Vacuum correlators at short distances from lattice QCD

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Short-distance contributions

Many observables involve correlators integrated over *all* separations x_0
$\Rightarrow$ large cutoff effects[1] ?

e.g. the HVP in terms of the current correlator

$$G(x_0) = -\int d^3x \, \langle J_1(x) J_1(0) \rangle, \qquad J_\mu = \sum_f Q_f \bar{\psi}_f \gamma_\mu \psi_f$$

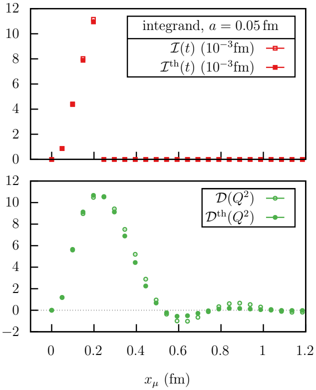
Consider two short-distance (SD) observables

- the SD contribution to the HVP $\boxed{t \sim 0.2\,\text{fm}}$

$$I(t) = \int_0^t dx_0 \, x_0^4 G(x_0)$$

- the Adler function at large virtuality $Q \sim 2.4\,\text{GeV}$

$$D(Q^2) = \int_0^\infty dx_0 \, K(x_0, Q^2) G(x_0)$$



[1]Della Morte et al. 2009.

Key idea

SD observables are insensitive to the temperature

Define the **thermal** observable at finite temperature T in the x_3 direction

$$I^{\mathrm{th}}(t) = \int_0^t \mathrm{d}x_3\, x_3^4 G^{\mathrm{th}}(x_3)$$

For SD observables $tT \ll 1$ apply the OPE

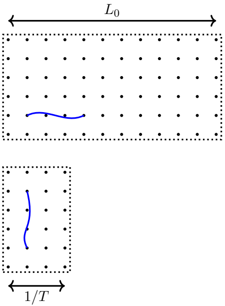
$$\frac{I - I^{\mathrm{th}}}{I} \sim (tT)^3 + \dots$$

while the lattice artifacts on the estimators $\mathcal{I}$ and $\mathcal{I}^{\mathrm{th}}$ are

$$\mathcal{I}^{\mathrm{th}} = I^{\mathrm{th}} + \mathrm{O}(a^2)$$
$$\mathcal{I} - \mathcal{I}^{\mathrm{th}} = I - I^{\mathrm{th}} + \mathrm{O}((tT)^3 a^2)$$

$\Rightarrow$ suggests decomposition

$$\mathcal{I} = \underbrace{\mathcal{I}^{\mathrm{th}}}_{\text{thermal}} + \underbrace{\left[\mathcal{I} - \mathcal{I}^{\mathrm{th}}\right]}_{\text{correction}}$$



Choosing $t \ll 1/T \ll L_0$

$\Rightarrow$ thermal effects suppressed and factor $L_0 T \gg 1$ cheaper

Lattice artifacts at LO

Short-distance quantity $\Rightarrow$ lattice perturbation theory

LO with $N_\mathrm{f} = 2$ free massless Wilson fermions
$\rightsquigarrow$ automatic $\mathrm{O}(a)$-improvement

We find a logarithmically-enhanced $\mathrm{O}(a^2)$ artifact at LO

$$\mathcal{I} = I + c_\mathcal{I} a^2 \log(1/a) + \mathrm{O}(a^2)$$

where

$$c_\mathcal{I} = \frac{7N_\mathrm{c}}{60\pi^2}$$

Power-counting suggests[2] for $x_0\Lambda \ll 1$ the cutoff effects on the lattice correlator

$$\mathcal{G}(x_0) = G(x_0) + a^2 \frac{c_2}{x_0^5} + \dots$$

c.f. NLO logarithms[3]

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[2] Michele Della Morte et al. "On cutoff effects in lattice QCD from short to long distances". In: *Phys. Lett. B* 672 (2009), pp. 407–412. arXiv: 0807.1120 [hep-lat].

[3] Nikolai Husung et al. "Asymptotic behavior of cutoff effects in Yang–Mills theory and in Wilson's lattice QCD". In: *Eur. Phys. J. C* 80.3 (2020), p. 200. arXiv: 1912.08498 [hep-lat].

Continuum limit of thermal at LO

"Set the scale" by setting $T = 248\,\mathrm{MeV}$

$$\Rightarrow \begin{cases} 1/aT = N_\mathrm{t} = 12, 16, 20, 24 \\ a = 0.03 - 0.06\,\mathrm{fm} \end{cases}$$

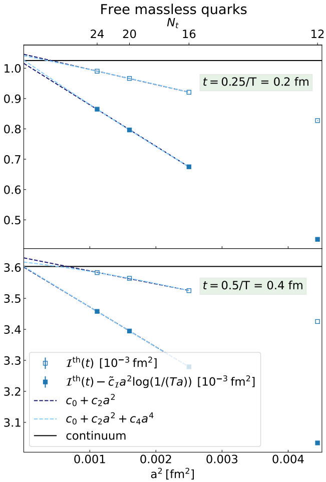
Study $t = 1/4T = 0.2\,\mathrm{fm}$ and $t = 1/2T = 0.4\,\mathrm{fm}$

Different Anšatze for continuum limit of $\mathcal{I}^{\mathrm{th}}$

- use a^2 or $a^2 + a^4$
- with/without subtracting log

$\Rightarrow$ log important for precise continuum limit

extrapolation error $2\% \to 0.2\%$ for $t = 0.2\,\mathrm{fm}$



Continuum limit of correction at LO

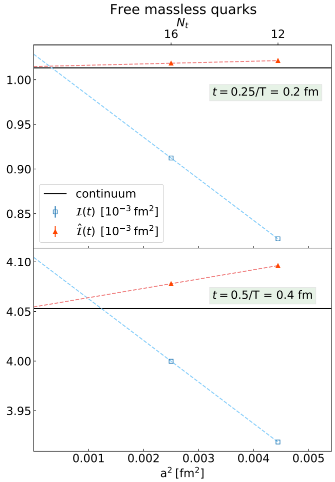
Define improved vacuum estimator

$$\widehat{\mathcal{I}} = I^{\rm th}_{\rm extrap.} + [\mathcal{I} - \mathcal{I}_{\rm th}]$$

where $I^{\rm th}_{\rm extrap.}$ computed using fine thermal lattices

$N_{\rm t} = 12, 16$ for the expensive vacuum correction

- original $\mathcal{I}$ has long extrapolation
- improved $\widehat{\mathcal{I}}$ has flatter behaviour



Study with $N_\mathrm{f} = 2$ Wilson fermions

Same strategy as for LO computation with $N_\mathrm{f} = 2$ $\mathrm{O}(a)$-improved Wilson fermions[4]

(currents are **not** $\mathrm{O}(a)$-improved)

However, SD quantity and...

Above T_c no $\chi\mathrm{SB} \rightsquigarrow \mathrm{O}(am_\mathrm{q})$ cutoff effects[5]

	L/a	N_t	L_0/a	$a\,(\mathrm{fm})$	$6/g_0^2$	N_conf
F7	48	12	96	0.0658	5.3	482
O7	64	16	128	0.049	5.5	305
W7	80	20	–	0.039	5.685 727	1566
X7	96	24	–	0.033	5.827 16	511

thermal is a factor $TL_0 = 8$ cheaper!

[4] CLS ensembles

[5] Mattia Dalla Brida et al. "Non-perturbative definition of the QCD energy-momentum tensor on the lattice". In: *JHEP* 04 (2020), p. 043. arXiv: 2002.06897 [hep-lat]; Bastian B. Brandt et al. "An estimate for the thermal photon rate from lattice QCD". In: *EPJ Web Conf.* 175 (2018). Ed. by M. Della Morte et al., p. 07044. arXiv: 1710.07050 [hep-lat].

Continuum limit in $N_\mathrm{f} = 2$ QCD

Exclude $N_\mathrm{t} = 12$ lattice for $\mathcal{I}^{\mathrm{th}}$

Also use LO result to improve thermal limit

$$\mathring{\mathcal{I}} = \mathcal{I}^{\mathrm{th}} - \left[\mathcal{I}^{\mathrm{th}} - I^{\mathrm{th}}\right]_{\mathrm{LO}}$$

- thermal contribution benefits from fine lattices
- improved has flatter continuum limit than original

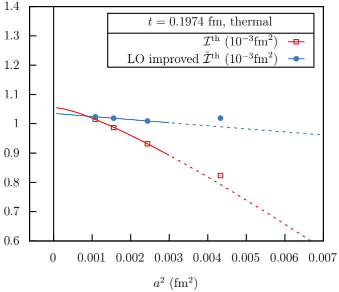
Final result

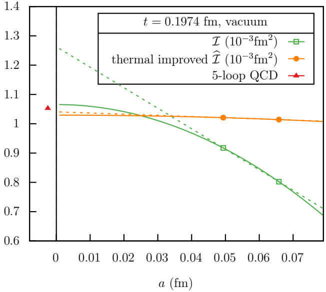
$$I(t = 0.1974\,\mathrm{fm}) = 1.035(9)_{\mathrm{stat}}(19)_{\mathrm{sys}} \times 10^{-3}\mathrm{fm}^2$$

compared with 5-loop perturbative result

$$I_{\mathrm{pert.}}(t = 0.1974\,\mathrm{fm}) = 1.059^{+1}_{-6} \times 10^{-3}\mathrm{fm}^2$$

(similar picture for the Adler function)





Running of $\alpha_{\rm em}$ at large Q^2

Can we compute running of $\alpha_{\rm em}$ at the Z pole?

The discrete running $\Delta_2\Pi(Q^2) = \Pi(Q^2) - \Pi((Q/2)^2)$ is a SD observable for large Q^2

Invoking the OPE as before

$$\Delta_2\Pi(Q^2) - \Delta_2\Pi^{\rm th}(Q^2, T) \sim \text{const.} \times \left(\frac{\pi T}{Q}\right)^4$$

so that a good estimate could be obtained by computing

$$\underbrace{\Delta_2\Pi^{\rm th}(Q^2, T)}_{\text{thermal (fine)}} - \underbrace{\left[\Delta_2\Pi^{\rm th}(Q^2, T/2) - \Delta_2\Pi^{\rm th}(Q^2, T)\right]}_{\text{correction (coarse or PT)}}$$

with $T = Q/8\pi$, c.f. step-scaling[6]

Confirmed at LO achievable with 48×192^3 lattices

[6] Giulia Maria de Divitiis et al. "Heavy quark masses in the continuum limit of quenched lattice QCD". In: *Nucl. Phys. B* 675 (2003), pp. 309–332. arXiv: hep-lat/0305018; Martin Luscher et al. "A Numerical method to compute the running coupling in asymptotically free theories". In: *Nucl. Phys. B* 359 (1991), pp. 221–243.

Summary

- For SD contributions, use thermal lattices
  - study using OPE (up to NLO!)
  - confirmed at LO in QCD
- Logarithmically-enhanced $\mathrm{O}(a^2)$ lattice artifact
- Improved estimators for SD HVP/Adler function
  - with free massless Wilson fermions
  - with $N_\mathrm{f} = 2$ Wilson fermions

  Using only lattices $a \geq 0.05\,\mathrm{fm}$
  $\rightsquigarrow$ might overestimate SD HVP $g-2$ by 0.4×10^{-10}
- Proposal for the running of α_em at large energies
  $\rightsquigarrow$ link Q to T (c.f. step-scaling)