Calculation of kaon semileptonic form factor with the PACS10 configurations

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Introduction

Urgent task: search for signal beyond standard model (BSM)

Muon $g-2$ @ FNAL 2021 : 4.2σ away from SM

$|V_{us}|$: a candidate of BSM signal

Most accurate $|V_{us}|$ from K_{l3} decay [FNAL/MILC19]

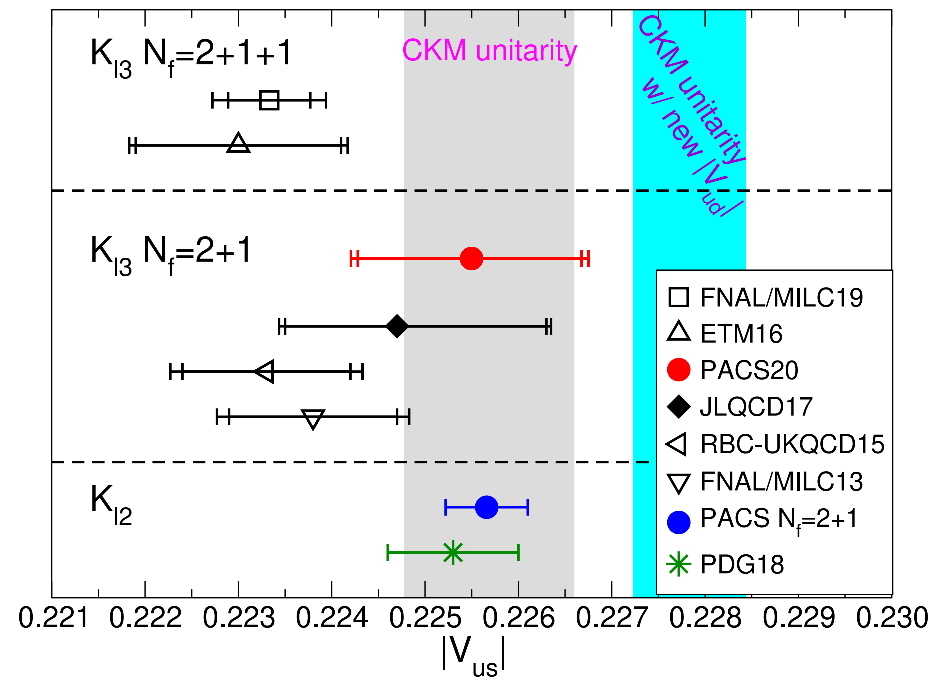
$\sim 2\sigma$ from SM (gray band)

using CKM unitarity $|V_{us}| \approx \sqrt{1-|V_{ud}|^2}$

$\sim 5\sigma$ from SM w/ new $|V_{ud}|$ (cyan band) [Seng *et al.*:PRL121,241804(2018)]

Important to confirm by several different calculations

[PACS:PRD101,9,094504(2020)]



K_{l3} form factors with one of PACS10 configurations [PACS20]

$L = 10.9$[fm] at physical point, but single a

Negligible finite L effect, tiny q^2 region, without chiral extrapolation

Largest uncertainty: finite a effect $\rightarrow$ smaller a calculation

Simulation parameters

$N_f = 2+1$ six-stout-smeared non-perturbative $O(a)$ Clover action
+ Iwasaki gauge action

PACS10 configurations: $L \gtrsim 10$[fm] at physical point

β	$L^3 \cdot T$	L[fm]	a^{-1}[GeV]	M_π[MeV]	M_K[MeV]	$N_{\rm conf}$
1.82	128^4	10.9	2.3162	135	497	20
2.00	160^4	10.2	3.09	137	501	20

Parameters for K_{l3} form factors

β	source	$t_{\rm sep}$[fm]	current
1.82	R-local	3.1, 3.6, 4.1	local*, conserved
2.00	R-local	3.2, 3.7, 4.1	local, conserved
	R-smear	2.3, 2.7, 3.1, 3.5	local, conserved

R-local: $Z(2) \times Z(2)$ random source spread in spatial volume, spin, color spaces

R-smear: R-local + exponential smearing [RBC-UKQCD:JHEP07,112(2008)]

*reported in PACS:PRD101,9,094504(2020)

Calculation at smaller a + continuum extrapolation

All results are preliminary.

Calculation method R-local source with local current

Details in PACS:PRD101,9,094504(2020)

3-point function*

$$C_{V_\mu}(t,p) = \langle 0|O_K(t_{\text{sep}},\mathbf{0})V_\mu(t,\mathbf{p})O_\pi^\dagger(0,\mathbf{p})|0\rangle$$
$$= \frac{Z_\pi Z_K}{Z_V}\frac{M_\mu(p)}{4E_\pi M_K}e^{-E_\pi t}e^{-M_K(t_{\text{sep}}-t)} + \cdots$$

$p \equiv |\mathbf{p}| = \left|\frac{2\pi}{L}\mathbf{n}\right|, \ |\mathbf{n}| = 0\text{–}6$

with periodic boundary

2-point function* $X = \pi, K$

$$C_X(t,p) = \langle 0|O_X(t,\mathbf{p})O_X^\dagger(0,\mathbf{p})|0\rangle = \frac{Z_X^2}{2E_X}\left(e^{-E_X t} + e^{-E_X(2T-t)}\right) + \cdots$$

*Averaging ones with periodic, anti-periodic temporal boundary conditions reducing wrapping around effect in 3pt, and doubling periodicity in 2pt

$Z_V = \sqrt{Z_V^\pi Z_V^K}$ from electromagnetic form factor $F_{\pi,K}(0) = 1$

Ratio $R_\mu(t,p)$

$$R_\mu(t,p) = \frac{Z_\pi Z_K Z_V C_{V_\mu}(t,p)}{C_\pi(t,p)C_K(t_{\text{sep}}-t,0)} = M_\mu(p) + \cdots$$

Constant in $R_\mu(t,p)$ corresponds to $M_\mu(p) = \langle \pi(p)|V_\mu|K(0)\rangle$

Conserved current case: $V_\mu \to \widetilde{V}_\mu$ and $Z_V = 1$

R-smear source case: $Z_\pi, Z_K \to Z_\pi(p), Z_K(0)$

Calculation method

Details in PACS:PRD101,9,094504(2020)

Constant in $R_\mu(t,p)$ corresponds to $M_\mu(p) = \langle \pi(p)|V_\mu|K(0)\rangle$

K_{l3} form factors $f_+(q^2), f_0(q^2)$

$$\langle \pi(p)|V_\mu|K(0)\rangle = (p_K + p_\pi)_\mu f_+(q^2) + (p_K - p_\pi)_\mu f_-(q^2)$$

$$f_0(q^2) = f_+(q^2) - \frac{q^2}{M_K^2 - M_\pi^2} f_-(q^2)$$

$p_K = (M_K, \mathbf{0}), p_\pi = (E_\pi, \vec{p})$
$q^2 = -(M_K - E_\pi)^2 + p^2$

Calculation of physical quantities

$R_4(t,p), R_i(t,p) \to M_4(p), M_i(p) \to f_+(q^2), f_0(q^2)$ at each q^2

except for $p = 0$, where only $f_0(q^2)$

$\to q^2$ interpolations to $q^2 = 0$ for $f_+(q^2), f_0(q^2)$

1. $f_+(0)\,(= f_0(0)) \to |V_{us}|$

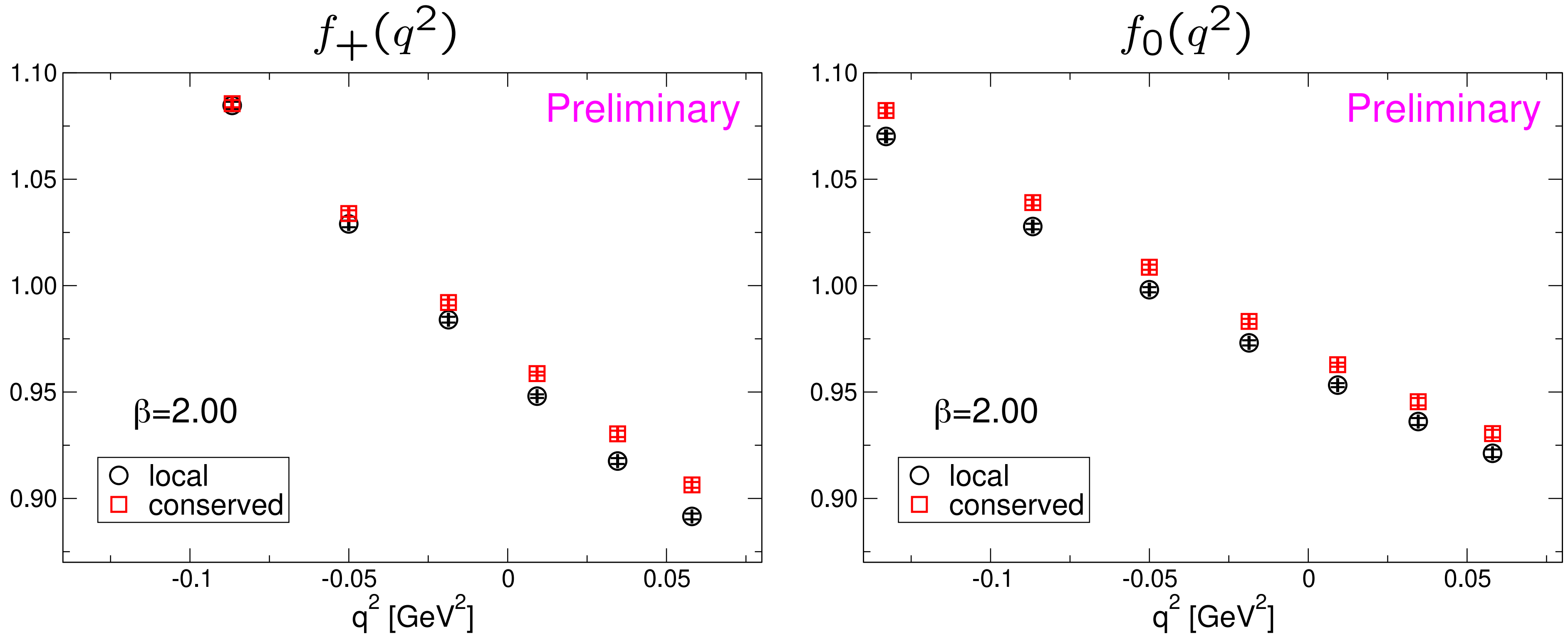
$$|V_{us}|f_+(0) = 0.21654(41)$$ [Moulson:PoS(CKM2016)033(2017)]

2. slope and curvature

$$\lambda_+^{(n)} = \frac{M_{\pi^-}^{2n}}{f_+(0)} \frac{d^n f_+(0)}{d(-q^2)^n}, \quad \lambda_0^{(n)} = \frac{M_{\pi^-}^{2n}}{f_+(0)} \frac{d^n f_0(0)}{d(-q^2)^n}$$

Result: $f_+(q^2)$ and $f_0(q^2)$ at $\beta = 2.00$

Local and conserved currents



Access tiny q^2 region thanks to $L \sim 10$[fm]

Clear discrepancy between local and conserved currents due to different finite lattice spacing effects

Calculation with unimproved local and conserved currents

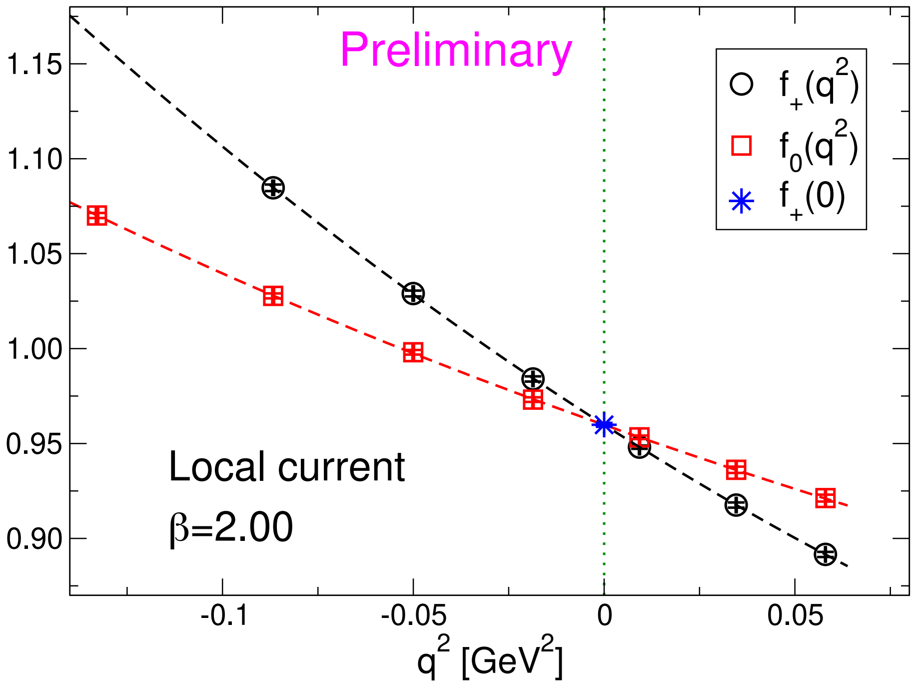
Similar, but larger discrepancy at $\beta = 1.82$

Result: q^2 interpolation at $\beta = 2.00$

Fit based on SU(3) NLO ChPT with $f_+(0) = f_0(0)$ [PACS:PRD101,9,094504(2020)]

$$f_+(q^2) = 1 - \frac{4}{F_0^2} L_9(\mu) q^2 + K_+(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^+ q^4$$

$$f_0(q^2) = 1 - \frac{8}{F_0^2} L_5(\mu) q^2 + K_0(q^2, M_\pi^2, M_K^2, F_0, \mu) + c_0 + c_2^0 q^4$$



free parameters:

$L_5(\mu), L_9(\mu), c_0, c_2^+, c_2^0$

fixed parameters:

$F_0 = 0.11205$ GeV, $\mu = 770$ MeV

FLAG $F^{SU(2)}/F_0 = 1.08$–$1.23 \to 1.15$

with Our $F^{SU(2)} = 0.129$ GeV

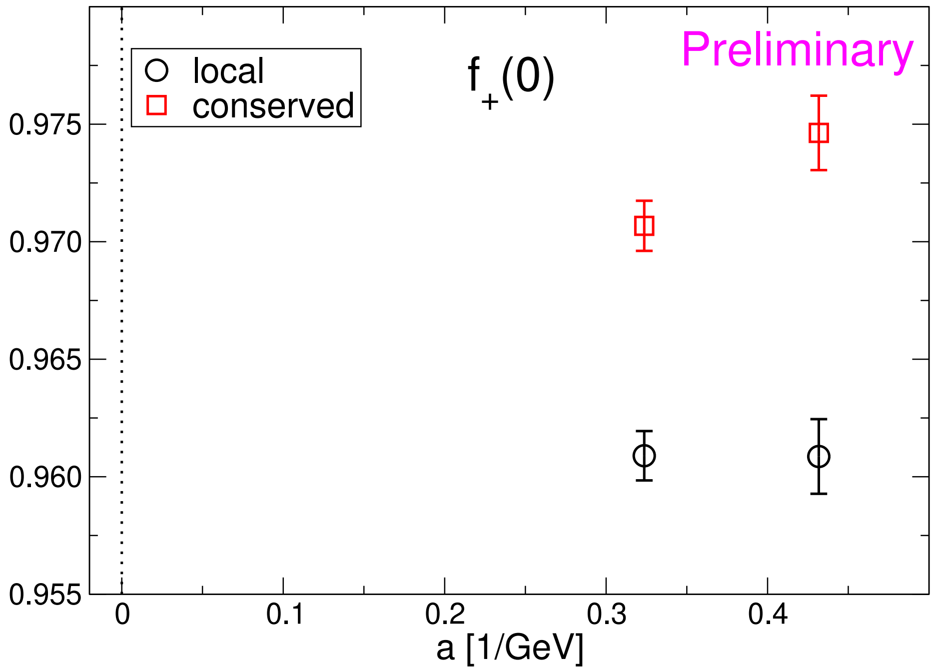
K_+, K_0: known functions

[Gasser, Leutwyler:NPB250,465 and 517(1985)]

Simultaneous fit works well.

Tiny extrapolation to physical M_{π^-} and M_{K^0} using same formulas

Result: Continuum extrapolation of $f_+(0)$



local current: almost flat

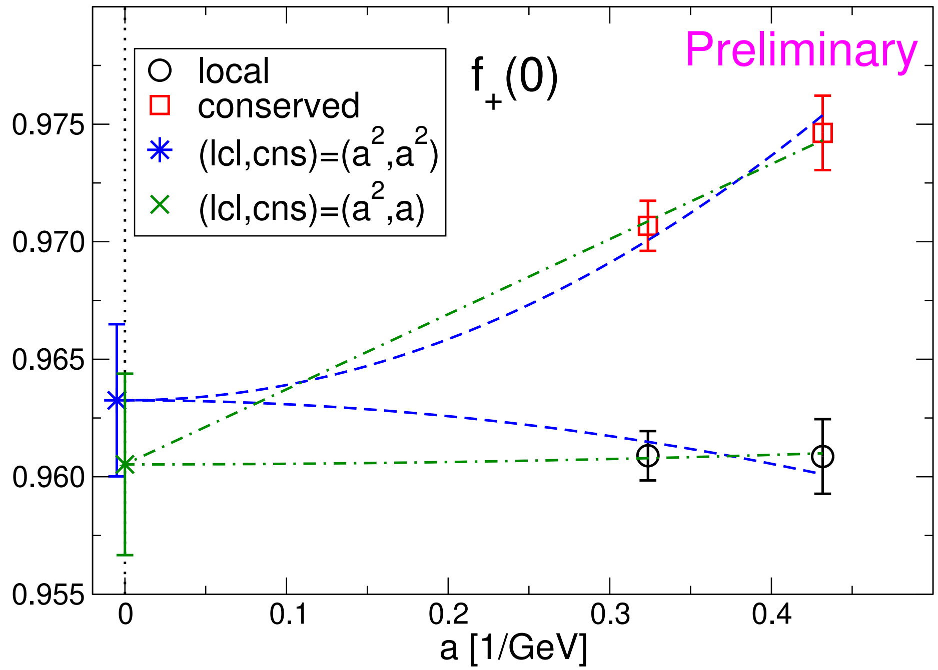
conserved current: clear a dependence

Similar trend seen in HVP calculation [PACS:PRD100,3,034517(2019)]

Simultaneous fit: *e.g.* (lcl,cns)=(a^2,a)

$$f_+^{\text{lcl}}(0) = A + B^{\text{lcl}} a^2, \quad f_+^{\text{cns}}(0) = A + B^{\text{cns}} a$$

Result: Continuum extrapolation of $f_+(0)$



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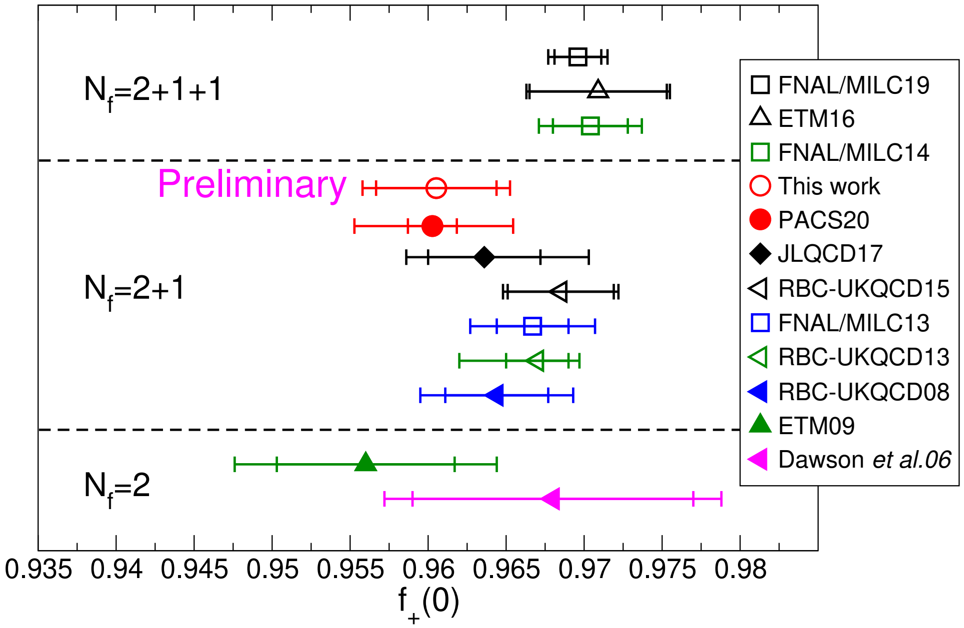
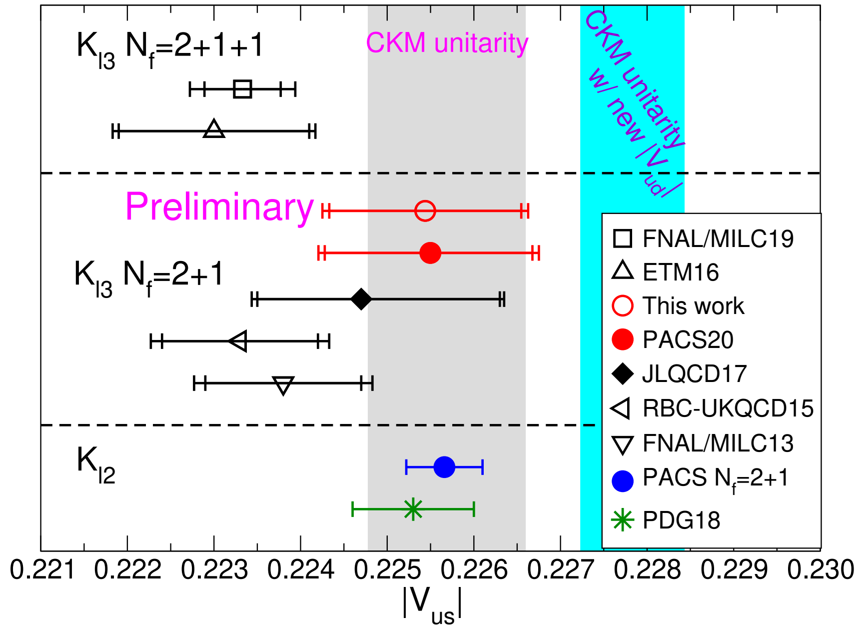
Preliminary $f_+(0) = 0.9605(39)(27)$

central value + statistical from (a^2, a); systematic from difference from (a^2, a^2)

Need smaller a data for more reliable extrapolation

Result: Continuum extrapolation of $f_+(0)$





inner, outer = statistical, total(stat.+sys.) inner, outer = lattice, total(lat.+exp.)

gray band: Standard model (SM) prediction using $|V_{us}| \approx \sqrt{1 - |V_{ud}|^2}$

cyan band: same prediction with new $|V_{ud}|$ [Seng *et al.*:PRL121,241804(2018)]

$f_+(0)$: Consistent with PACS20 result

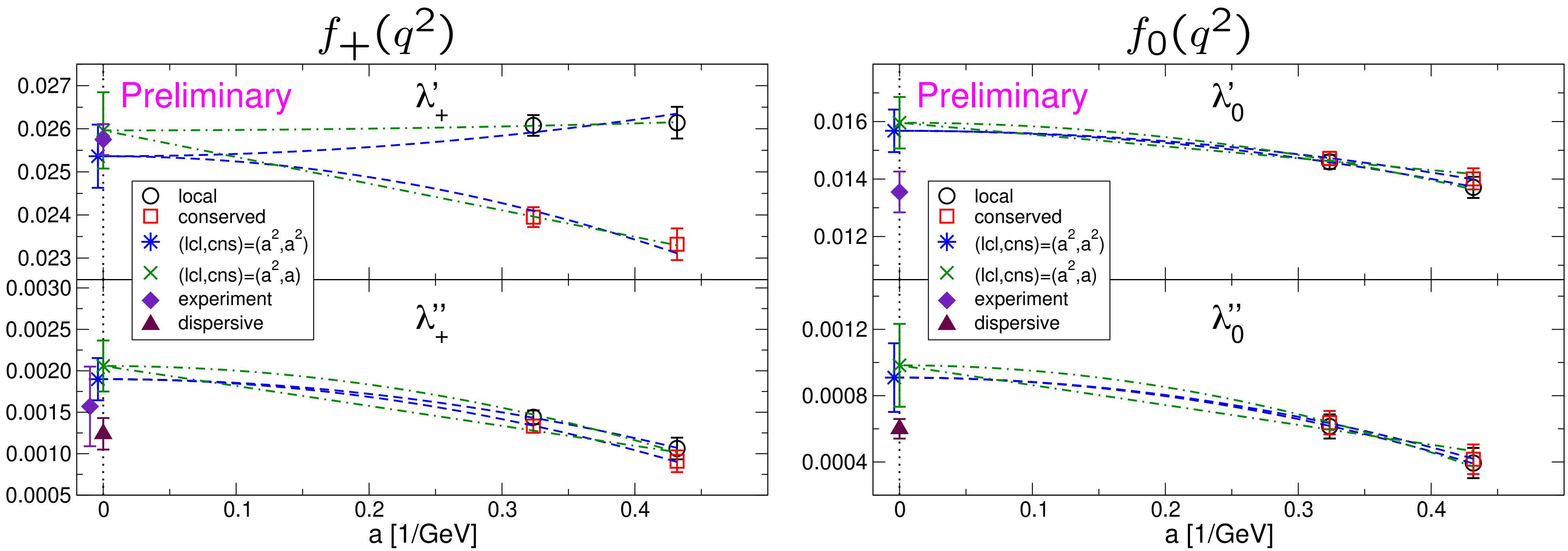
little smaller than recent lattice calculations

$|V_{us}|$ using $|V_{us}|f_+(0) = 0.21654(41)$ [Moulson:PoS(CKM2016)033(2017)]

agree with $|V_{us}|$ from K_{l2} (f_K/f_π) and SM prediction (gray band)

$\sim 2\sigma$ difference from new SM prediction (cyan band)

Result: $a \to 0$ of shape of $f_+(q^2), f_0(q^2)$ $\lambda_{+,0}^{(n)} = \frac{M_{\pi^-}^{2n}}{f_+(0)} \frac{d^n f_{+,0}(0)}{d(-q^2)^n}$



local current: flat behavior in λ'_+, but little a dependence in others

Smaller a data is important for $a \to 0$ extrapolation of λ''_+, λ'_0, λ''_0

Comparable with experiment and dispersive representation,

[Antonelli *et al*.10; Moulson17; Bernard *et al*.09]

and also previous lattice calculations of λ'_+, λ'_0 [ETM09,16; JLQCD17]

Summary

K_{l3} form factors with 2nd PACS10 configuration at $a^{-1} = 3.09$[GeV]

$L = 10.2$[fm] at physical point smaller a than [PACS:PRD101,9,094504(2020)]

- $f_+(q^2), f_0(q^2)$ in tiny q^2 region
- Clear difference in local and conserved currents
- Reasonable q^2 interpolation
- Flat(clear) a dependence for local(conserved) current for $f_+(0)$
- Preliminary result $f_+(0) = 0.9605(39)(27)$ in continuum limit

  Consistent with PACS20, little smaller than recent lattice calculations
- Slopes and curvatures: comparable with experiment and previous lattice calculations

Future works

- $a \to 0$ extrapolation of $f_+(q^2), f_0(q^2)$
- Estimate systematic uncertainties
- Calculations with 3rd PACS10 configuration in smaller a

  more reliable $a \to 0$ extrapolations