Towards determining the short-distance contribution to neutrinoless double-beta decay from lattice QCD.

Saurabh V. Kadam

with

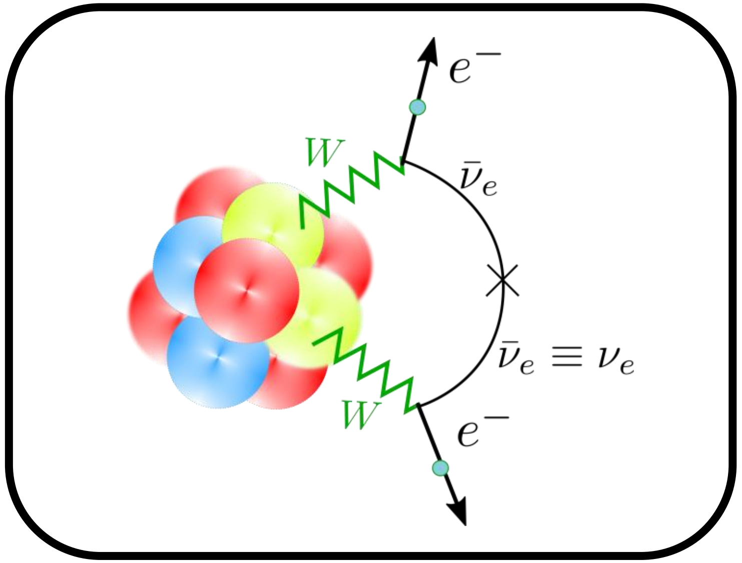
Zohreh Davoudi

Phys. Rev. Lett. 126, 152003 (2021),

Motivation

Neutrinoless double beta decay ($0\nu\beta\beta$)

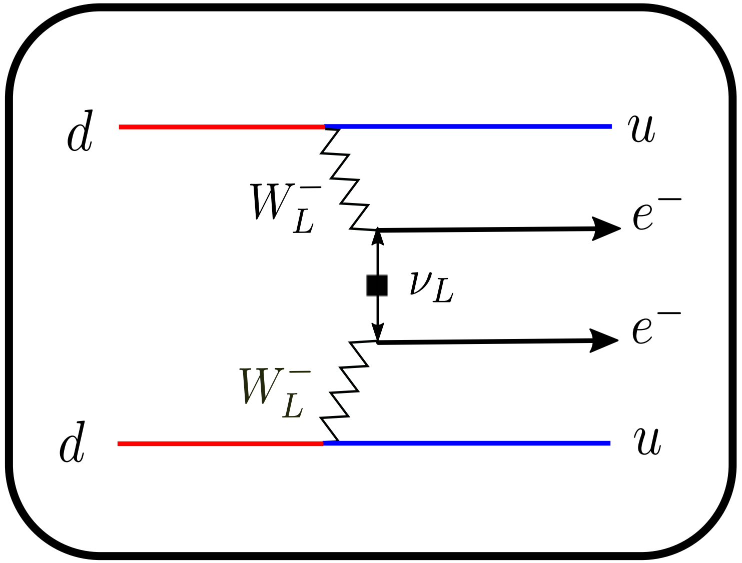
- Total lepton number is violated by two units
- Forbidden in the standard model (SM)
- Not yet observed
- Neutrinos are their own anti-particles



Racah (1937)
Furry (1939)

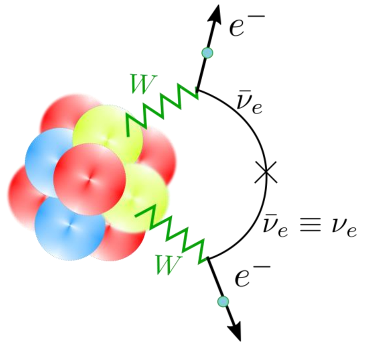
Light Neutrino Exchange

- Minimal deviation from the SM
- The SM neutrinos are promoted to Majorana neutrinos
- Mass term allows emitted neutrino to be reabsorbed



Avignone, Elliott and Engel
Reviews of Modern Physics, 80 (2008)

Light Neutrino Exchange



$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)\, |M_{0\nu}|^2\, \langle m_{\beta\beta}\rangle^2$$

Half life

Phase Space Factor

Nuclear Matrix Element (NME)

Effective Majorana Mass

$m_{\beta\beta} = \sum_k m_k U_{ek}^2$

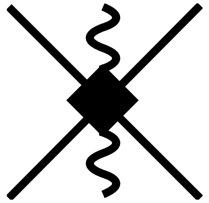
Mixing matrix U_{ek} transforms mass eigenstates to electron flavor eigenstate

- Calculations of the NME in different frameworks differ by up to a factor of 3.
  Vergados, Ejiri and Simkovic, Rep. Prog. Phys. 75 106301 (2012)
- A major source of uncertainty is the contribution from few-nucleon amplitudes.
- Two-nucleon amplitude in pionless EFT is undetermined at LO due to an unknown short-range low energy constant (LEC), g_ν^{NN}.
- An indirect estimate of g_ν^{NN} suggests an enhancement of $\sim$ 43% in NME for Ca nucleus

Cirigliano, Dekens, de Vries, Hoferichter, Mereghetti
Phys. Rev. Lett. 126, 172002

Wirth, Yao, Hergert arXiv: 2105.05415

Need direct and accurate constraints on LEC g_ν^{NN}



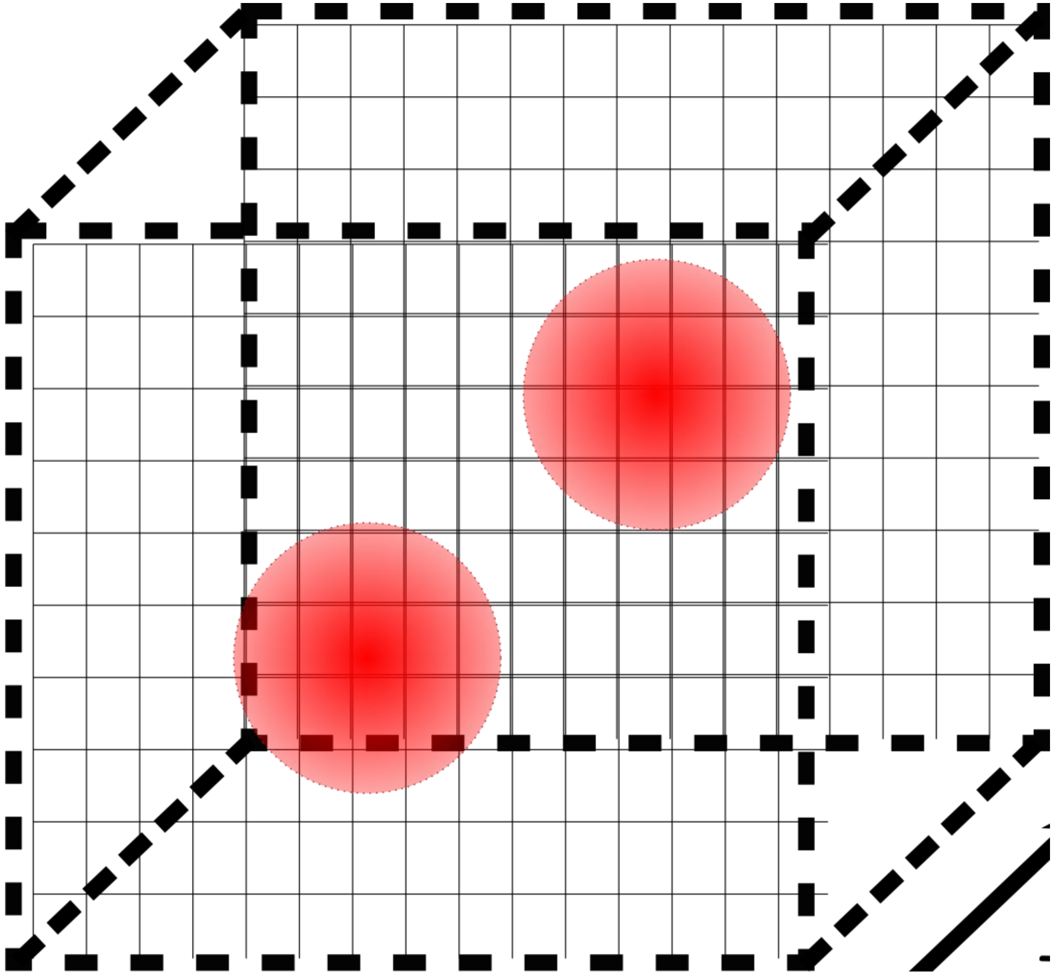
$\sim g_\nu^{NN}$

Cirigliano, Dekens, de Vries, Graesser, Mereghetti, Pastore, van Kolck
Phys. Rev. Lett. 120, 202001

How? $\longrightarrow$ Lattice QCD

QCD formulated on

- Discrete Euclidean spacetime grid
- Lattice spacing
- Finite volume



Finite vs. Infinite Volume Physics

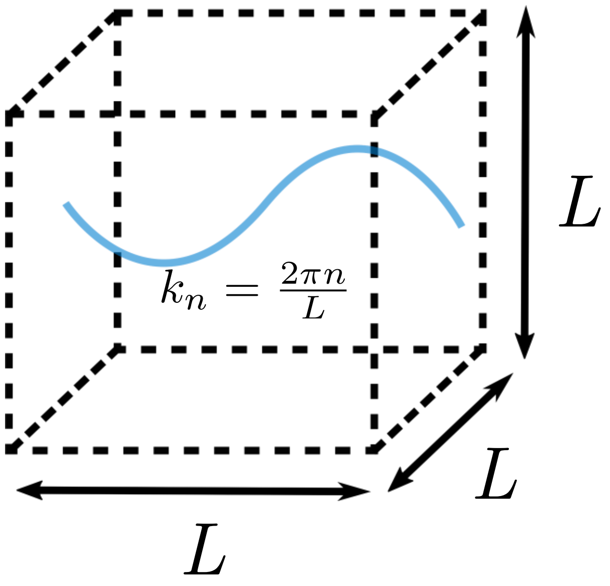
Luscher (1986a), Commun. Math. Phys. 104, 177

Lellouch, and Luscher (LL) (2001), Commun. Math. Phys. 219,

Kim, Sachrajda, and Sharpe (2005), Nucl. Phys. B727

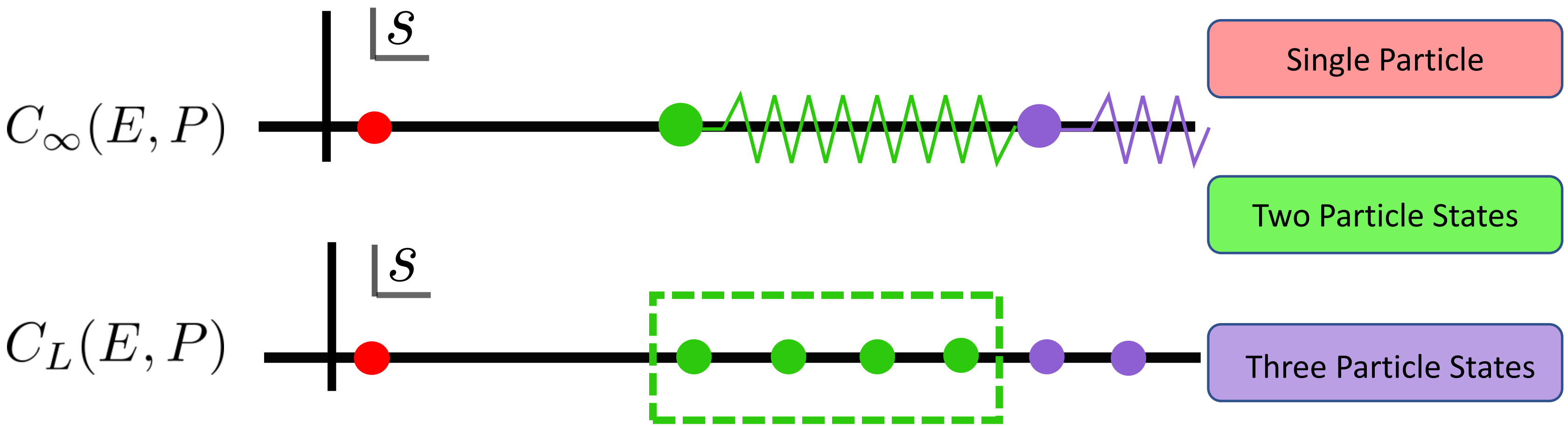
Particle Spectrum

Finite Volume (FV)



Periodic Boundary Condition

Quantization of momenta



Assumptions

- Temporal extent of Euclidean space much larger than L
- Ignoring discretization effects
- Center of mass energy below three hadron threshold.

Luscher's Formalism

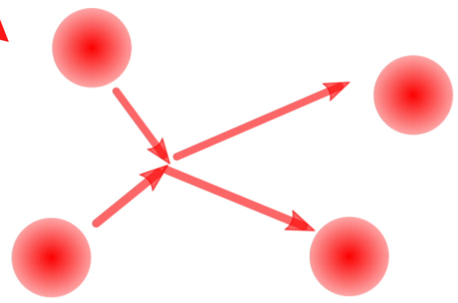
Poisson Summation Formula

- $f(\vec{k})$ UV convergent and non-singular in $\vec{k}$ $\quad \sum_{\vec{k}} f(\vec{k}) = \int_{\vec{k}} f(\vec{k}) + \mathcal{O}(e^{-m_\pi L})$
- Power-law finite volume (FV) corrections from singularities in $\vec{k}$ $\quad \sum_{\vec{k}} \frac{f(\vec{k})}{k^2-p^2} = \int_{\vec{k}} \frac{f(\vec{k})}{k^2-p^2+i\epsilon} + F(p)$
- Identifying FV corrections leads to Luscher's quantization condition

Finite Volume Two Particle Spectrum

Luscher's Quantization Condition

$$\det\left[F_0^{-1}(E_n) + \mathcal{M}(E_n)\right] = 0$$



$i\mathcal{M}_{NN\to NN}$

Infinite Volume Scattering Amplitude

First derived for $K \to \pi\pi$ by Lellouch and Luscher

Luscher, Commun. Math. Phys. 104, 177 (1986)
Luscher, Commun. Math. Phys. 105, 153 (1986)
Lellouch and Luscher Commun.Math.Phys., 219, 31 (2001)
Review: Briceño, Dudek and Young Rev. Mod. Phys. 90 (2018)

Background

- Extended towards electro-weak current ($\mathcal{J}$) interactions: Review: Davoudi et al. arXiv:2008.11160v1
  - Formalism for generalized $0 + \mathcal{J} \rightarrow 2$ and $1 + \mathcal{J} \rightarrow 2$ processes. Briceno and Hansen (2015),Phys. Rev. D92 (7), 074509

    From LQCD: $\gamma^* \rightarrow \pi\pi$ and $\pi\gamma^* \rightarrow \pi\pi$ amplitudes.

    Feng et al. (2015) Phys. Rev. D91 (5), 054504
    Briceno et al. (2015a) Phys. Rev. Lett. 115, 242001
  - Formalism for $2 + \mathcal{J} \rightarrow 2$ processes. Briceño and Davoudi (2013) Phys. Rev. D 88, 094507

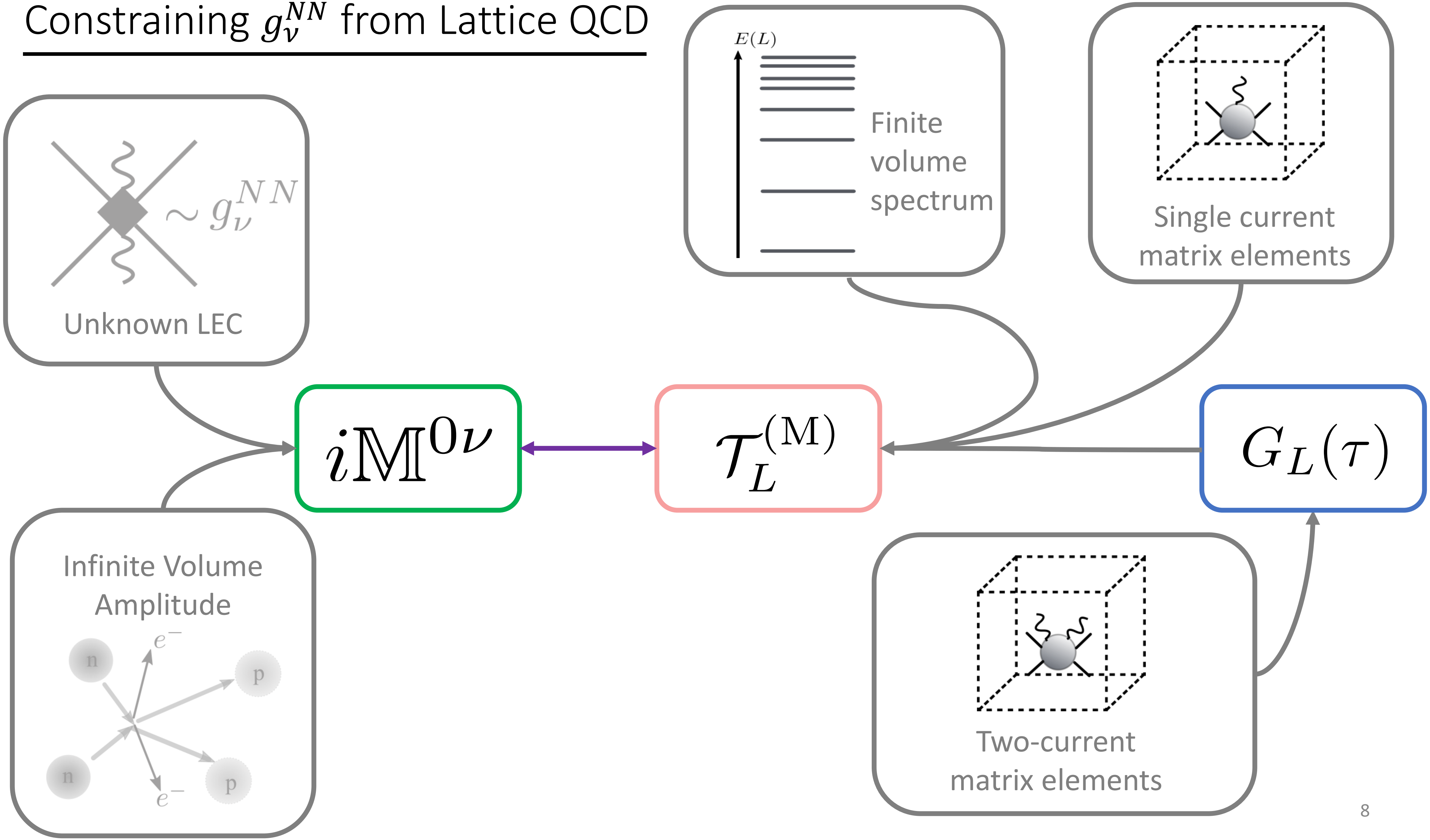
    Value of $L_{1,A}$ from LQCD via studying pp fusion $pp \rightarrow de^+\nu$ process. Briceno and Hansen (2016) Phys. Rev. D94 (1), 013008

    NPLQCD Collaboration Phys. Rev. Lett. 119 (6) (2017) 62002.
  - Formalism for $1 + 2\mathcal{J} \rightarrow 1$ processes. Briceño, Davoudi, Hansen, Schindler and Baroni Phys. Rev. D 101, 014509

    $2\nu\beta\beta$ matrix elements (MEs) at $m_\pi \sim 800$ MeV
    NPLQCD Collaboration Phys. Rev. D 96, 054505.
  - $K_L - K_S$ mass difference. Christ, Izubuchi, Sachrajda, Soni, and Yu (RBC and UKQCD Collaborations) Phys. Rev. D 88, 014508 (2013)
  - Formalism for $2 + 2\mathcal{J} \rightarrow 2$ processes for $2\nu\beta\beta$. Feng, Jin, Wang, and Zhang Phys. Rev. D 103, 034508 (2021)
  - Formalism for $1 + 2\mathcal{J} \rightarrow 1$ processes with massless leptonic propagators. Davoudi and Kadam Phys. Rev. D 102, 114521 (2020)

    Christ, Feng, Jin, and Sachrajda Phys. Rev. D 103, 014507 (2021)

Constraining g_ν^{NN} from Lattice QCD

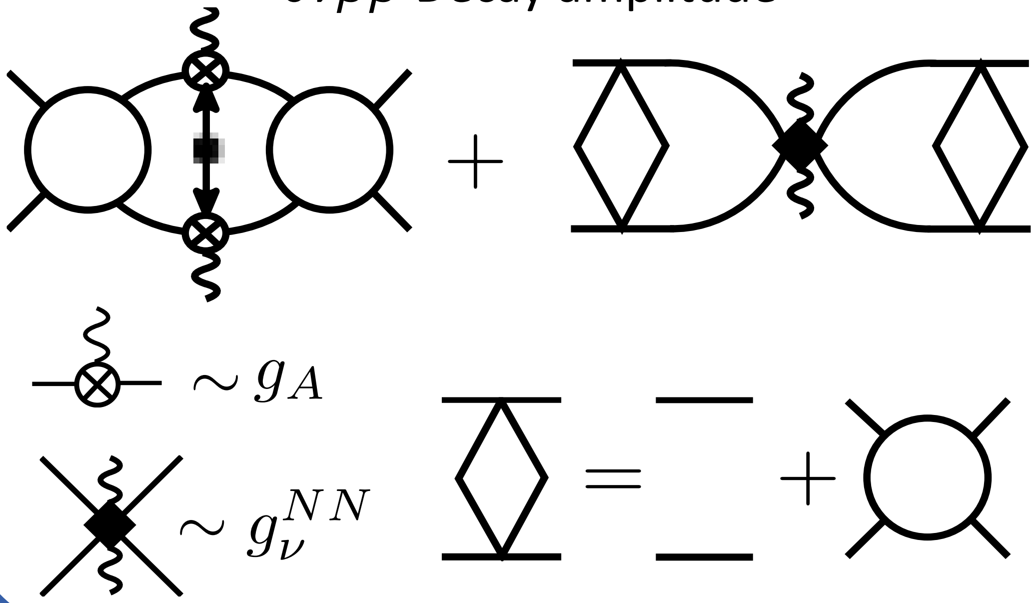


Constraining g_ν^{NN} from Lattice QCD

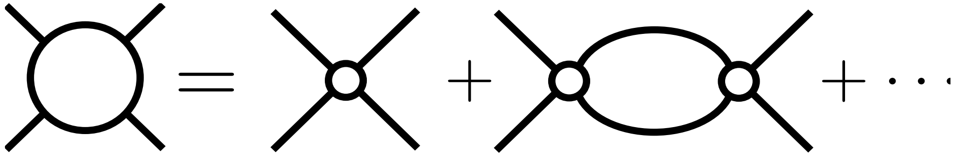
At leading order (LO) in non-relativistic Pionless EFT

$$i\mathbb{M}^{0\nu} = i\mathcal{M}^{(\text{Int.})}_{nn\to pp} - m_{\beta\beta}(1+3g_A^2)\mathcal{M}_{nn}\,\delta J^V \mathcal{M}_{pp}$$

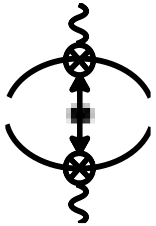
$0\nu\beta\beta$ Decay amplitude



LO two-nucleon amplitude



Finite volume corrections



$$\sum_{\vec{k}} = \int_{\vec{k}} + \delta J^V$$

For details see supplemental of:
Davoudi and Kadam Phys. Rev. Lett. 126, 152003 (2021)

Constraining g_ν^{NN} from Lattice QCD

For details see:
Davoudi and Kadam Phys. Rev. D 102, 114521 (2020)
Davoudi and Kadam Phys. Rev. Lett. 126, 152003 (2021)

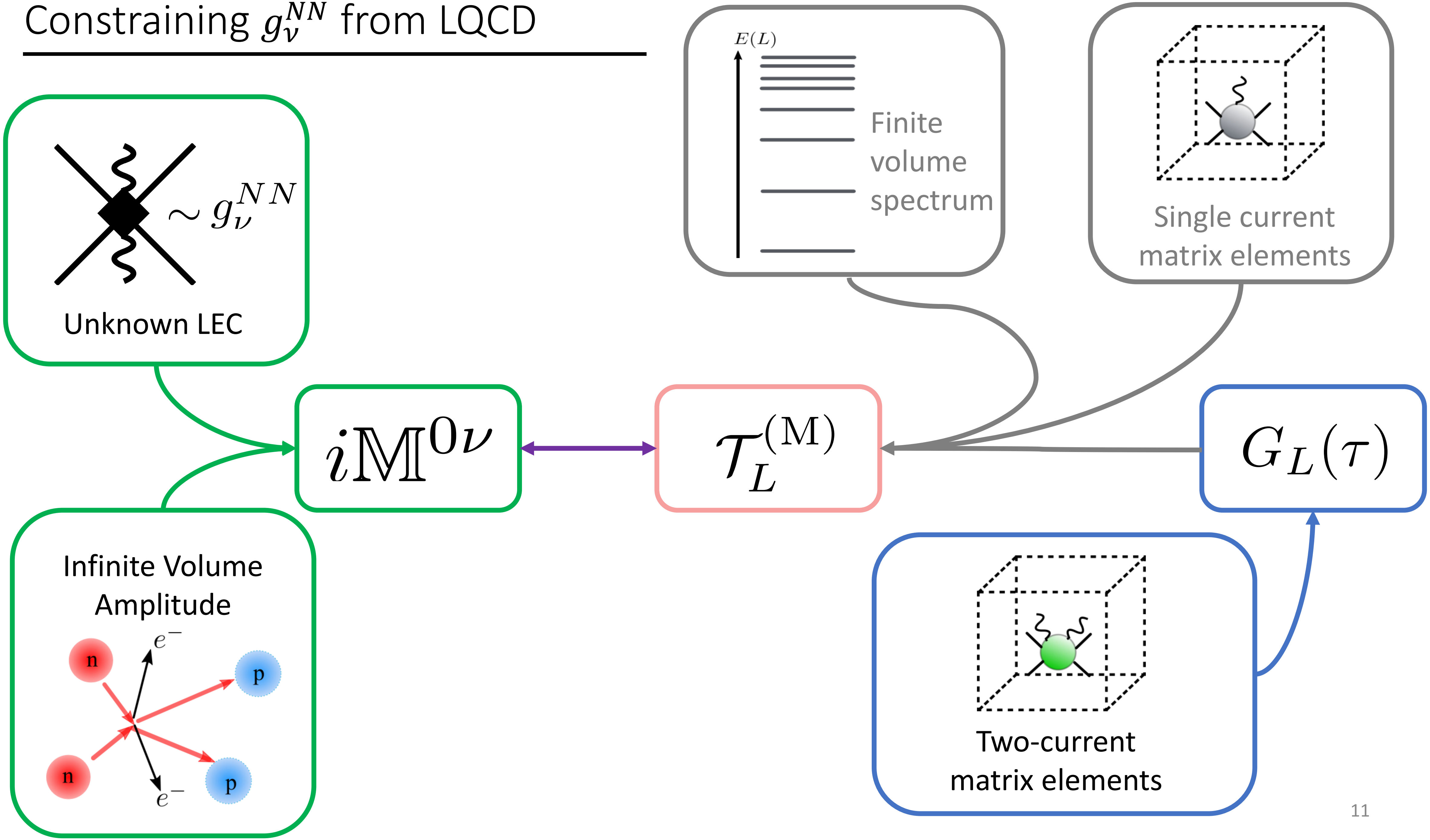
$$i\mathbb{M}^{0\nu} = i\mathcal{M}^{(\text{Int.})}_{nn\to pp} - m_{\beta\beta}(1+3g_A^2)\mathcal{M}_{nn}\delta J^V \mathcal{M}_{pp}$$

$$L^6 \left|\mathcal{T}_L^{(\text{M})}\right|^2 = \left|\mathcal{R}^*(E_{n_f})\right| \left|i\mathbb{M}^{0\nu}\right|^2 \left|\mathcal{R}^*(E_{n_i})\right|$$

Lellouch Luscher residue matrix

$$\mathcal{T}_L^{(M)} = \int_L d^3z \int dz_0\, e^{iE_1 z_0} \left[\langle E_{n_f}|\, T^{(M)}[\mathcal{J}(z)\, S_\nu(z)\, \mathcal{J}(0)]\, |E_{n_i}\rangle\right]_L$$

Constraining g_ν^{NN} from LQCD



$\mathcal{T}_L^{(\mathrm{M})}$

Minkowski Signature Correlation Function

$$\mathcal{T}_L^{(M)} = \int_L d^3z \int dz_0 \, e^{iEz_0} \left[\langle E_{n_f} | \, T^{(M)}[\mathcal{J}(z) \, S_\nu(z) \, \mathcal{J}(0)] \, | E_{n_i} \rangle \right]_L$$

?

$G_L(\tau)$

Euclidean Time Four-point Correlation Function from LQCD

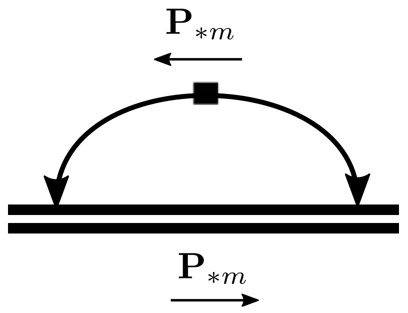
$$G_L(\tau) = \int_L d^3z \left[\langle E_f, L | \, T^{(\mathrm{E})}[\mathcal{J}^{(\mathrm{E})}(\tau, z) \, S_\nu^E(\tau, z) \mathcal{J}^{(\mathrm{E})}(0)] \, | E_i, L \rangle \right]_L ,$$

Plugging back the missing time integral

$$\mathcal{T}_L^{(\mathrm{E})} \stackrel{?}{=} \int d\tau e^{E\tau} G_L(\tau) \quad \sim \int_0^\infty d\tau \, e^{-(|\mathbf{P}_{*m}| + E_{*m} - E_*)\tau}$$

Diverges for $|\mathbf{P}_{*m}| + E_{*m} < E_*$

$$E_* = E_i - E$$



Diverges for intermediate states that can go on-shell

Need to remove these divergences for analytic continuation!!

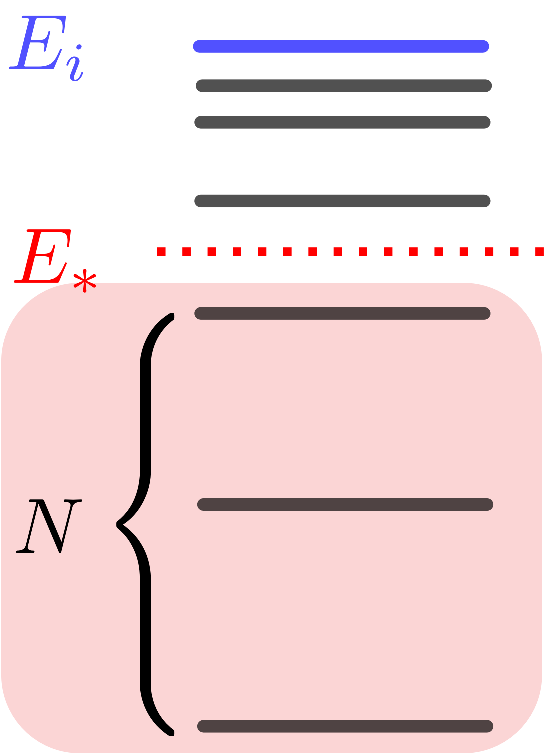
Removing Divergences from $G_L(\tau)$

Davoudi and Kadam
Phys. Rev. Lett. 126, 152003 (2021)

Where is the divergence coming from?

Two particle FV states which can go on-shell

$$|\mathbf{P}_{*m}| + E_{*m} < E_*$$



Finite Volume Two Particle Spectrum

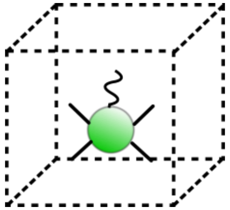
Constructing divergent contributions

Spectral representation by integrating over time

$$\mathcal{T}_L^{(\mathrm{M})} = i \sum_{m=0}^{\infty} \frac{c_m}{E_* - E_{*m} - |\mathbf{P}_{*m}| + i\epsilon}$$

Two-body spectrum to identify N low-lying states

$c_m \sim$



Finite volume matrix elements of single hadronic current between the initial (final) and intermediate states

$$G_L^{<}(\tau) \equiv \sum_{m=0}^{N-1} c_m\, \theta(\tau)\, e^{-(|\mathbf{P}_{*m}| + E_{*m} - E_f)|\tau|}$$

Can be analytically continued to Minkowski space

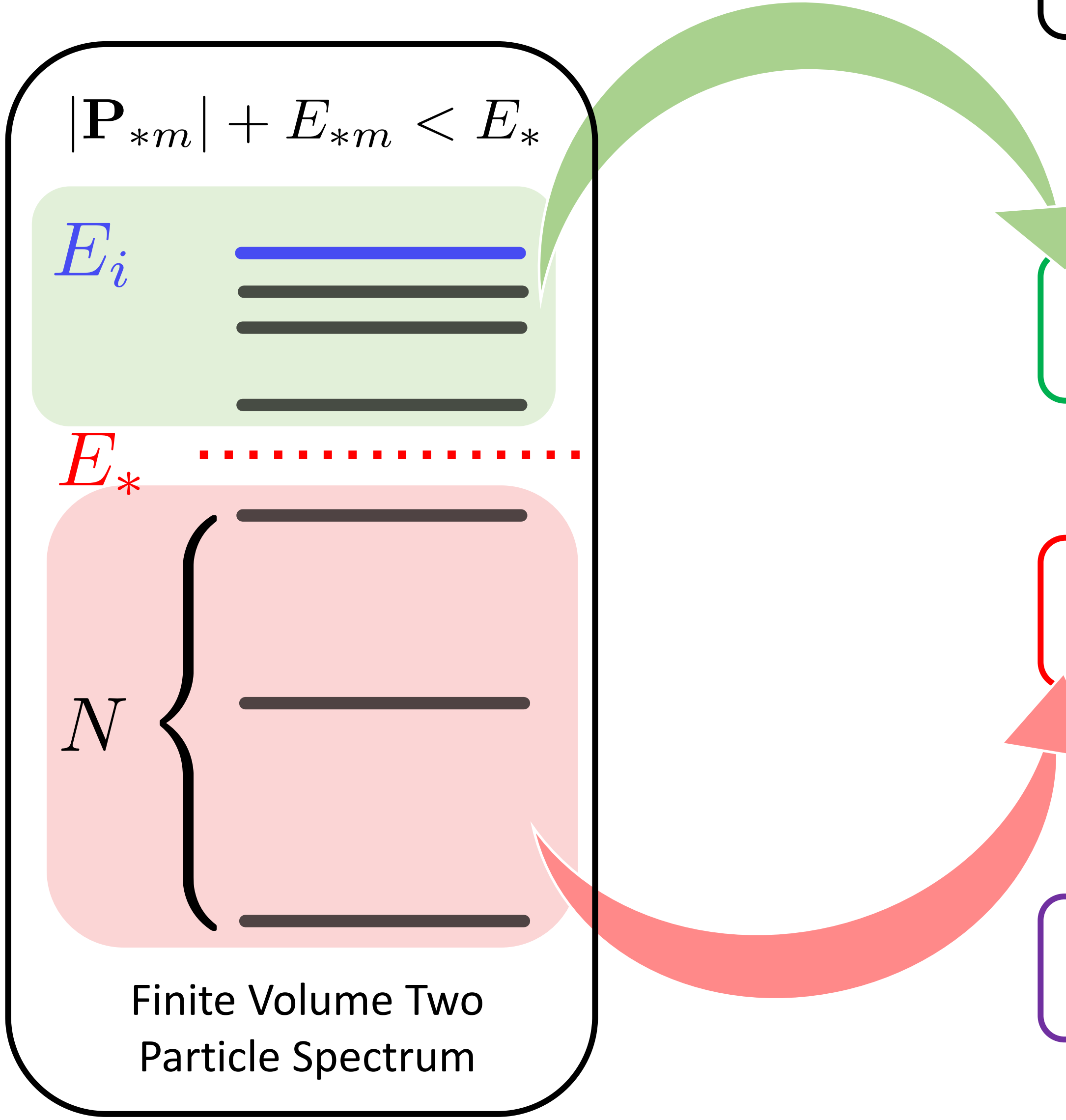
$G_L(\tau)$

Four-point Function From LQCD

$$\mathcal{T}_L^{(\mathrm{E})\,\geq} \equiv \int d\tau\, e^{E\tau} \left[G_L(\tau) - G_L^{<}(\tau)\right]$$

What do we want?

$$\mathcal{T}_L^{(\mathrm{M})} = i\sum_{m=0}^{\infty} \frac{c_m}{E_* - E_{*m} - |\mathbf{P}_{*m}| + i\epsilon}$$



So far, we have

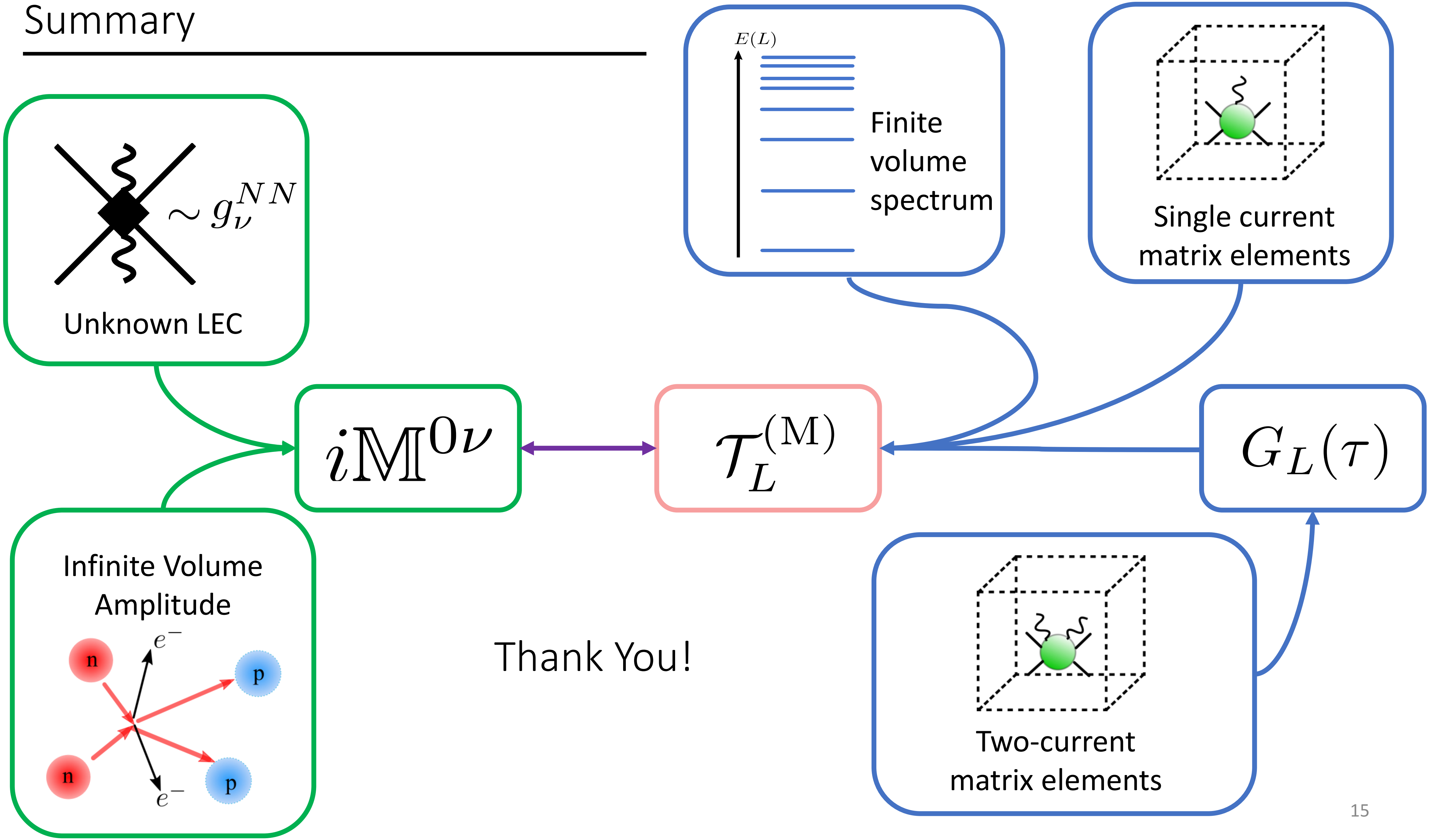
$$\mathcal{T}_L^{(\mathrm{E})\,\geq} \equiv \int d\tau\, e^{E\tau}\left[G_L(\tau) - G_L^{<}(\tau)\right]$$

Missing Piece

$$\mathcal{T}_L^{(\mathrm{E})<} \equiv \sum_{m=0}^{N-1} \frac{c_m}{E_* - E_{*m} - |\mathbf{P}_{*m}|}$$

$$\mathcal{T}_L^{(\mathrm{M})} = i\mathcal{T}_L^{(\mathrm{E})\,<} + i\mathcal{T}_L^{(\mathrm{E})\,\geq}$$

Summary



Thank You!