The muon g-2 with four flavors of staggered quarks

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LATTICE 21

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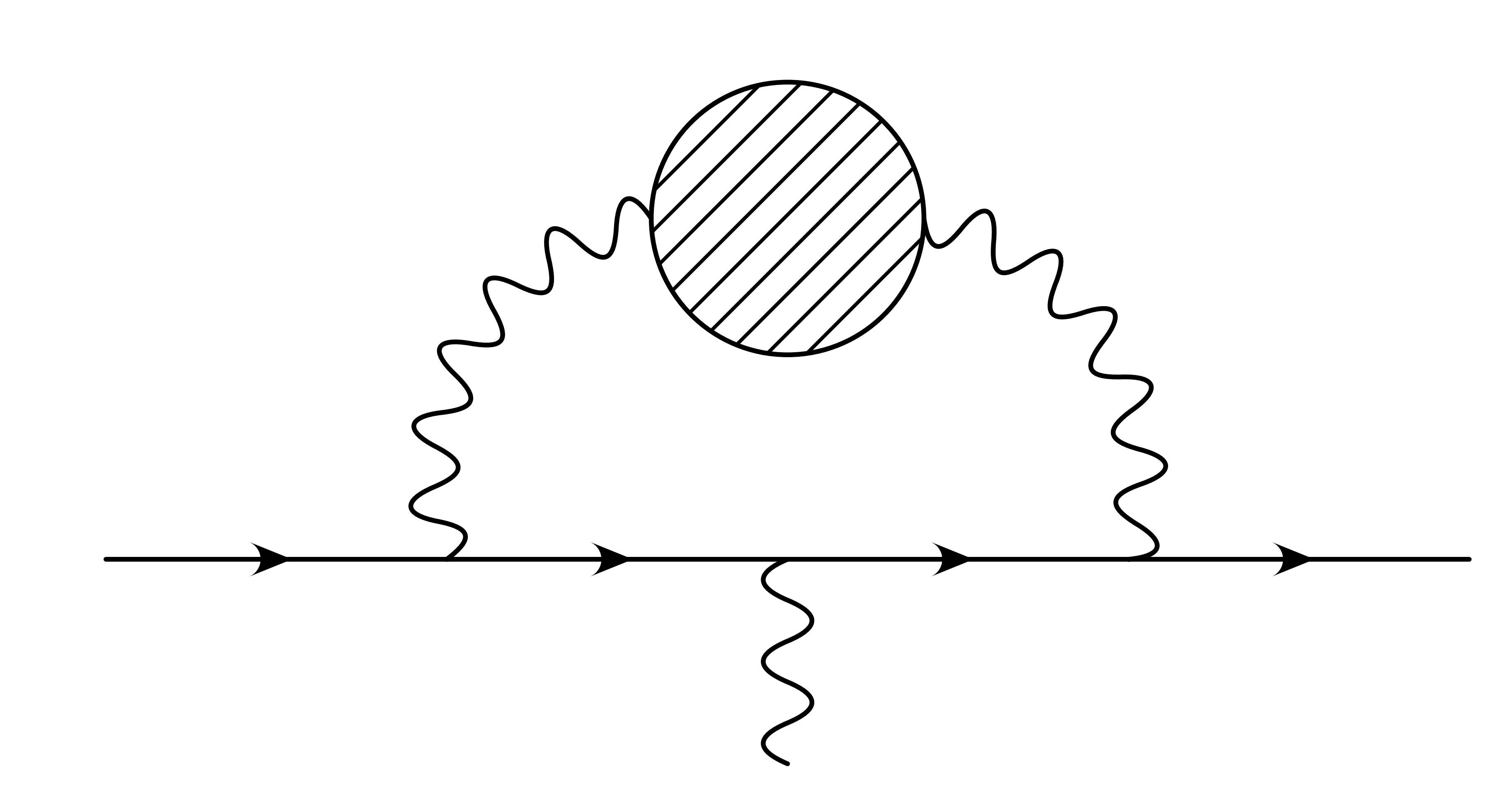
OUTLINE

- Introduction
- Lattice Details
- Old & New Results
- Summary

INTRODUCTION TO THE MUON g-2

Familiar to everyone here (light-quark connected only)
j^μ is the EM current [isospin limit]



$$a_\mu^{\rm HVP} = 4\alpha^2 \int_0^\infty dq^2 f(q^2)\hat{\Pi}(q^2),$$

$$f(q^2) = \frac{m_\mu^2 q^2 Z^3 (1 - q^2 Z)}{1 + m_\mu^2 q^2 Z^2},$$

$$Z = -\frac{q^2 - \sqrt{q^4 + 4m_\mu^2 q^2}}{2m_\mu^2 q^2}.$$

$$\Pi^{\mu\nu}(q) = \int d^4x e^{iqx}\langle j^\mu(x) j^\nu(0)\rangle$$

$$= \Pi(q^2)(-q^\mu q^\nu + q^2\delta^{\mu\nu}),$$

Time-momentum representation
[Bernecker & Meyer, EPJA 47, 148]

$$\Pi(q^2) - \Pi(0) = \sum_t \left(\frac{\cos qt - 1}{q^2} + \frac{1}{2}t^2\right) C(t),$$

$$C(t) = \frac{1}{3}\sum_{\vec{x},i}\langle j^i(\vec{x},t) j^i(0)\rangle,$$

$$w(t) = 4\alpha^2 \int_0^\infty d\omega^2 f(\omega^2)\left[\frac{\cos \omega t - 1}{\omega^2} + \frac{t^2}{2}\right],$$

$$a_\mu^{\rm HVP}(T) = \sum_{t=-T/2}^{T/2} w(t) C(t)$$

NNLO FINITE VOLUME CORRECTIONS

$$\Delta a_\mu^{\rm HVP} = \lim_{L\to\infty} a_\mu^{\rm HVP}(L) - a_\mu^{\rm HVP}(L)$$

$$C(t) = \frac{1}{48L^3}\sum_{\vec p}\sum_X \frac{\vec p^{\,2}}{E_X^2(p)} e^{-2E_X(p)t}\Bigg(1 - \frac{1}{4f^2}\sum_Y D_Y(0) - \frac{16\ell_6(\vec p^{\,2}+m_X^2)}{f^2} + \frac{1}{24f^2}\frac{1}{L^3}\sum_{\vec q}\sum_Y \frac{\vec q^{\,2}}{E_Y(q)}\frac{1}{\vec q^{\,2}-\vec p^{\,2}+m_Y^2-m_X^2}\Bigg)$$

$$E_Y(k) = \sqrt{k^2+m_Y^2}$$

$$D_Y(0) = \frac{1}{L^3}\sum_{\vec k}\frac{1}{2E_Y(k)}$$

- Finite-volume effects can be significant even on large volumes (L~6 fm, m_π ~ physical)
- These long-distance effects can be largely corrected for using Chiral Perturbation Theory
- Can't extract *g*-2 with ChPT — calculate difference between the infinite-volume and the FV result
- Have done so with full staggered ChPT (shown) to NNLO

Above the sums over *X*, *Y* include the eight pion tastes for staggered quarks

Problem with ChPT? No! — Aubin *et al*, PRD 102 094511, arXiv:2008.03809

LATTICE DETAILS - CONFIGURATIONS

All (near) physical pion masses, HISQ ensembles generated by the MILC collaboration

We use a combination of all-mode mode averaging and low-mode averaging

Old results

m_π (MeV)	a (fm)	L^3	L (fm)	$m_\pi L$	LM	# confs	traj. sep.
133	0.12121(64)	48^3	5.82	3.91	4000	26	40
130	0.08787(46)	64^3	5.62	3.66	4000	36(40)	12
134	0.05684(30)	96^3	5.46	3.73	6000	22(23)	48

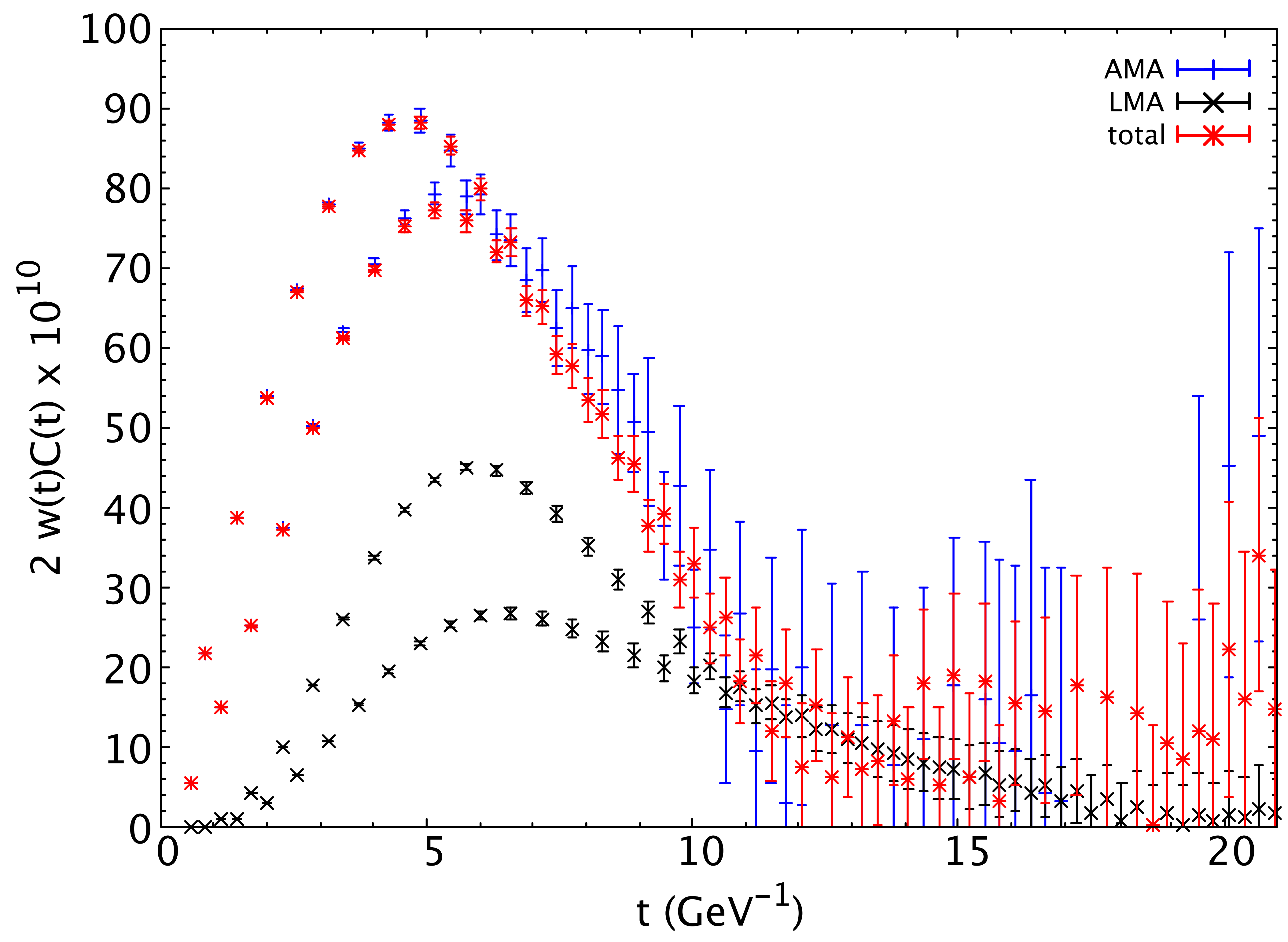
New results [no updated 0.12 fm results]

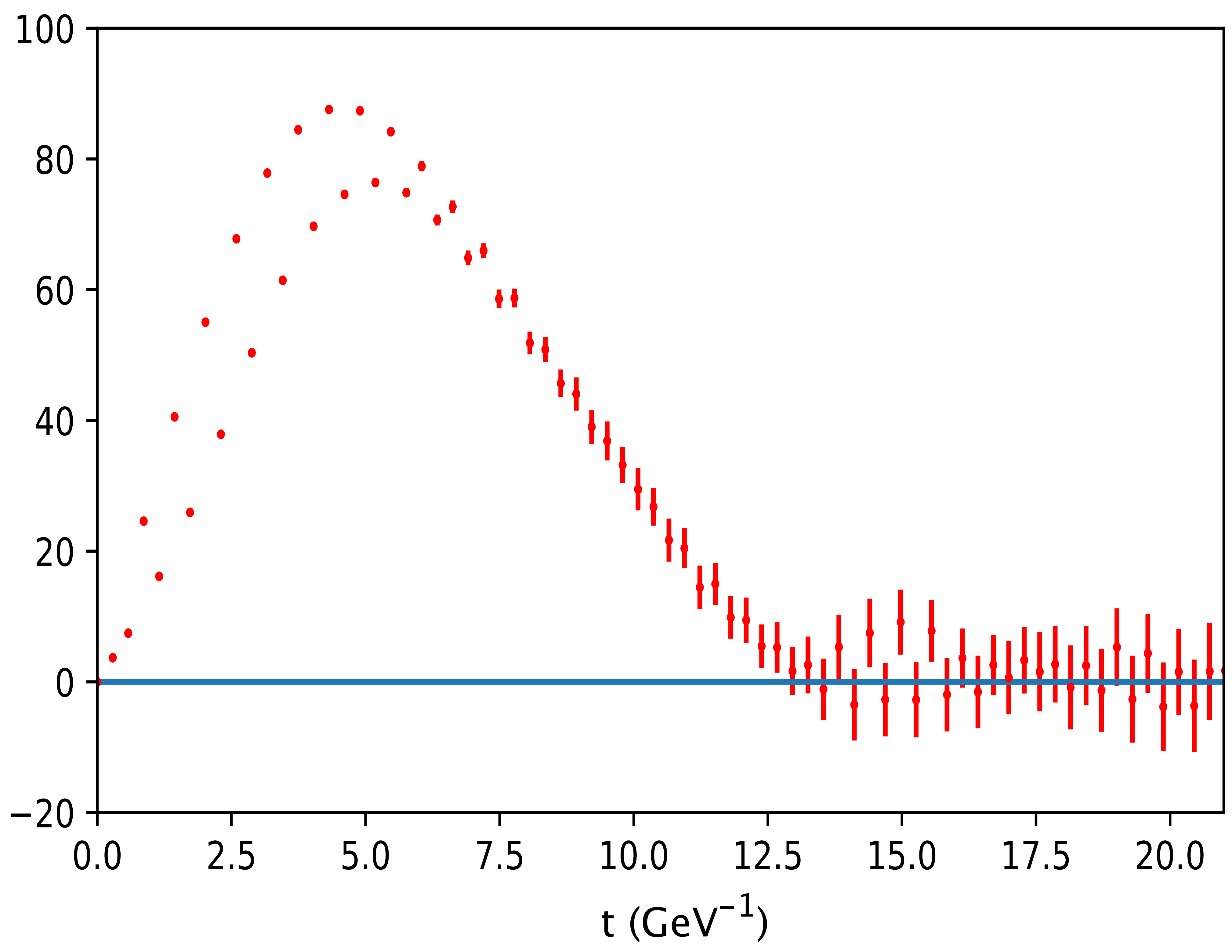
m_π (MeV)	a (fm)	L^3	L (fm)	$m_\pi L$	LM	# confs	traj. sep.
133	0.15148(80)	32^3	4.83	3.26	8000	48	40
130	0.08787(46)	64^3	5.62	3.66	8000	38	100
134	0.05684(30)	96^3	5.46	3.73	8000	35	60

Have not yet combined old + new 64^3, 96^3 data

RESULTS

Compare summand for original 96^3 data vs. new data:





RESULTS

FV corrections for the window (x10^{10})

a	Volume	m_{π_5}(MeV)	m_{π_5} FV corr.	taste-breaking FV corr.	m_π retuning	total
0.06	96^3	134	0.727538	0.759853	−0.0687552	1.4186
0.09	64^3	130	0.697276	3.51669	−0.516077	3.6979
0.12	48^3	133	0.560572	7.99304	−0.21689	8.3367
0.15	32^3	133	1.24171	10.3470	−0.186605	11.4021

These corrections are NLO for the window method only (we also have the full NNLO corrections)

Intermediate window method

$$a_\mu^W = 2\sum_{t=0}^{T/2} C(t)w(t)(\Theta(t,t_0,\Delta) - \Theta(t,t_1,\Delta))$$

$$\Theta(t,t',\Delta) = \frac{1}{2}(1+\tanh((t-t')/\Delta)),$$

$t_0 = 0.4,\ t_1 = 1.0$ fm, Δ=0.15

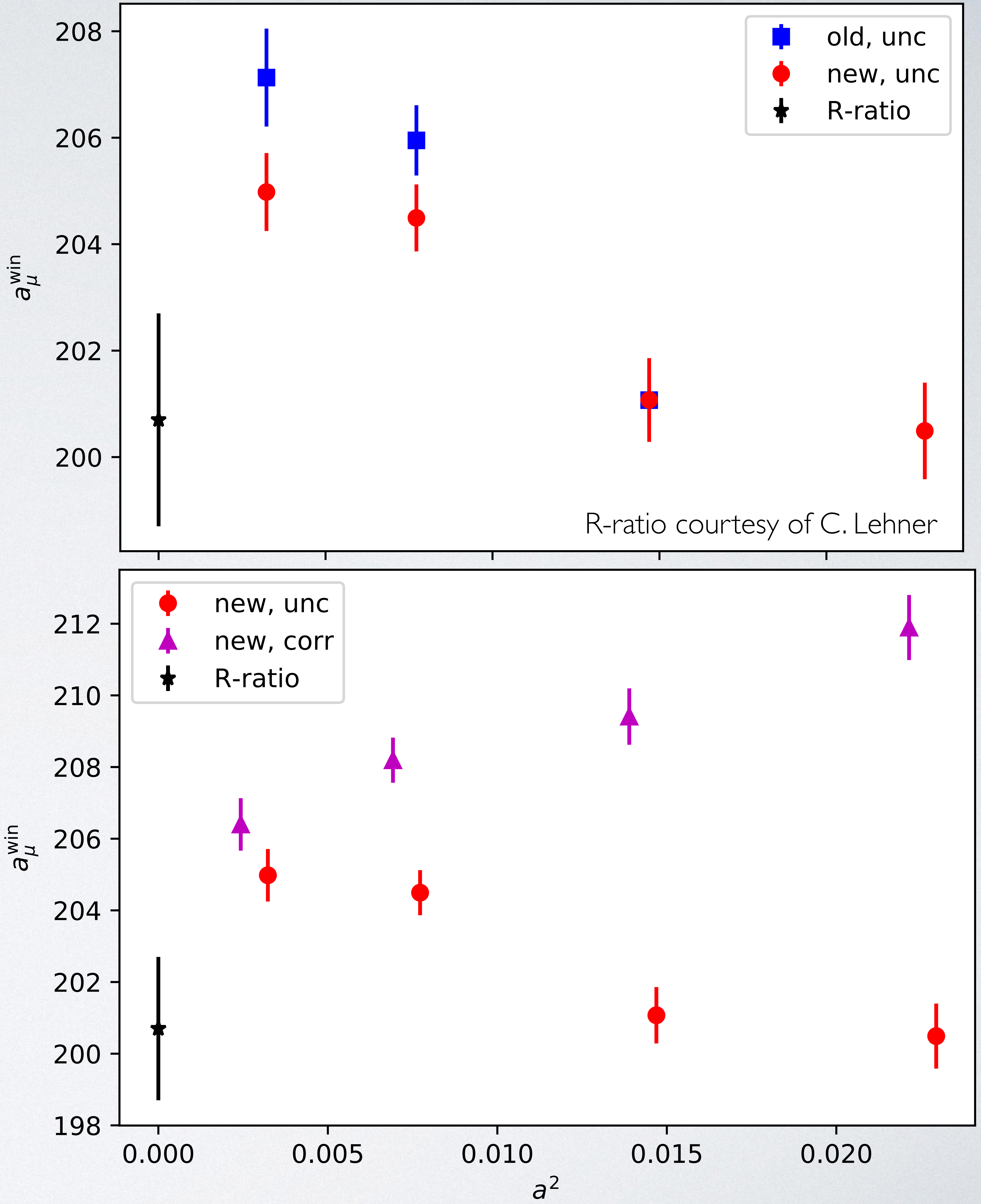
RESULTS

Updated 0.06 fm & 0.09 fm + 0.15 fm results

Difference in the lowest two results could be caused by autocorrelations, scale setting? (still studying)

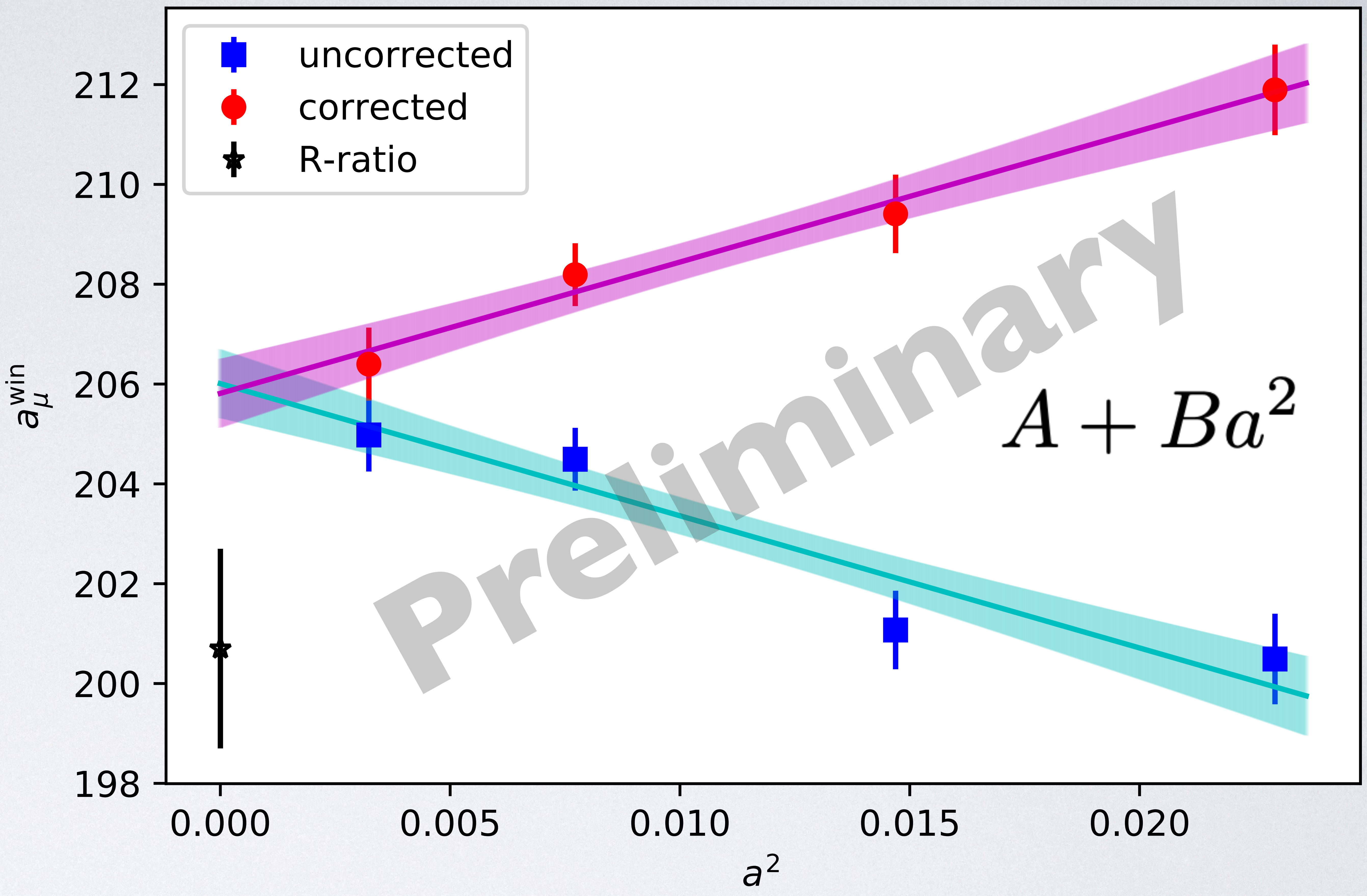
(a=0.12 fm result isn't "new," but is included in fits to the new data)

With the FV corrections added in, our negative slope becomes positive, but "cleaner" remaining a^2 effects?



RESULTS

A simple linear-in-a^2 fit gives consistent results for the corrected vs. uncorrected data



The uncorrected data does not show a^2 scaling

Both fits are inconsistent with the *R*-ratio result

Does this mean the lattice result is inconsistent with the *R*-ratio?

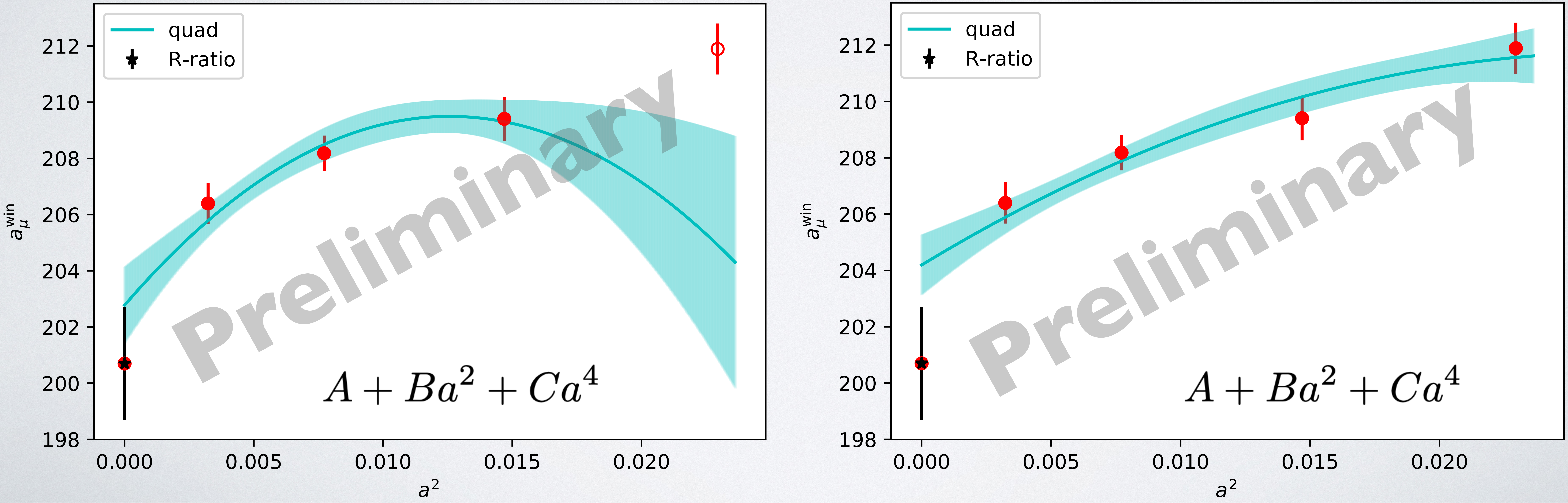
RESULTS

What if we include the *R*-ratio in the fits? Would the lattice result be consistent?

Simple quadratic fit gives mixed results.

If we include the coarsest data point, the agreement with the *R*-ratio is not great

(better than the linear fit)



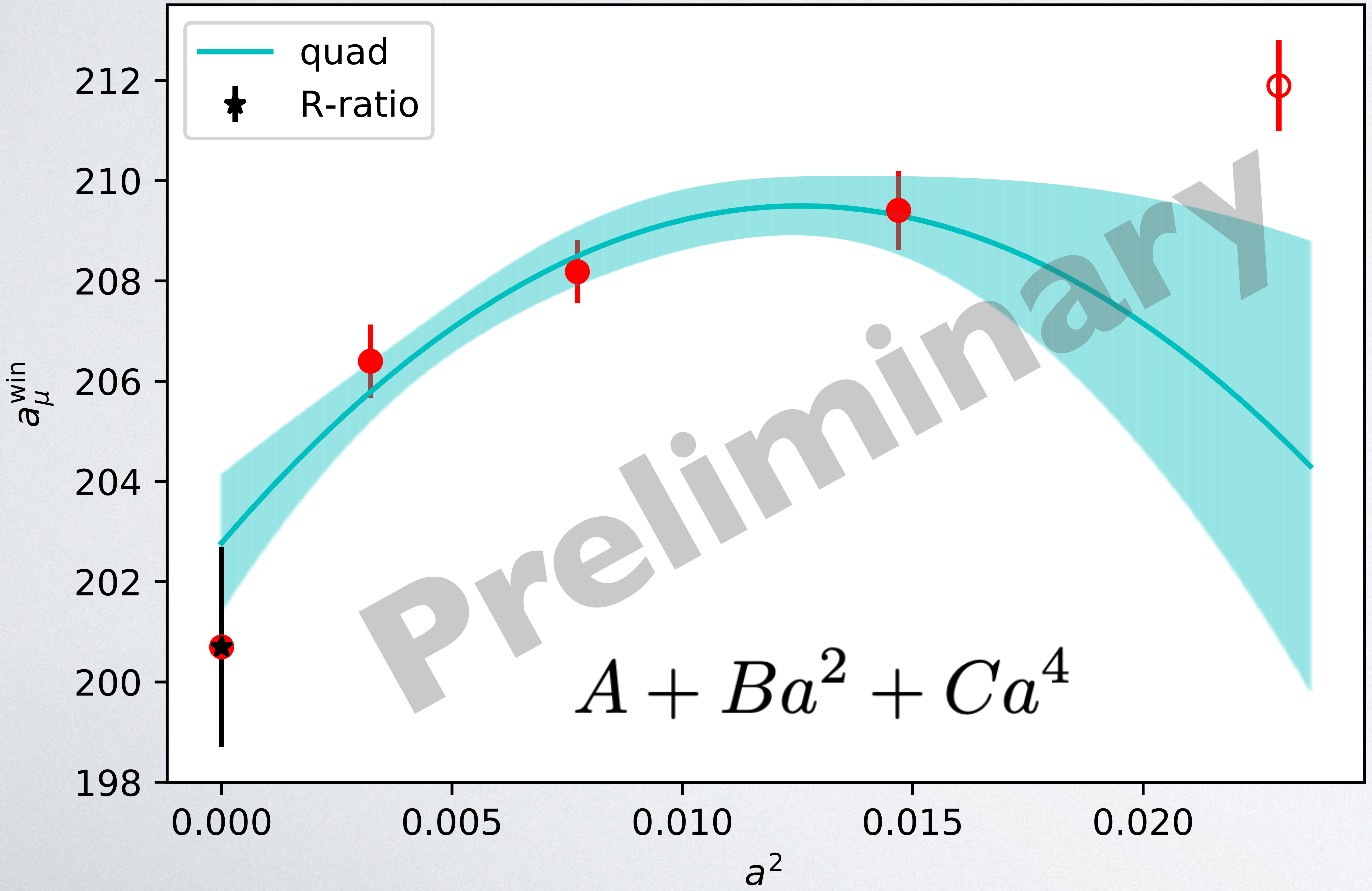
RESULTS

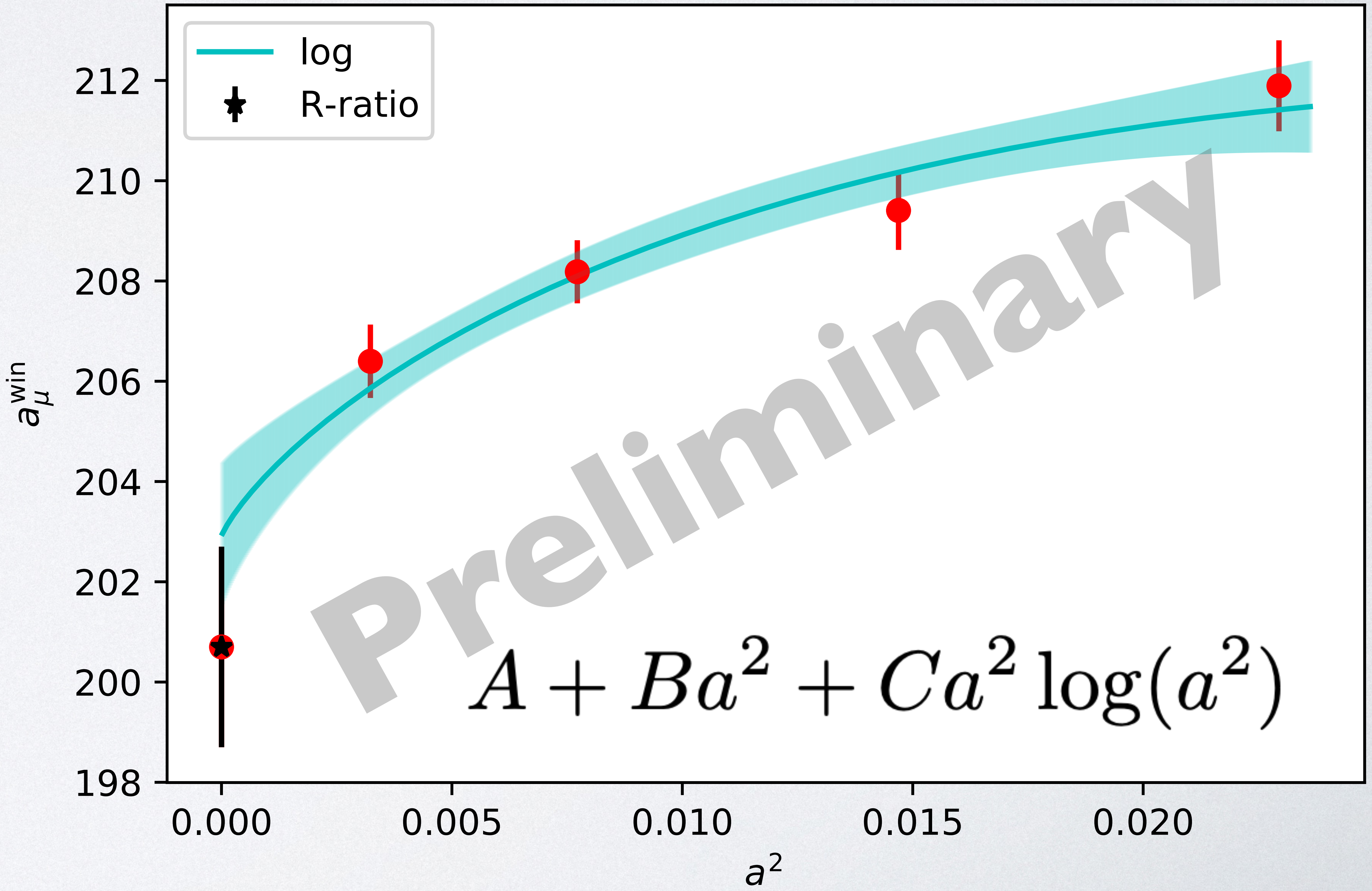
What about a "perturbation-theory-inspired" fit?

Polynomials are simple but not physically motivated [compare "old-school" chiral fits]

Expansions of quantities in a^2 would have $\log(a^2)$ terms which we could in principle calculate exactly [e.g., Husung, Marquard, Sommer , EPJC 80, 200 (2020)]

As a simple first step:





SUMMARY & OUTLOOK

- All fits have p-values > 0.10
- Results are not inconsistent with each other or the R-ratio
- Important to simulate at smaller a to understand cutoff effects (the uncorrected data is evidence for this)
- To fully understand the FV effects, we are currently studying an a=0.15 fm lattice with 48^3 (as opposed to the 32^3 here) [thanks to CalLat]
- Thanks as always to the MILC collaboration for their lattices, as well as the many fruitful discussions from the g-2 theory initiative.