

# Towards a lattice determination of the form factors of the rare hyperon decay $\Sigma^+ \rightarrow p\ell^+\ell^-$

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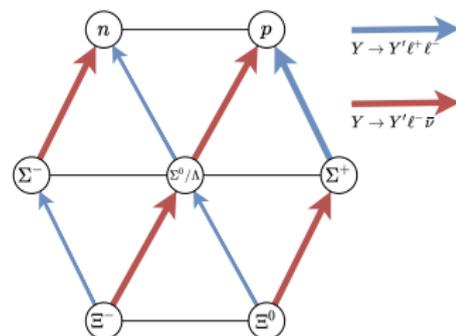
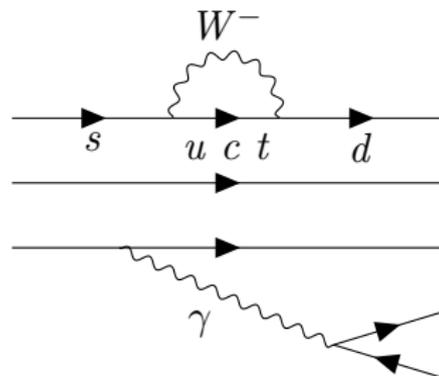
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## Main topic

- $\Sigma^+ \rightarrow p \ell^+ \ell^-$  is an  $s \rightarrow d$  FCNC process
- Sensitive to new physics
- Baryonic equivalent to the rare kaon decay  $K \rightarrow \pi \ell^+ \ell^-$   
[Fionn Ó hÓgáin Wed 5:30 EDT]

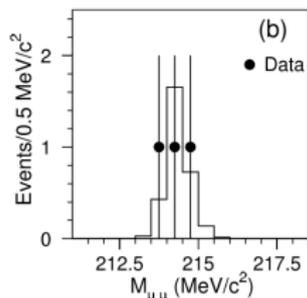
## Secondary topic

- $\Sigma^- \rightarrow n \ell^- \bar{\nu}$  et al. are  $s \rightarrow u$  processes
- Alternate measurement of  $V_{us}$



# Motivation: Experiment

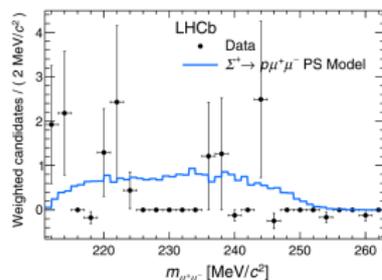
## Hyper CP (2005)



- Observed 3 events
- $\mathcal{B}(\Sigma^+ \rightarrow p\mu^+\mu^-)_{HCP} = 8.6_{-5.4}^{+6.6} \pm 5.5 \times 10^{-8}$
- Originally thought this could be signal for a new particle of mass  $\simeq 214 MeV$

[HyperCP 10.1103/PhysRevLett.94.021801]

## LHCb (2018)



- $\mathcal{O}(10)$  events
- $\mathcal{B}(\Sigma^+ \rightarrow p\mu^+\mu^-)_{LHCb} = 2.2_{-1.3}^{+1.8} \times 10^{-8}$
- No evidence for new particle
- Interest in improving measurement with the latest data, including first measurement of  $\mathcal{B}(\Sigma^+ \rightarrow pe^+e^-)$

[LHCb 10.1103/PhysRevLett.120.221803]

- SM prediction from combination of experimental input, unitarity cuts, vector meson dominance and Chiral Perturbation Theory

$$1.6 \times 10^{-8} < \mathcal{B}(\Sigma^+ \rightarrow p\mu^+\mu^-) < 9.0 \times 10^{-8}$$

- Short distance contribution  $\mathcal{O}(10^{-12})$
- Dominated by the long distance contribution

$$\Sigma^+ \rightarrow p\gamma^* , \gamma^* \rightarrow \mu^+\mu^-$$

- Large range mainly coming from different types of ChPT

[X. He 10.1103/PhysRevD.72.074003]

[X. He et al. 10.1007/JHEP10(2018)040]

- Long distance part  $\Sigma^+ \rightarrow p\gamma^*$  given by amplitude

$$\mathcal{A}_\mu = \langle p | T \{ H_w J_\mu \} | \Sigma \rangle$$

- $J_\mu$ : EM current       $H_w$ :  $s \rightarrow d$  effective weak Hamiltonian

$$H_w(x) = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} [C_1(Q_1^u - Q_1^c) + C_2(Q_2^u - Q_2^c) + \dots] + \text{c.c.}$$

with the 4-quark operators

$$Q_1^q = (\bar{s}\gamma_\mu^L d)(\bar{q}\gamma_\mu^L q) \quad Q_2^q = (\bar{s}\gamma_\mu^L q)(\bar{q}\gamma_\mu^L d)$$

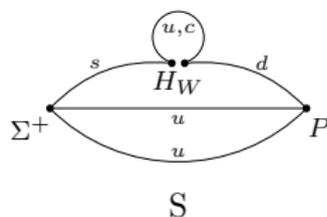
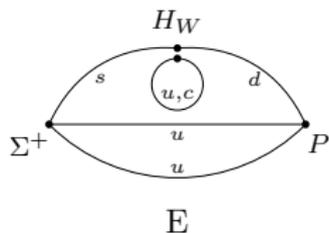
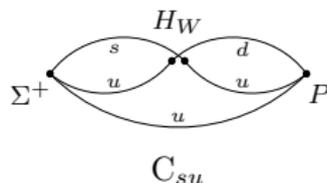
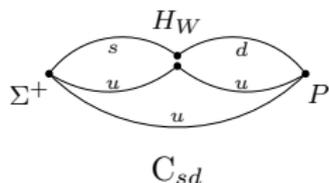
- Form factor decomposition

$$\mathcal{A}_\mu = \bar{u}_p [(q^2 \gamma_\mu - \not{q} q_\mu) (f_1 + g_1 \gamma_5) + \sigma_{\mu\nu} q^\nu (f_2 + g_2 \gamma_5)] u_\Sigma$$

- Can be extracted from the 4pt correlation function

$$C^4 = \langle \psi_N(\Delta t) H_w(t_H) J_\mu(t_J) \bar{\psi}_\Sigma(0) \rangle$$

- Requires computation of diagrams of the type



[[github.com/paboyle/Grid](https://github.com/paboyle/Grid)]



[[github.com/aportelli/Hadrons](https://github.com/aportelli/Hadrons)]

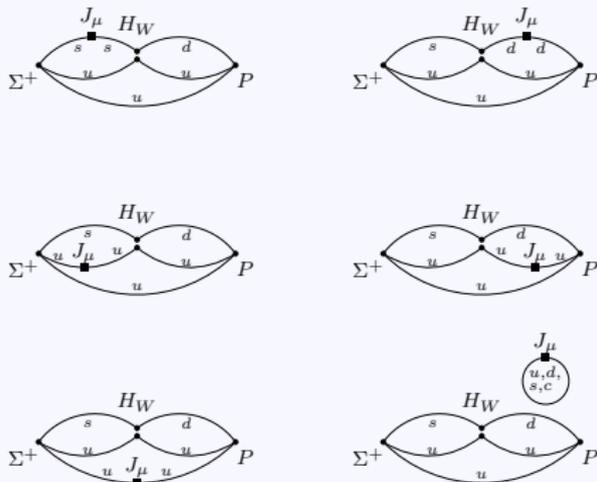
# Lattice Theory

- Can be extracted from the 4pt correlation function

$$C^4 = \langle \psi_N(\Delta t) H_w(t_H) J_\mu(t_J) \bar{\psi}_\Sigma(0) \rangle$$

Including vector current insertions e.g.

- Requi



[[github.com/paboyle/Grid](https://github.com/paboyle/Grid)]



[[github.com/aportelli/Hadrons](https://github.com/aportelli/Hadrons)]

# Rare K Style Approach

- Can use method very similar to the rare Kaon decay  $K \rightarrow \pi \ell^+ \ell^-$   
[Fionn Ó hÓgáin Wed 5:30 EDT] [\[N. Christ et al. 10.1103/PhysRevD.92.094512\]](#)  
[\[N. Christ et al. 10.1103/PhysRevD.94.114516\]](#)
- Fix EM current position and integrate  $t_H$  in a window around  $t_J$

$$I(T_a, T_b) = \int_{t_J - T_a}^{t_J + T_B} dt_H \langle \psi_N(\Delta t) H_w(t_H) J_\mu(t_J) \bar{\psi}_\Sigma(0) \rangle$$

- Growing exponential contamination in  $T_a$  from intermediate states  $N$  and  $N\pi$  with  $E < E_\Sigma$
- Can construct and remove  $N\pi$  contribution similarly to  $\pi\pi$  state in  $K \rightarrow \pi \nu \bar{\nu}$  decay [\[N. Christ et al. 10.1103/PhysRevD.100.114506\]](#)
- Understanding of finite volume corrections from  $N\pi$  state in progress

# 3pt Summed Method

Summed method where operator insertion is summed over all time

[L. Maiani et al. 10.1016/0550-3213(87)90078-2]

[C. Bouchard et al. 10.1103/PhysRevD.96.014504]

$$\begin{aligned} I^3(\Delta t) &= \sum_{t=0}^T \langle \psi_f(\Delta t) J(t) \bar{\psi}_i(0) \rangle \\ &= \sum_{n,m} \left[ Z_{0n}^i Z_{m0}^f J_{nm} \frac{1 - e^{-(E_n - E_m)(\Delta t - 1)}}{e^{E_n - E_m} - 1} e^{-E_m \Delta t} \right] \\ &+ \sum_n \left[ Z_{0n}^i Z_{n0}^f J + \sum_l \frac{Z_{0n}^i Z_{nl}^f J_{l0}}{e^{E_l} - 1} \right] e^{-E_n \Delta t} \\ &+ \sum_m \left[ Z_{0m}^i Z_{m0}^f J + \sum_l \frac{J_{0l} Z_{lm}^i Z_{m0}^f}{e^{E_l} - 1} \right] e^{-E_m \Delta t} + \sum_k \cancel{Z_{0k}^i Z_{k0}^f J_{00}} e^{-E_k \Delta t} \end{aligned}$$

- First line gives regular linear  $\Delta t$  behaviour for  $n = m$  states
- Second and third lines give combination of contact terms ( $t = 0$  and  $t = \Delta t$ ) and out of order terms ( $t > \Delta t$ )

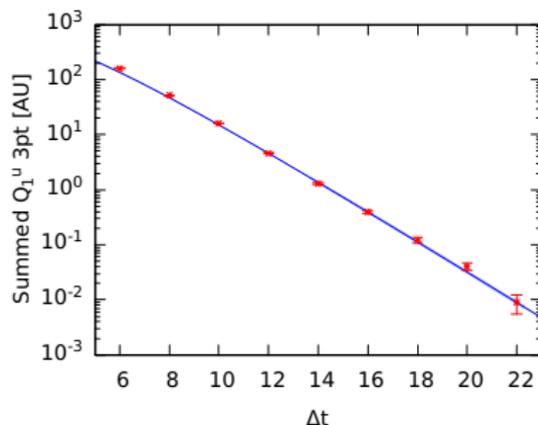
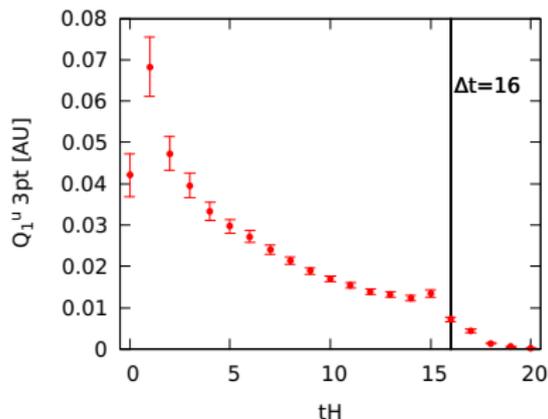
- Can extend the summed method to  $\Sigma^+ \rightarrow p\gamma^*$  4pt function

$$I_\mu^4(\Delta t) = \int_0^T dt_H \int_0^T dt_J \langle \psi_N(\Delta t) H_w(t_H) J_\mu(t_J) \bar{\psi}_\Sigma(0) \rangle$$

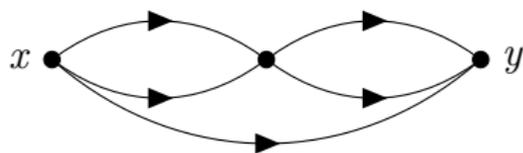
- Get contamination from intermediate states  $N, \Sigma, N\pi, \Sigma\pi \dots$   
Can in theory be constructed and removed
- Full understanding of contact and out of order terms in progress
- Investigating practicality of this method
- Could reduce cost of simulations of the rare hyperon decay and rare Kaon decay

# Preliminary Results

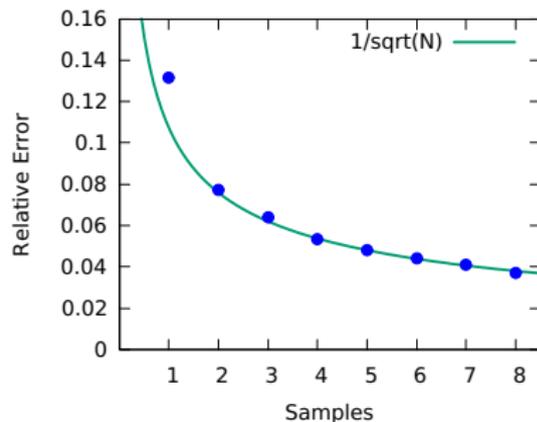
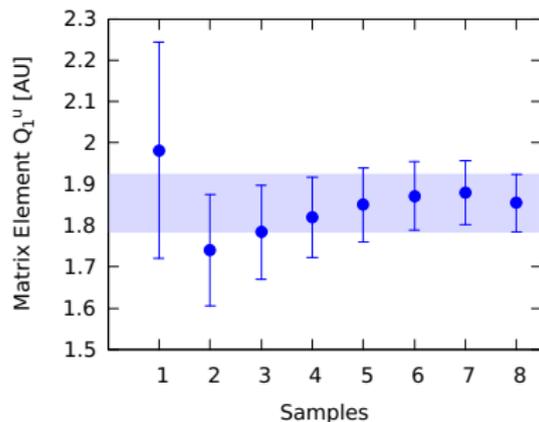
- Working towards a first exploratory calculation of this matrix element on a larger than physical pion mass ensemble
- RBC UKQCD 2 + 1 flavour  $24^3 \times 64$  Iwasaki gauge configurations  $a^{-1} = 1780 \text{ MeV}$  and  $m_\pi \simeq 340 \text{ MeV}$  using Shamir domain wall fermions
- 3pt correlator  $\langle \psi_N(\Delta t) H_w(t_H) \bar{\psi}_\Sigma(0) \rangle$  using Gaussian smearing without summing (left) and summed (right)



# Source-Sink Sampling

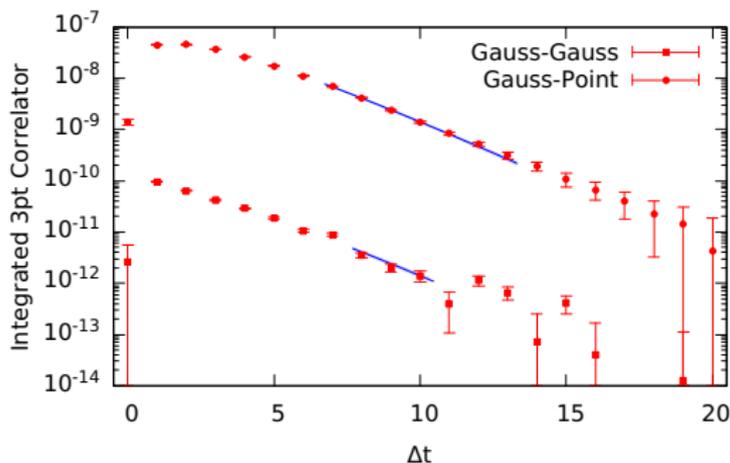


- Positions  $x$  and  $y$  fixed in solves
- Volume sum for momentum projection requires  $\sim 14,000$  solves on  $24^3 \times 64$  lattice
- Approximate with sum over  $N$  random position samples
- Could get up to  $1/N$  error scaling [Y. Li et al. 10.1103/PhysRevD.103.014514]
- Only observe approx.  $1/\sqrt{N}$  scaling for this quantity



# $s \rightarrow u \ell \bar{\nu}$ Semileptonics

- Similar project to compute  $SU(3)$  breaking corrections to  $s \rightarrow u \ell \bar{\nu}$  hyperon transitions at close to physical pion mass
- RBC UKQCD 2 + 1 flavour  $48^3 \times 96$  Iwasaki gauge configurations with  $a^{-1} = 1730 \text{ MeV}$  and  $m_\pi \simeq 140 \text{ MeV}$  using Möbius domain wall fermions
- Relevant for a baryonic determination of CKM matrix element  $V_{us}$
- Using 3pt summed method



## Summary

- SM prediction of  $\Sigma^+ \rightarrow p\ell^+\ell^-$  is a quantity of interest
- Have a method to compute on the lattice and working towards a calculation at larger than physical pion mass
- Also working on  $s \rightarrow u$  hyperon transitions at the physical point

## Future outlook

- Compute  $s \rightarrow u$  hyperon transitions with second lattice spacing
- Compute  $\Sigma^+ \rightarrow p\ell^+\ell^-$  decay at the physical point



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