singlet channel.

- Results complement our recent calculation of the scattering amplitude in the vector channel and our prediction of its coupling to two GBs.

[Drach, Janowski, Pica & Prelovsek, *JHEP* 04 (2021) 117 2012.09761]

Thank you for you attention !