Power divergences of the quark-chromo electric dipole moment operator with the gradient flow [arXiv:2106.07633]

Jangho Kim   Thomas Luu   Matthew D. Rizik   Andrea Shindler

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SYMLAT

Outline

Introduction

- CP-violation based on the Standard Model doesn't cover the amount needed for the observed baryon asymmetry.
- New CP-violation sources are needed.
- Via the electric dipole moment (EDM) study, we can investigate the CP-violation.
- quark-chromoelectric dipole moment (qCEDM) is CP-violating operator and the one of the operators contributing to the EDM.
- Power divergences of lower dimensional operators are introduced to the qCEDM by operator mixing.
- Using gradient flow, we can control the power divergences and extract the mixing coefficient of the qCEDM operator.

Quark Chromo Electric Dipole Moment (qCEDM)

- Quark chromoelectric dipole moment (qCEDM) operator is defined by

$$O_{\mathrm{C}}^{ij}(x;t) = \overline{\chi}_i(x;t)\gamma_5\sigma_{\mu\nu}G_{\mu\nu}(x;t)\chi_j(x;t)\,, \tag{1}$$

  - i,j : flavor
  - $\sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu \gamma_\nu]$
  - $G_{\mu\nu}(x;t)$: the gauge field tensor with flowed.
  - $\chi(x;t)$, $\overline{\chi}(x;t)$: flowed quark field, $\chi(x;0) = \psi(x)$
- Mixed with P^{ij}, $\partial^2 P^{ij}$, $\frac{e(q_i+q_j)}{2}\overline{\psi}_i\gamma_5\sigma_{\mu\nu}F^{\mathrm{em}}_{\mu\nu}\psi_j$
- In this talk, we focus on the power divergence of the qCEDM which comes from the operator mixing with pseudoscalar density P^{ij}.

Short Flow Time Expansion

- By short flow time expansion [Talk by M.Rizik@BSM session,Wed 7:30AM]

$$\left[O_{\mathrm{C}}^{ij}\right]_R(x;t) \overset{t\to 0}{\sim} c_{\mathrm{CP}}(t) P_R^{ij}(x)\,, \tag{2}$$

- where $c_{\mathrm{CP}} \sim 1/t$ and the contributions comes from higher dimensional operators are neglected.
- Perturbative calculation at one-loop order[M.Rizik et al., PRD 102, 034509] gives

$$c_{\mathrm{CP}} = c_{\mathrm{CP}}^{(1)}\overline{g}^2 + O(\overline{g}^4)\,, \qquad c_{CP}^{(1)} = \frac{1}{2\pi^2 t}\,, \tag{3}$$

where $\overline{g}$ is the strong coupling renormalized at a scale $\mu = (8t)^{-1/2}$.

Lattice Correlators

- We compute

$$\Gamma_{CP}(x_4;t) = a^3 \sum_{\mathbf{x}} \left\langle O_{\mathrm{C}}^{ij}(x_4,\mathbf{x};t) P^{ji}(0,\mathbf{0};0) \right\rangle \quad (4)$$

$$\Gamma_{PP}(x_4) = \Gamma_{PP}(x_4;t=0) = a^3 \sum_{\mathbf{x}} \left\langle P^{ij}(x_4,\mathbf{x};t=0) P^{ji}(0,\mathbf{0}) \right\rangle \quad (5)$$

- By spectral decomposition,

$$\Gamma_{CP}(x_4;t) = \frac{1}{2m_\pi} \left\langle 0|O_{\mathrm{C}}^{ij}(t)|\pi \right\rangle \left\langle \pi|P^{ij}|0 \right\rangle \mathrm{e}^{-m_\pi x_4} \quad (6)$$

$$\Gamma_{PP}(x_4) = \frac{1}{2m_\pi} \left\langle 0|P^{ij}|\pi \right\rangle \left\langle \pi|P^{ij}|0 \right\rangle \mathrm{e}^{-m_\pi x_4}\,, \quad (7)$$

  where $|\pi\rangle$ is the pion state.

- For large Euclidean times x_4,

$$R_P(t) = t\frac{[\Gamma_{CP}(x_4;t)]}{[\Gamma_{PP}(x_4)]} = t\frac{\left\langle 0|\,[O_{\mathrm{C}}]_R^{ij}(t)|\pi \right\rangle}{\left\langle 0|P_R^{ij}|\pi \right\rangle}\,. \quad (8)$$

Renormalized Ratio

- By SFTE,

$$[R_P(x_4;t)]_R = t\frac{[\Gamma_{CP}(x_4;t)]_R}{[\Gamma_{PP}(x_4)]_R} = t \cdot c_{\mathrm{CP}} \text{ for small } t \tag{9}$$

- Renormalization of the pseudoscalar density and flowed any operators.

$$P_R^{ij} = Z_P P^{ij}\,, \qquad [O_{\mathrm{C}}]_R^{ij}(t) = Z_\chi O_{\mathrm{C}}^{ij}(t)\,, \tag{10}$$

where $Z_\chi^{1/2}$ is the renormalization factor of the flowed fermion field.

Strategies to determine c_{CP} (I)

- Replace the denominator with the flowed pseduscalar density

$$\Gamma_{PP}(x_4;t) = a^3 \sum_{\mathbf{x}} \left\langle P^{ij}(x_4,\mathbf{x};t) P^{ji}(0,\mathbf{0}) \right\rangle \,, \tag{11}$$

where $P^{ij}(x;t) = \overline{\chi}_i(x;t)\gamma_5\chi_j(x;t)\,.$

$$\left[\overline{R}_P(x_4;t)\right]_R = t\,\frac{[\Gamma_{CP}(x_4;t)]_R}{[\Gamma_{PP}(x_4;t)]_R}\,, \tag{12}$$

- It is possible to perform the continuum limit of $\overline{R}_P(x_4;t)$ because this is scheme independent and free of renormalization ambiguities.
- Need the expansion coefficient, c_P, of $P^{ij}(x;t) = c_{\mathrm{P}}(t)P^{ij}(x) + \mathcal{O}(t)$

$$c_{\mathrm{CP}}(t) = \frac{1}{t}\Delta(\overline{g}^2)c_{\mathrm{P}}(t) + \mathcal{O}(t)\,, \quad \Delta(\overline{g}^2) = \left[\overline{R}_P(x_4;t)\right]_R\,, \quad x_4 \gg \sqrt{8t}\,. \tag{13}$$

- We leave the determination of c_P for a future work. [Talk by C.Monahan@Theory, Wed 21:15PM]
- In this talk, we analyze $\overline{R}$ as a function of $\overline{g}$.

Strategies to determine c_{CP} (II)

- Define the effective expansion coefficient

$$c_\chi \equiv \frac{t}{Z_\chi} c_{\mathrm{CP}} = \frac{1}{Z_P} R_P(x_4; t)\,, \tag{14}$$

- We study the dependence of c_χ on the bare coupling g_0 leaving the determination of Z_χ to the future.

Lattice Setup

- Iwasaki gauge action and nonperturbatively clover-improved fermions [S.Aoki et al.,JHEP 08,101].

Ens	β	κ_l	κ_s	L/a	T/a	c_{sw}	N_G	a [fm]	m_π [MeV]	m_N [GeV]	Z_P
M_1	1.90	0.13700	0.1364	32	64	1.715	399	0.0907(13)	699.0(3)	1.585(2)	0.49605
M_2	1.90	0.13727	0.1364	32	64	1.715	400	0.0907(13)	567.6(3)	1.415(3)	0.49605
M_3	1.90	0.13754	0.1364	32	64	1.715	450	0.0907(13)	409.7(7)	1.219(4)	0.49605
A_1	1.83	0.13825	0.1371	16	32	1.761	800	0.1095(25)	710(1)	1.65(1)	0.44601
A_2	1.90	0.13700	0.1364	20	40	1.715	790	0.0936(33)	676.3(7)	1.549(6)	0.49605
A_3	2.05	0.13560	0.1351	28	56	1.628	650	0.0684(41)	660.4(7)	1.492(5)	0.51155

- N_G is the number of gauge configurations
- M_1 & M_2 & M_3 : mass dependence
- M_1 & A_2 : volume dependence
- A_1 & A_2 & A_3 : lattice spacing dependence

Scale Setting

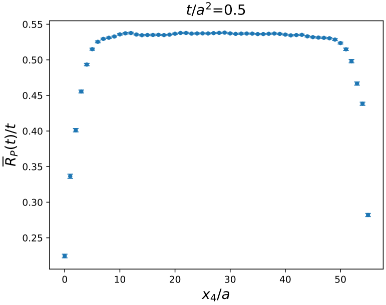
- Here we present the $t_0^{(M)}/a^2$ values with improvements in [Z.Fodor et al.,JHEP09,018], where M is the order of the improvements.

$$\left(t_0^{(M)}\right)^2 \left\langle E(t_0^{(M)})\right\rangle_{\rm lat} \times \frac{1}{C(t_0^{(M)})} = 0.3\,, \tag{15}$$

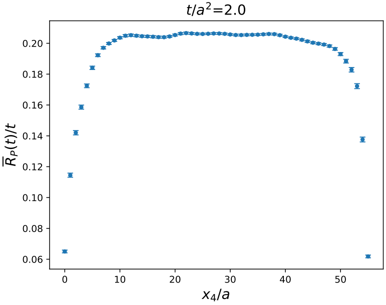
$$C(t) = 1 + \sum_{m=1}^{M} C_m \left(\frac{a^2}{t}\right)^m \,. \tag{16}$$

Designation	$t_0^{(0)}/a^2$	$t_0^{(1)}/a^2$	$t_0^{(2)}/a^2$	$t_0^{(3)}/a^2$	$t_0^{(4)}/a^2$
M_1	2.2586(12)	2.1344(12)	2.1723(12)	2.1655(12)	2.1680(12)
M_2	2.3993(12)	2.2739(11)	2.3094(11)	2.3033(11)	2.3054(11)
M_3	2.5371(15)	2.4088(15)	2.4435(15)	2.4379(15)	2.4397(15)
A_1	1.3627(15)	1.2397(15)	1.3028(14)	1.2849(15)	1.2951(14)
A_2	2.2378(24)	2.1145(23)	2.1526(23)	2.1457(23)	2.1482(23)
A_3	4.9879(65)	4.8652(64)	4.8815(64)	4.8802(64)	4.8804(64)

Analysis in terms of renormalized coupling



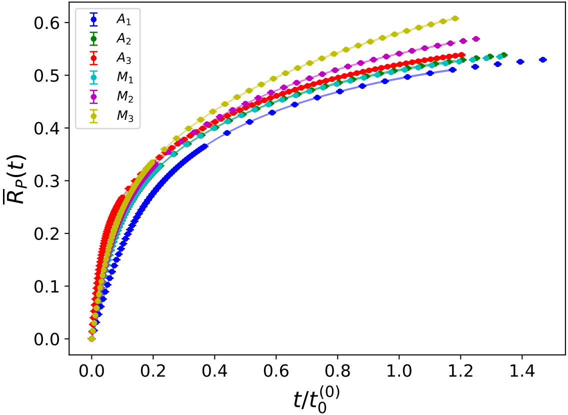
(a) t=0.5



(b) t=2.0

- $\overline{R}_P(t)/t$ on the ensemble A_3 for values of the flow time $t/a^2 = 0.5, 2.0$, corresponding to a flow time radius $r_f = \sqrt{8t} = 2a, 4a$.

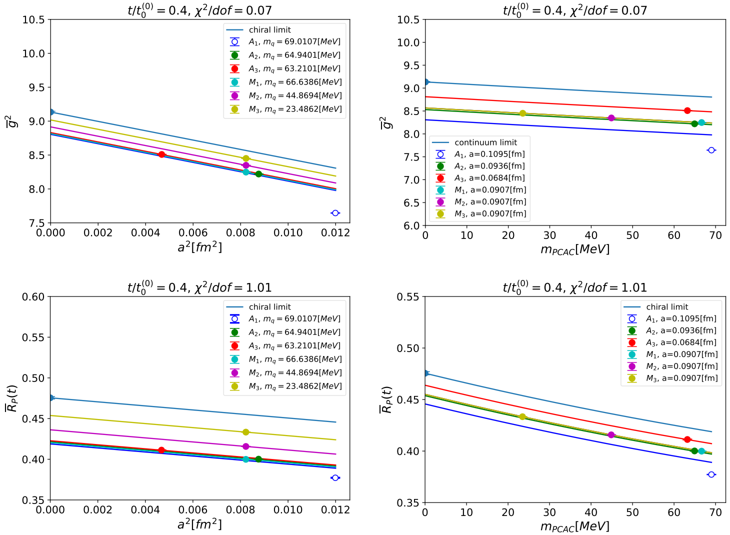
Flow time dependence of the $\overline{R}_P$



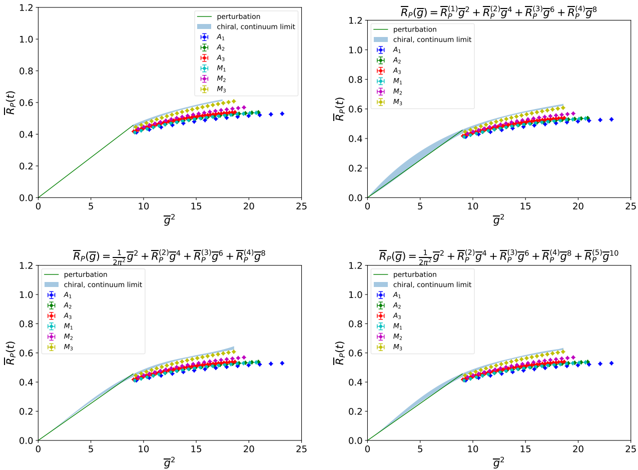
- Flow time dependence of the $\overline{R}_P$ for all our ensembles.
- Interpolation with a cubic spline.
- Perform the continuum & chiral extrapolation at fixed values of $t/t_0^{(M)}$, $M = 0, 1, 2, 3, 4$ for the renormalized coupling $\overline{g}$ and $\overline{R}$.

Continuum & chiral extrapolation

- $f = A(t) + B(t)a^2 + C(t)m_q + D(t)m_q^2$



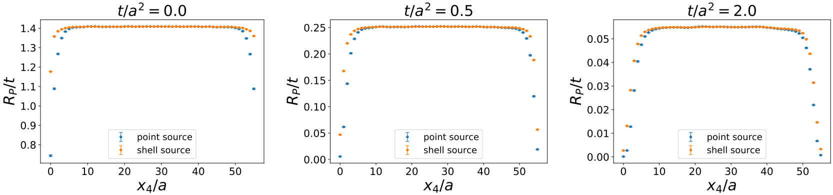
Comparison with perturbative result



- This curve is universal and renormalization group invariant.
- It can be applied to any determination of the expansion coefficient of the pseudoscalar density and to any corresponding hadronic matrix element.

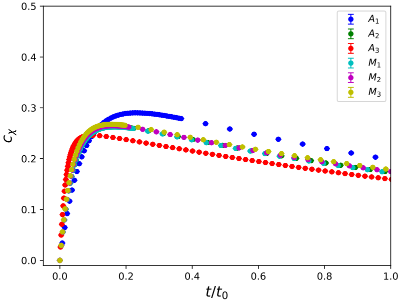
Analysis in terms of bare coupling

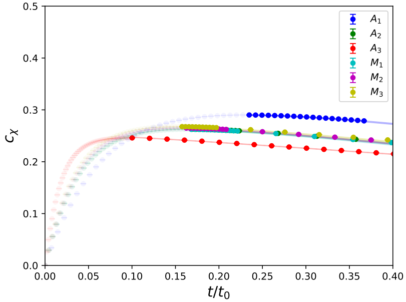
$$c_\chi \equiv \frac{t}{Z_\chi} c_{\rm CP} = \frac{1}{Z_P} R_P(x_4;t) = \frac{t}{Z_P} \frac{a^3 \sum_{\mathbf{x}} \left\langle O_{\rm C}^{ij}(x_4,\mathbf{x};t) P^{ji}(0,\mathbf{0};0) \right\rangle}{a^3 \sum_{\mathbf{x}} \langle P^{ij}(x_4,\mathbf{0}) P^{ji}(0,\mathbf{0}) \rangle} \tag{17}$$



- The ratio R_P/t on the ensemble A_3 for several values of the flow time $t/a^2 = 0, 0.5, 2.0$, corresponding to a flow time radius $r_f = \sqrt{8t} = 0, 2a, 4a$.
- We perform a simple constant fit for plateau.

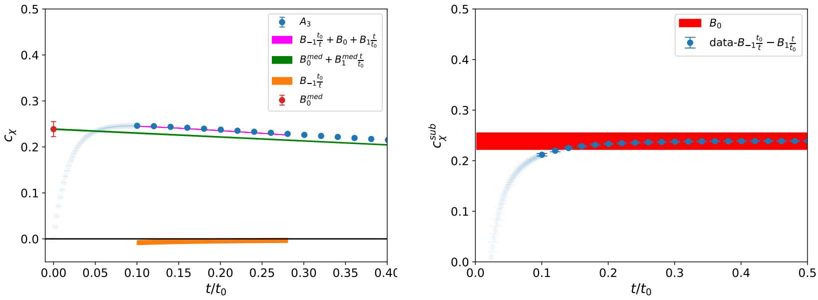
Flow time dependence of $c_\chi = \frac{R_P}{Z_P} = \frac{t}{Z_\chi} c_{\text{CP}}$





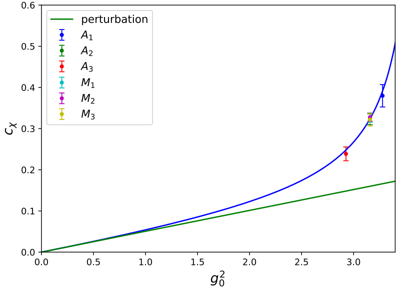
- The left plot is the ratio R_P as a function of t/t_0.
- Perform a simple fit using $R_{\text{fit}}(t) = B_{-1}\frac{t_0}{t} + B_0 + B_1\frac{t}{t_0}$
- B_{-1} parametrizes O(a^2/t) effects. B_1 parametrizes effects from higher dimensional operators. B_0 provides the value of c_χ.
- In the right plot, error bands are reconstructed based on the final fitting results and solid data points are belonged in the selected fitting ranges.

Flow time dependence of c_χ for the ensemble A_3



- Magenta: example of a single fit satisfying the p-value condition.
- Orange: the contribution of the cutoff effect, B_{-1}.
- Green: the fit result obtained removing cutoff effects.
- Red data point: the final value of B_0, the error represents the sum in quadrature of the statistical and systematic uncertainties on B_0.
- Right: The blue data points represent the raw data after subtracting the cutoff effects and the higher dimensional operator contributions.

Nonperturbative dependence of c_χ as a function of the bare coupling g_0



- Green line is one-loop perturbation theory.

$$c_\chi = c_\chi^{(1)} g^2 + O(g^4)\,, \quad c_\chi^{(1)}(t) = \frac{1}{2\pi^2}\,. \tag{18}$$

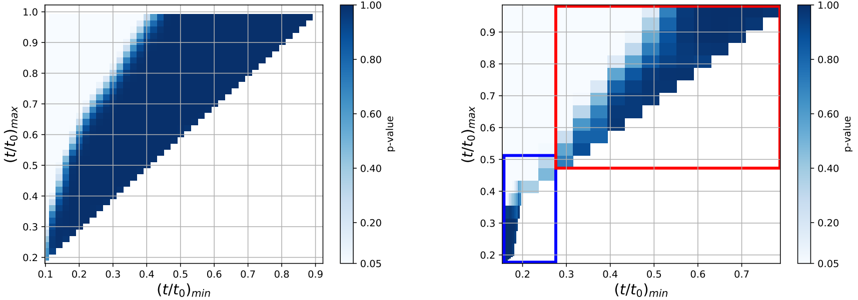
- The blue band represents the Padé approximant.

$$c_\chi(g_0^2) = \frac{\frac{1}{2\pi^2} g_0^2 + c_2 g_0^4}{1 + c_4 g_0^2}\,, \quad c_2 = -0.01115(63)\,, \quad c_4 = -0.2690(61) \tag{19}$$

Summary

- We have analyzed the quark chromo-EDM operator using the gradient flow method and determine the effective mixing coefficient nonperturbatively.
- Once the expansion parameter of pseudoscalar density is determined, it is possible to determine the leading and subleading logarithmic corrections of power divergences nonperturbatively and in the continuum.
- We plan to expand this study to stabilized Wilson fermion ensembles which have larger volumes and smaller pion masses (200MeV, 300MeV 400MeV) to reduce the uncertainty comes from continuum and chiral extrapolations. [Talks by A.Francis and G.Pederiva@Hadron Spectroscopy Wed 13:30 & 13:45PM]

Backup

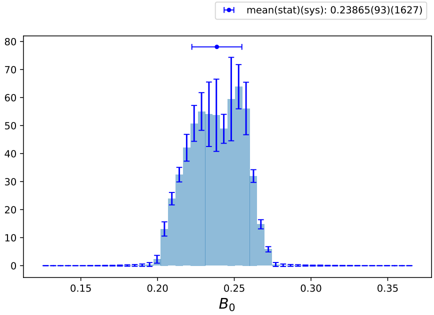


(c) A_3 ensemble

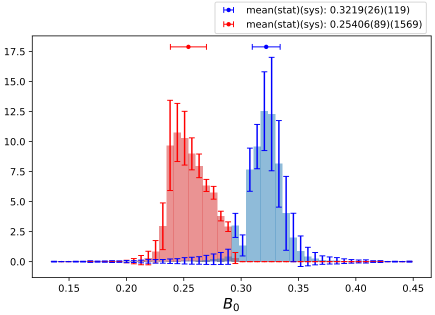
(d) M_3 ensemble

- p-values obtained from the $\chi^2/\text{d.o.f.}$
- The darker regions correspond to the acceptable fit ranges.

Histogram analysis



(e) A_3 ensemble



(f) M_3 ensemble

- Distribution of the fit parameter B_0 for all the fit ranges satisfying $p > 0.05$.