BSM $B_{(s)} - \bar{B}_{(s)}$ mixing on domain-wall lattices

Peter Boyle, Luigi Del Debbio, **Felix Erben**, Andreas Jüttner,
Takashi Kaneko, Michael Marshall, Antonin Portelli,
J Tobias Tsang, Oliver Witzel
RBC/UKQCD and JLQCD

28 July 2021

THE UNIVERSITY of EDINBURGH

European Research Council
Established by the European Commission

LATTICE 21
JULY 26-30 2021, ZOOM/GATHER@MIT

RBC/UKQCD collaboration

The RBC & UKQCD collaborations

UC Berkeley/LBNL
Aaron Meyer

BNL and BNL/RBRC
Yasumichi Aoki (KEK)
Peter Boyle (Edinburgh)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Christopher Kelly
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

CERN
Andreas Jüttner (Southampton)

Columbia University
Norman Christ
Duo Guo
Yikai Huo
Yong-Chull Jang
Joseph Karpie
Bob Mawhinney
Ahmed Sheta
Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut
Tom Blum
Luchang Jin (RBRC)
Michael Riberdy
Masaaki Tomii

Edinburgh University
Matteo Di Carlo
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tim Harris
Raoul Hodgson
Nelson Lachini
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
James Richings
Azusa Yamaguchi
Andrew Z.N. Yong

KEK
Julien Frison

University of Liverpool
Nicolas Garron

Michigan State University
Dan Hoying

Milano Bicocca
Mattia Bruno

Peking University
Xu Feng

University of Regensburg
Davide Giusti
Christoph Lehner (BNL)

University of Siegen
Matthew Black
Oliver Witzel

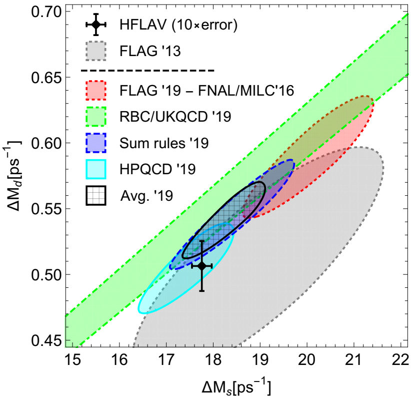
University of Southampton
Nils Asmussen
Alessandro Barone
Jonathan Flynn
Ryan Hill
Rajnandini Mukherjee
Chris Sachrajda

University of Southern Denmark
Tobias Tsang

Stony Brook University
Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Motivation

- $B_{(s)} - \bar{B}_{(s)}$ mixing gives access to CKM matrix elements $|V_{ts}|$ and $|V_{td}|$
- current tension between ΔM_d, ΔM_s lattice determinations
  - RBC/UKQCD 2019 result is missing renormalization factors
- mass splitting with SM described by $VV + AA$ flavour-changing current
  - $\Rightarrow$ 4 additional currents interesting for some SM extensions



[Di Luzio et al. arxiv 1909.11087]

- Data produced on RBC-UKQCD and JLQCD ensembles using Grid and Hadrons.

GRID

`[github.com/paboyle/Grid]`

Hadrons

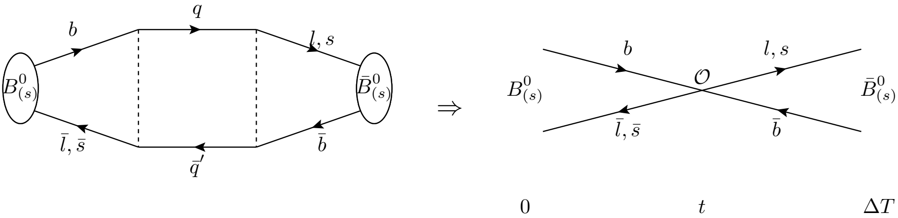
`[github.com/aportelli/Hadrons]`

Related talks by RBC/UKQCD and JLQCD

Related RBC/UKQCD and JLQCD talks (US/Eastern time):

- Takashi Kaneko: $B \to D^{(*)}\ell\nu$ semileptonic decays in lattice QCD with domain-wall heavy quarks [Mon 21:15]
- Shoji Hashimoto: Composition of the inclusive semi-leptonic decay of B meson [Wed 21:45]
- Ryan Hill: Semileptonic form factors for $B \to \pi\ell\nu$ decays [Thu 13:45]
- Jonathan Flynn: Form factors for semileptonic $B_s \to K$ and $B_s \to D_s$ decays [Thu 14:00]
- Michael Marshall: Semileptonic $D \to \pi\ell\nu$, $D \to K\ell\nu$ and $D_s \to K\ell\nu$ decays with 2+1f Domain Wall Fermions [Thu 14:30]

neutral meson mixing



$$C_3^{\mathcal{O}}(t, \Delta T) = \sum_{i,j} \frac{P_i P_j}{4E_i E_j} \langle i|\mathcal{O}|j\rangle e^{-(E_j - E_i)(t - \Delta T/2)} e^{-(E_j + E_i)\Delta T/2}$$

$$\approx \frac{P_0^2}{4E_0^2} \langle 0|\mathcal{O}|0\rangle e^{-E_0 \Delta T} \times$$

$$\left[1 + 2\frac{P_1 E_0}{P_0 E_1} \frac{\langle 0|\mathcal{O}|1\rangle}{\langle 0|\mathcal{O}|0\rangle} e^{-\Delta E \Delta T/2} \cosh\left[\Delta E\left(t - \Delta T/2\right)\right]\right]$$

[Boyle et al. arxiv 1812.08791]

neutral meson mixing

Noise reduction through ratios designed to approach bag parameters :

$$C_3^{\mathcal{O}}(t,\Delta T) \approx \frac{P_0^2}{4E_0^2}\langle 0|\mathcal{O}|0\rangle e^{-E_0\Delta T}\times \left[1+2\frac{P_1E_0}{P_0E_1}\frac{\langle 0|\mathcal{O}|1\rangle}{\langle 0|\mathcal{O}|0\rangle}e^{-\Delta E\Delta T/2}\cosh\left[\Delta E\left(t-\Delta T/2\right)\right]\right]$$

$$C_2^{PA}(t)C_2^{PA}(\Delta T-t) \approx \frac{P_0^2}{4E_0^2}A_0^2 e^{-E_0\Delta T}\times \left[1+2\frac{P_1E_0}{P_0E_1}\frac{A_1}{A_0}e^{-\Delta E\Delta T/2}\cosh\left[\Delta E\left(t-\Delta T/2\right)\right]\right]$$

$$R^{\mathcal{O}}(t,\Delta T)=\frac{C_3^{\mathcal{O}}(t,\Delta T)}{C_2^{PA}(t)C_2^{PA}(\Delta T-t)} \rightarrow \frac{\langle 0|\mathcal{O}|0\rangle}{A_0^2}=N^{\mathcal{O}}\times B^{\mathcal{O}}$$

[Boyle et al. arxiv 1812.08791]

neutral meson mixing

Noise reduction through ratios designed to approach bag parameters:

$$C_3^{\mathcal{O}}(t,\Delta T) \approx \frac{P_0^2}{4E_0^2}\langle 0|\mathcal{O}|0\rangle e^{-E_0\Delta T}\times$$

$$\left[1 + 2\frac{P_1 E_0}{P_0 E_1}\frac{\langle 0|\mathcal{O}|1\rangle}{\langle 0|\mathcal{O}|0\rangle}e^{-\Delta E\Delta T/2}\overbrace{\cosh\left[\Delta E\left(t-\Delta T/2\right)\right]}^{=1 \text{ for } t=\Delta T/2}\right]$$

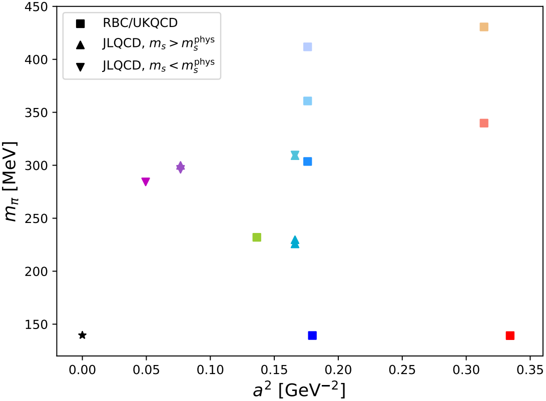
$$C_2^{PA}(t)C_2^{PA}(\Delta T - t) \approx \frac{P_0^2}{4E_0^2}A_0^2 e^{-E_0\Delta T}\times$$

$$\left[1 + 2\frac{P_1 E_0}{P_0 E_1}\frac{A_1}{A_0}e^{-\Delta E\Delta T/2}\overbrace{\cosh\left[\Delta E\left(t-\Delta T/2\right)\right]}^{=1 \text{ for } t=\Delta T/2}\right]$$

$$R^{\mathcal{O}}(t,\Delta T) = \frac{C_3^{\mathcal{O}}(t,\Delta T)}{C_2^{PA}(t)C_2^{PA}(\Delta T - t)} \to \frac{\langle 0|\mathcal{O}|0\rangle}{A_0^2} = N^{\mathcal{O}}\times B^{\mathcal{O}}$$

[Boyle et al. arxiv 1812.08791]

Landscape plot of our ensembles



- 2 ensembles at $m_\pi^{\rm phys}$
- JLQCD ensembles are very fine
  - $\Rightarrow$ almost reach $m_b^{\rm phys}$
- 2 very similar ensembles with $m_\pi L = 3.0$ and $m_\pi L = 4.4$
- 6 different lattice spacings from $a^{-1} = 1.7\mathrm{GeV}$ to $a^{-1} = 4.5\mathrm{GeV}$

$\Rightarrow$ These strongly constrain the relevant limits we will take in a final global fit to data on all ensembles.

Fit strategy

- We have studied a number of different strategies to fit all these parameters and settled on a **simultaneous**, **fully correlated** fit to:
  $C_2^{PP}(t), C_2^{PA}(t), C_2^{AA}(t), R^{\mathcal{O}}(\Delta T)$
- We define a vector with all data points entering the fit

$$C = (C_2^{PP}(t_{\mathrm{min}}^{PP}), \ldots, C_2^{PA}(t_{\mathrm{min}}^{PA}), \ldots, C_2^{AA}(t_{\mathrm{min}}^{AA}), \ldots, R^{\mathcal{O}}(\Delta T_{\mathrm{min}}^{\mathcal{O}}), \ldots)$$

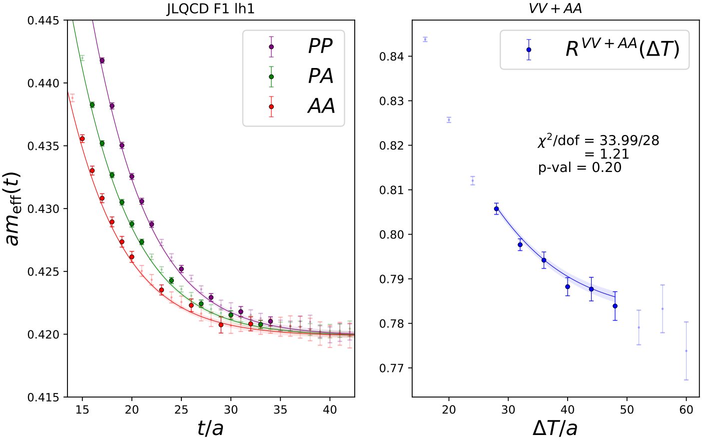
$$\Delta = C^{\mathrm{data}} - C^{\mathrm{model}}$$

- From this we define and minimise a χ^2 function

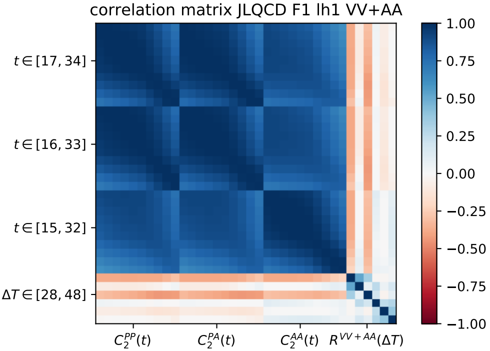
$$\chi^2 = \Delta C_{cov}^{-1} \Delta^T$$

Fit strategy

- We thin the data in the 2pt functions. for e.g. C_2^{PP}, the fit takes every timeslice from 17 to 22, and then every $3^{\rm rd}$ from 22 to 34.

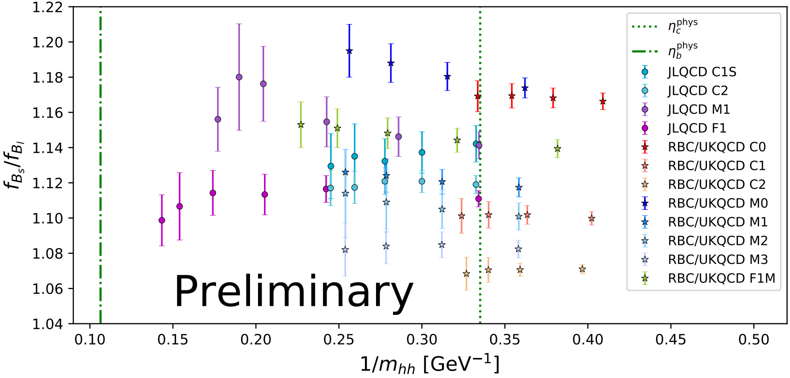


Correlation matrix



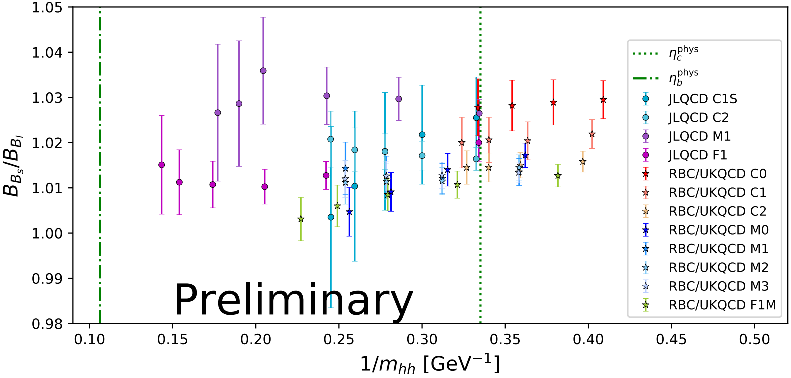
- 2pt functions are highly correlated
- ratios are decorrelated from the 2pt functions
- We improved upon an earlier attempt of fitting 2pt and raw 3pt functions simultaneously, which had high correlations

Ratio of decay constants



- illustration in the heavy-quark mass reach of the JLQCD ensembles
- dependence on heavy-quark mass is very mild
- RBC/UKQCD values are taken from an earlier analysis on the same dataset, using a different fit technique. **[Boyle et al. arxiv 1812.08791]**

Ratio of bag parameters - $VV + AA$



- this $SU(3)$-breaking ratio is close to 1
- dependence on heavy-quark mass is very mild
- RBC/UKQCD values are taken from an earlier analysis on the same dataset, using a different fit technique. **[Boyle et al. arxiv 1812.08791]**

Domain-wall operator-mixing matrix

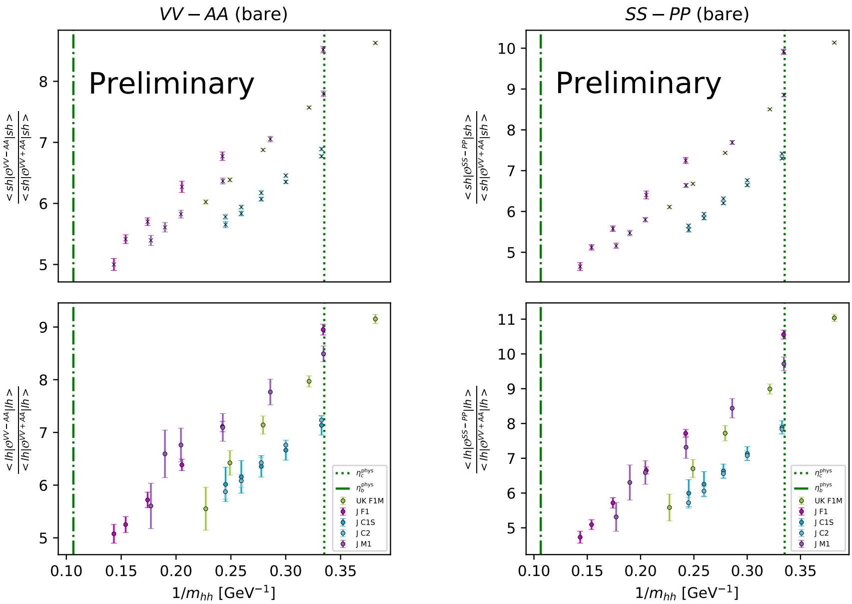
Based on chiral symmetry of our domain-wall fermions, a very simple mixing pattern of the 5 operators arises:

$$\begin{aligned}
\mathcal{O}_1 &= \mathcal{O}^{VV+AA} \\
\mathcal{O}_2 &= \mathcal{O}^{VV-AA} \\
\mathcal{O}_3 &= \mathcal{O}^{SS-PP} \\
\mathcal{O}_4 &= \mathcal{O}^{SS+PP} \\
\mathcal{O}_5 &= \mathcal{O}^{TT} \,.
\end{aligned}
\qquad
\begin{pmatrix}
\mathcal{O}_1 & 0 & 0 \\
0 & \begin{pmatrix} \mathcal{O}_{2/3} & \mathcal{O}_{2/3} \\ \mathcal{O}_{2/3} & \mathcal{O}_{2/3} \end{pmatrix} & 0 \\
0 & 0 & \begin{pmatrix} \mathcal{O}_{4/5} & \mathcal{O}_{4/5} \\ \mathcal{O}_{4/5} & \mathcal{O}_{4/5} \end{pmatrix}
\end{pmatrix}$$

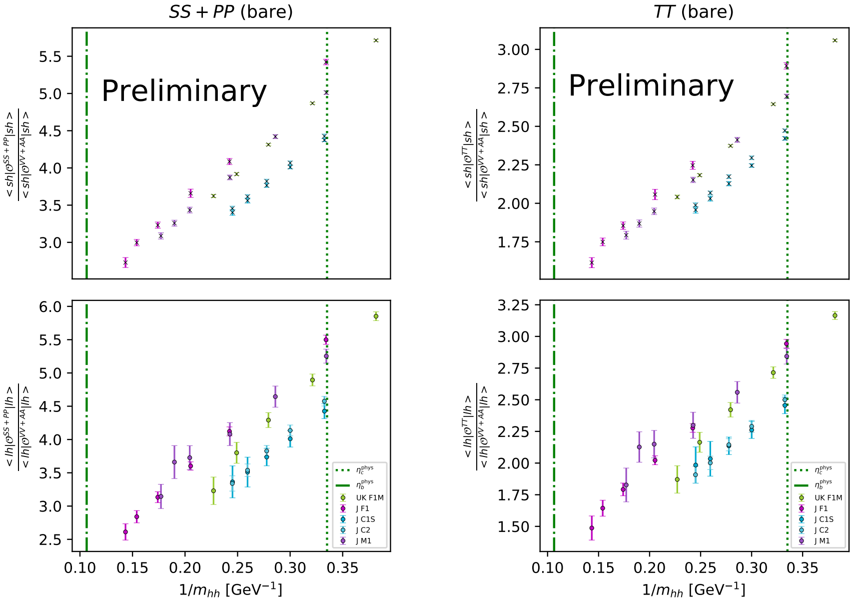
This block-structure means that only $\mathcal{O}_2, \mathcal{O}_3$ as well as $\mathcal{O}_4, \mathcal{O}_5$ mix, but they are linearly independent from each other and from $\mathcal{O}_1$.
This is a great advantage of clean and chiral domain wall fermions to other lattice discretisations, where a more complicated mixing pattern has to be dealt with.

[Boyle et al. arxiv 1708.03552]

VV-AA and SS-PP



SS+PP and TT



$SS + PP$ (bare)

TT (bare)

Conclusions & Outlook

Conclusions:

- We can extract bag parameters and matrix elements $\langle 0|\mathcal{O}|0\rangle$ using a fully correlated fit with a combined χ^2/dof for C_2 and C_3.
  - $\Rightarrow$ This can be done for the full BSM operator basis
- DWF leads to a very simple mixing pattern of the 5 operators due to chiral symmetry

Next steps

- Non-perturbative renormalisation (NPR)**[Boyle et al. arxiv 1812.08791]**
  - $\Rightarrow$ Code in Grid / Hadrons is production-ready and currently being used by other projects.
- We have measurements on 7 additional ensembles, and we will repeat this analysis on those.
  - 2 ensembles at m_π^{phys}
  - JLQCD ensembles almost reach m_b^{phys}
  - 2 very similar ensembles with $m_\pi L = 3.0$ and $m_\pi L = 4.4$
  - 6 different lattice spacings from $a^{-1} = 1.7\mathrm{GeV}$ to $a^{-1} = 4.5\mathrm{GeV}$
  - $\Rightarrow$ These strongly constrain the relevant limits we will take in a final global fit to data on all ensembles.

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme under grant agreement No 757646.

Backup

Lattice setup

- RBC-UKQCD's 2+1 flavour domain wall fermions **[Blum et al. arxiv 1411.7017]**
  - pion masses from $m_\pi = 139$ MeV to $m_\pi = 430$ MeV
  - several heavy-quark masses from below m_c to $0.5 m_b$, using a stout-smeared action ($\rho = 0.1$, $N = 3$) with $M_5 = 1.0$, $L_s = 12$ and Moebius-scale $= 2$ **[Boyle et al. arxiv:1812.08791]**
  - light and strange quarks: sign function approximated via:
    - Shamir approximation for heavier pion masses
    - Möbius approximation at m_π^{phys} and on the finest ensemble
- JLQCD's 2+1 flavour domain wall fermions **[Kaneko et al. arxiv 1711.11235]**
  - pion masses from $m_\pi = 226$ MeV to $m_\pi = 310$ MeV[1]
  - heavy-quark masses from m_c nearly up to m_b, using the same stout-smeared action.
  - light and strange quarks use the same action as the heavy quarks.
- We will account for different scaling trajectories due to the different light and strange quark actions in the global fit

---

[1] There are more JLQCD ensembles with heavier m_π, we just list the range of the subsets used in this analysis.

Lattice setup

	L/a	T/a	a^{-1} [GeV]	m_π [MeV]	$m_\pi L$	hits $\times N_{\rm conf}$
RBC-UKQCD						
C0	48	96	1.7295(38)	139.2	3.86	48 × 90
C1	24	64	1.7848(50)	339.8	4.57	32 × 100
C2	24	64	1.7848(50)	430.6	5.79	32 × 101
M0	64	128	2.3586(70)	139.3	3.78	64 × 82
M1	32	64	2.3833(86)	303.6	4.08	32 × 83
M2	32	64	2.3833(86)	360.7	4.84	32 × 76
M3	32	64	2.3833(86)	411.8	5.51	32 × 81
F1M	48	96	2.708(10)	232.0	4.11	48 × 72
JLQCD						
C1L	48	96	2.453(4)	225.8	4.4	24 × 100
C1S	32	64	2.453(4)	229.7	3.0	16 × 100
C2a	32	64	2.453(4)	309.7	4.0	16 × 100
C2b	32	64	2.453(4)	309.1	4.0	16 × 100
M1a	48	96	3.610(9)	296.2	3.9	24 × 50
M1b	48	96	3.610(9)	299.9	3.9	24 × 50
F1	64	128	4.496(9)	284.3	4.0	32 × 50