Calculation of neutron electric dipole moment due to the QCD topological term, Weinberg three-gluon operator and the quark chromoelectric moment

Tanmoy Bhattacharya

Vincenzo Cirigliano **Rajan Gupta** **Emanuele Metreghetti** **Boram Yoon**

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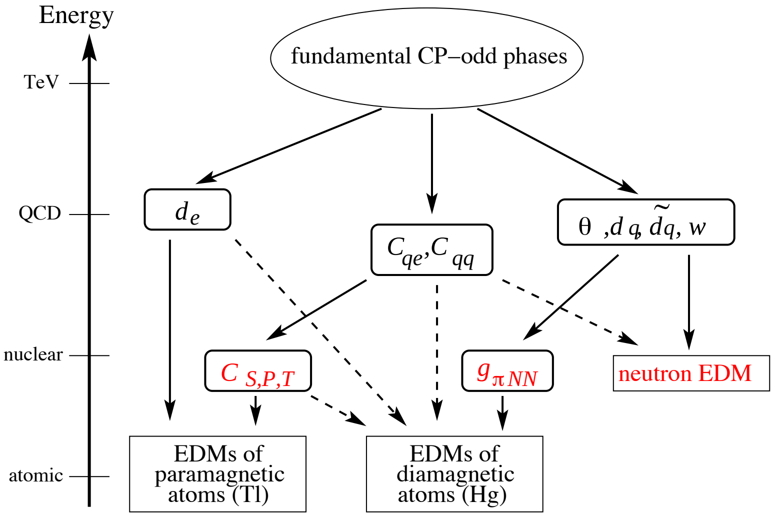
Introduction

Introduction
Effective Field Theory



Pospelov and Ritz, *Ann. Phys.* **318** (2005) 119.

Introduction
BSM Operators

Standard model CP violation in the weak sector.
Strong CP violation from dimension 3 and 4 operators anomalously small.

- Dimension 3 and 4:
  - CP violating mass $\bar{\psi}\gamma_5\psi$.
  - Toplogical charge $G_{\mu\nu}\tilde{G}^{\mu\nu}$.
- Suppressed by $v_{\mathrm{EW}}/M^2_{\mathrm{BSM}}$:
  - Electric Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{F}^{\mu\nu}\psi$.
  - Chromo Dipole Moment $\bar{\psi}\Sigma_{\mu\nu}\tilde{G}^{\mu\nu}\psi$.
- Suppressed by $1/M^2_{\mathrm{BSM}}$:
  - Weinberg operator (Gluon chromo-electric moment): $G_{\mu\nu}G_{\lambda\nu}\tilde{G}_{\mu\lambda}$.
  - Various four-fermi operators.

Introduction
Form Factors

Vector form-factors

Dirac F_1, Pauli F_2, Electric dipole F_3, and Anapole F_A

Sachs electric $G_E \equiv F_1 - (q^2/4M^2)F_2$ and magnetic $G_M \equiv F_1 + F_2$

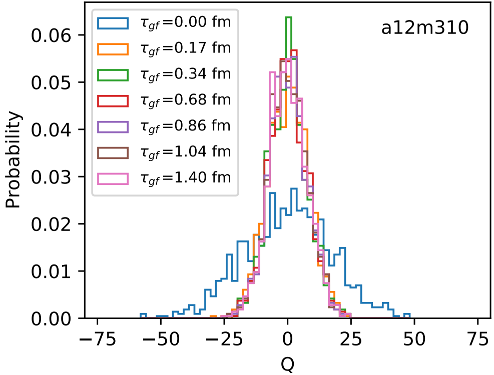
$$\begin{aligned}\langle N|V_\mu(q)|N\rangle &= \overline{u}_N\left[\gamma_\mu\, F_1(q^2) + i\frac{[\gamma_\mu,\gamma_\nu]}{2}q_\nu\,\frac{F_2(q^2)}{2m_N}\right.\\ &\quad + (2i\, m_N\gamma_5 q_\mu - \gamma_\mu\gamma_5 q^2)\,\frac{F_A(q^2)}{m_N^2}\\ &\quad \left. + \frac{[\gamma_\mu,\gamma_\nu]}{2}q_\nu\gamma_5\,\frac{F_3(q^2)}{2m_N}\right]u_N\end{aligned}$$

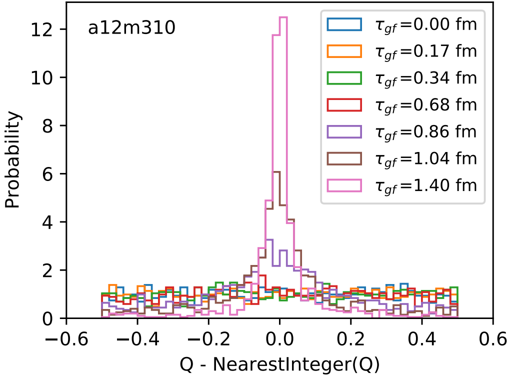
- In general $\langle\Omega|\hat{N}|N\rangle \propto e^{i\alpha_N\gamma_5}u_N$; CP implies $\mathrm{Im}\,\alpha_N = 0$, PT implies $\mathrm{Re}\,\alpha_N = 0$.
- F_A violates PT; F_3 violates CP.

QCD Topological Term

QCD Topological Term
Gradient Flow

A small amount of gradient flow smears away the ultraviolet fluctuations.
Charge distribution stabilizes fast, doesn't become an integer till later.





QCD Topological Term
Susceptibility: χPT

Witten-Veneziano relation, modified by SU(3) breaking:

$$\chi_Q^{\text{quench.}} \approx \frac{F_\pi^2 (M_{\eta'}^2 - M_\eta^2)}{6} \left(1 + 2\frac{M_\eta^2 - M_K^2}{M_{\eta'}^2 - M_\eta^2}\right)$$

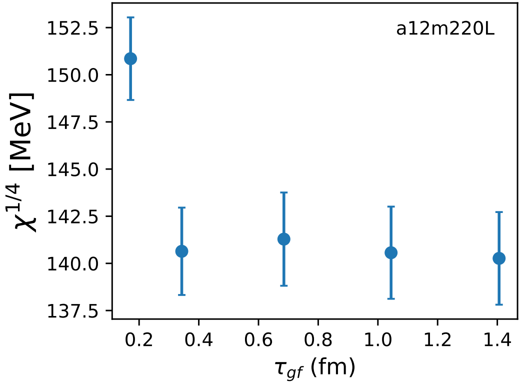
gives $\chi_Q^{\text{quench.}} \approx (179\ \text{MeV})^4$. With dynamical quarks this decreases to

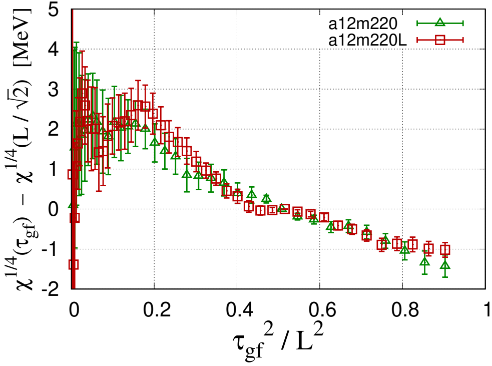
$$\frac{1}{\chi_Q} \approx \frac{1}{\chi_Q^{\text{quench.}}} + \frac{4}{M_\pi^2 F_\pi^2}\left(1 - \frac{M_\pi^2}{3M_\eta^2}\right)^{-1}$$

which gives $\chi_Q \approx (79\ \text{MeV})^4$.

QCD Topological Term
Susceptibility: Flow-time independence

Topological charge in gradient-flow scheme not renormalized.
Lattice data for topological susceptibility independent of flow-time up to finite volume effects.



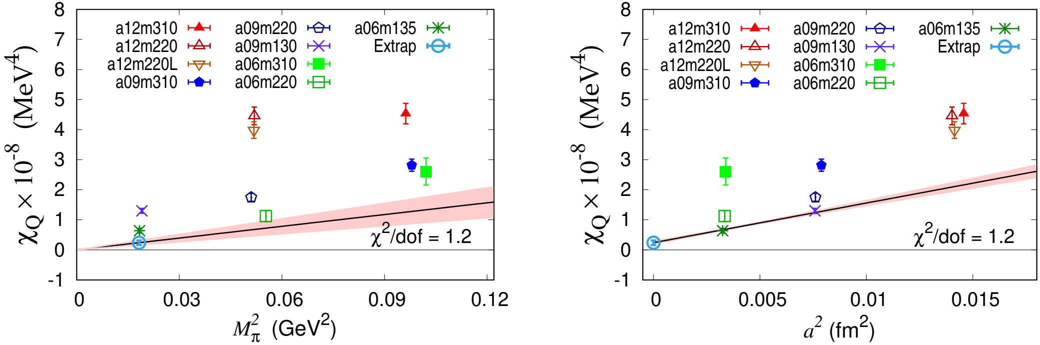


QCD Topological Term
Susceptibility: Chiral-Continuum Fits

Fit form:

$$\chi_Q \approx c_1 a^2 + c_2 M_\pi^2 + c_3 a^2 M_\pi^2$$

Neglecting coarse-lattice $a \approx 0.12$ fm ensembles.



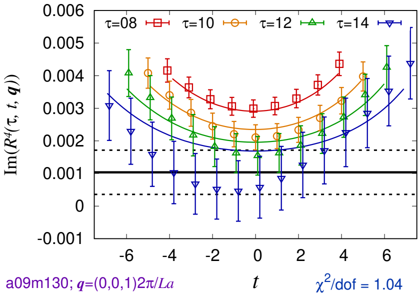
May also need to neglect a06m310 ensemble where topology may be frozen.
Our final result with systematics included $\chi_Q = (66(9)(4)\ \mathrm{MeV})^4$.

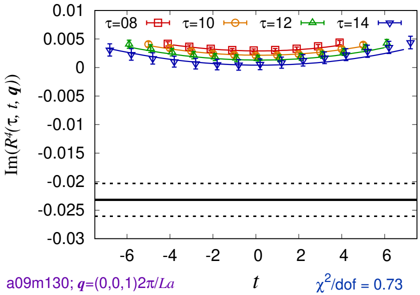
QCD Topological Term
3-pt functions: Excited States

Exponential fits not very sensitive to low-lying excited states:

$$A + Be^{-\Delta t} \approx (A+B) - (B\Delta)t \qquad \text{for} \qquad t \ll \Delta^{-1}\,.$$

Extrapolated result very sensitive to low-lying excited states:

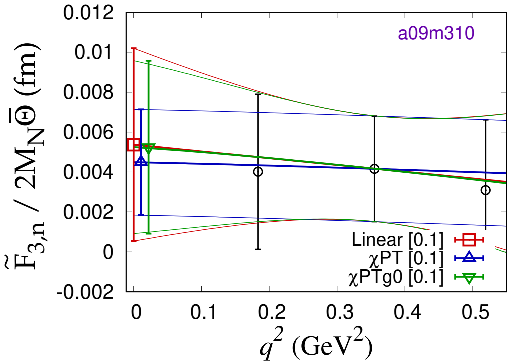




Chiral perturbation theory expects $N\pi$ intermediate state.

QCD Topological Term
Q^2 Extrapolation

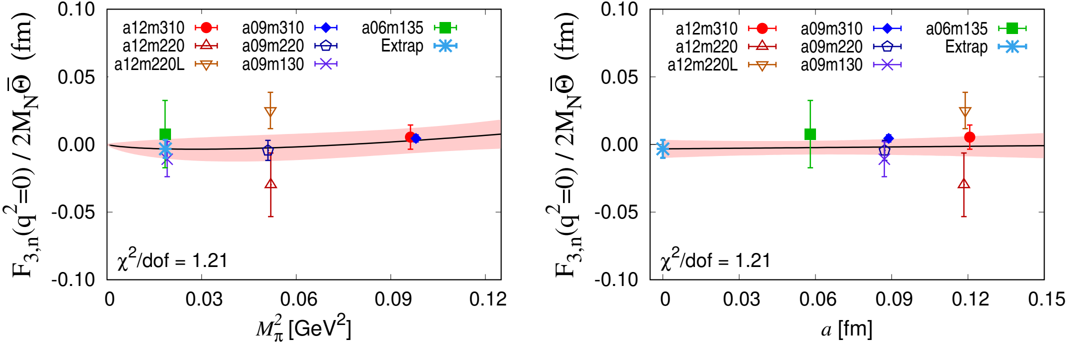
Chiral perturbation theory provides guidance for Q^2 fits.



Linear fits or fits holding the coefficient of the chiral log free make small differences.

QCD Topological Term
Chiral-Continuum Extrapolation

Chiral perturbation theory provides functional form for M_π dependence.



$$d_n = -0.003(7)(20)\ \overline{\Theta} e \cdot \mathrm{fm} \qquad d_p = 0.024(10)(30)\ \overline{\Theta} e \cdot \mathrm{fm} \qquad \text{(Standard analysis)}$$

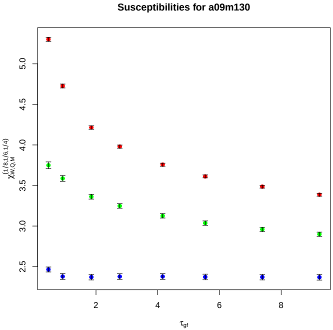
$$d_n = -0.028(18)(54)\ \overline{\Theta} e \cdot \mathrm{fm} \qquad d_p = 0.069(25)(120)\ \overline{\Theta} e \cdot \mathrm{fm} \qquad \text{(Assuming } N\pi \text{ state)}$$

Weinberg Three-Gluon Operator

Weinberg Three-Gluon Operator
Gradient Flow

Under gradient flow, Weinberg operator

- Has a finite limit at $a \to 0$
- Gets $1/t$ mixing with topological charge, and
- $O(t)$ mixing with higher dimension operators.

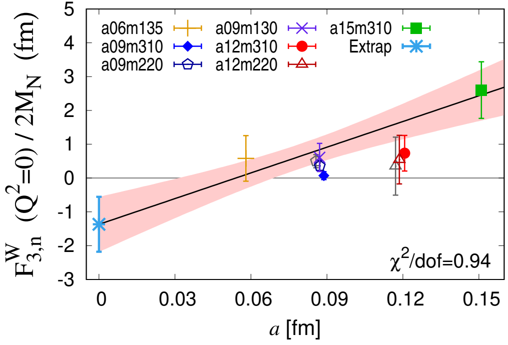


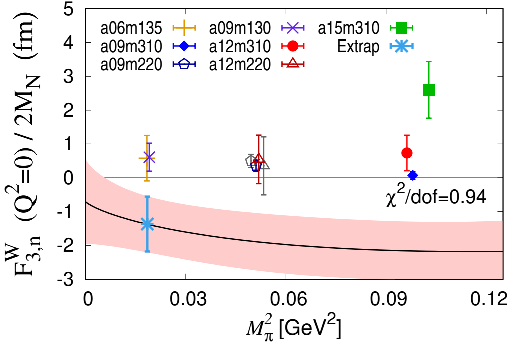
- Topological susceptibility flow-time independent.
- Weinberg and Mixed susceptibilities have power-law dependence on flow-time.

Weinberg Three-Gluon Operator

F_3

Weinberg operator mixes with topological charge: handle after continuum extrapolation.
Product of vector current and Weinberg operators mixes with quark-edm current.
Showing preliminary data for bare operators:





Is $a \approx 0.15$ fm too coarse?

Quark Chromoelectric Dipole Moment

Quark Chromoelectric Dipole Moment
Renormalization and mixing

Chromo-edm operator has power-divergent mixing with the pseudoscalar operator.

$$\tilde{C} \equiv i\bar{\psi}\sigma^{\mu\nu}\gamma_5 G_{\mu\nu} T^a \psi - \frac{iA}{a^2}\bar{\psi}\gamma_5 T^a \psi$$

has only logarithmic mixing for on-shell zero four-momentum quantities.
At zero four-momentum isovector P can be rotated away using non-anomalous Ward identity! For Wilson fermions, this leaves behind an $O(a)$ piece.

$$Z_A(1 + b_A ma)\partial \cdot A + iaZ_A c_A \partial^2 P + 2miP - iaK\tilde{C} \sim 0$$

A can be fixed by demanding, for example, $\langle \Omega | \tilde{C} | \pi(\vec{p} = 0) \rangle = 0$. Can also use pion correlators to fix K and express all these operators in terms of $\tilde{C}$ nonperturbatively.

Quark Chromoelectric Dipole Moment
Ward identity

$$\frac{ia\Delta_4\langle\pi|A_4\rangle - \bar{c}_A a^2\Delta_4^2\langle\pi|P\rangle + \bar{x}a^2\langle\pi|C\rangle}{\langle\pi|P\rangle} \approx \bar{y}$$

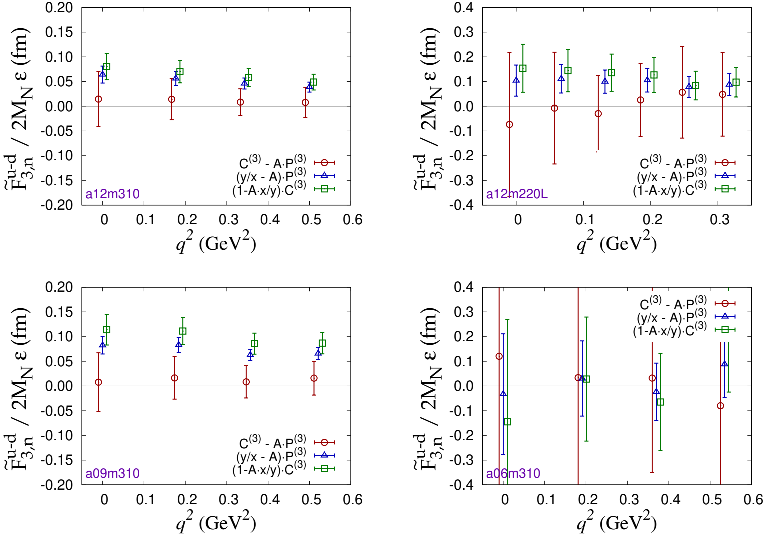
We can determine $\bar{x}$ and $\bar{y}$ by fitting the LHS to a constant. $\bar{c}_A \approx 0$ and is neglected. Then, the three isovector operators

$$\left(aC - A\frac{P}{a}\right)$$

$$\left(\frac{\bar{y}}{\bar{x}} - A\right)\frac{P}{a}$$

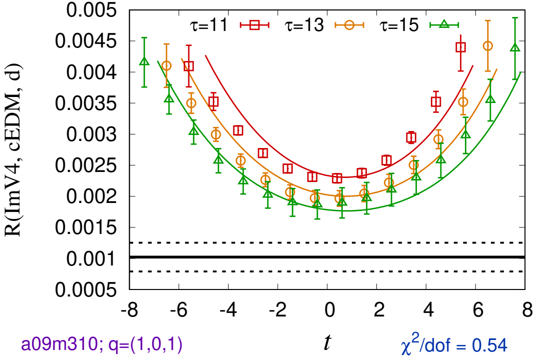
$$\left(1 - \frac{A\bar{x}}{\bar{y}}\right)aC$$

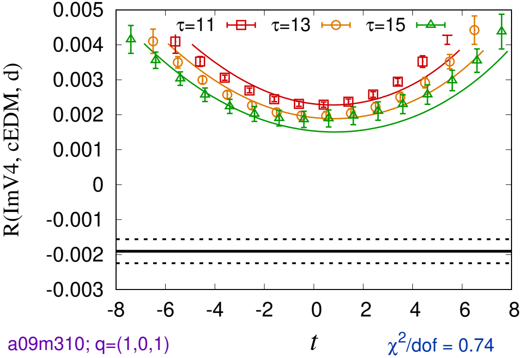
describe the same physics!


$\tilde{F}_{3,n}^{u-d}$ / $2M_N$ ε (fm)
q^2 (GeV^2)
$C^{(3)}$ - A·$P^{(3)}$
(y/x - A)·$P^{(3)}$
(1-A·x/y)·$C^{(3)}$
a12m310
a12m220L
a09m310
a06m310

Quark Chromoelectric Dipole Moment
Excited States

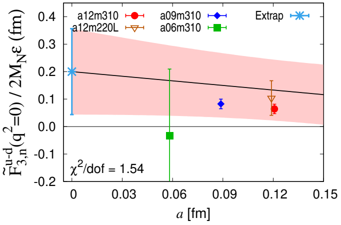
Assuming $N\pi$ intermediate state again makes a big difference.

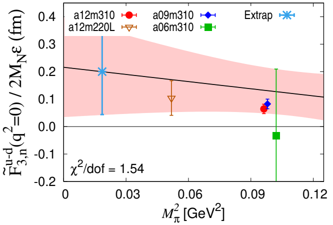




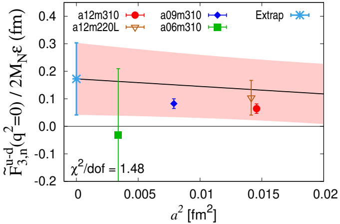
Quark Chromoelectric Dipole Moment
Extrapolation: very preliminary results

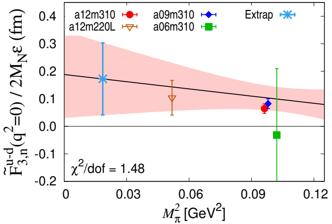
$$d_N[q^2\text{-Linear}] = c_1 + c_2 M_\pi^2 + c_3 a = 0.20(16) \quad [\chi^2/\text{dof} = 0.28]$$





$$d_N[q^2\text{-Linear}] = c_1 + c_2 M_\pi^2 + c_3 a^2 = 0.17(13) \quad [\chi^2/\text{dof} = 0.26]$$





Conclusions

Conclusions
Major take aways

- No determination yet for nEDM from topological term.
- Gradient flow good way of studying topological quantities.
- Ruling out low-lying excited states very important to get results.
- Power-divergence in isovector chromo-EDM under control.