Hadronic light-by-light contribution to $(g-2)_\mu$ from lattice QCD: a complete calculation

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Based on E.-H. Chao and R. J. Hudspith *et al*, arXiv:2104.02632

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Introduction

- Motivation: resolving the 4.2σ tension between theory and experiment for the anomalous magnetic moment of the muon a_μ. [T. Aoyama *et al*, Phys. Rept. '20; B. Abi *et al*, PRL '21]
- The Hadronic Light-by-Light (**HLbL**) contribution enters at subleading order ($O(\alpha_{\mathrm{QED}}^3)$) but with sizable error.
- Estimates from dispersive approach are available and a lattice determination with $\mathrm{QED_L}$ exists. [T. Blum *et al*, PRL '20]
- Our approach: QED in the continuum and infinite volume + QCD on the lattice to mitigate volume effects due to the photon. [N. Asmussen *et al*, LATTICE '19]

$$a_\mu^{\mathrm{hlbl}} = \frac{m_\mu e^6}{3} \int_{x,y} \mathcal{L}_{[\rho,\sigma];\mu\nu\lambda}(x,y) i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) \,,$$

$$i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma} = -\int_z z_\rho \tilde{\Pi}_{\mu\nu\sigma\lambda} \,, \quad \tilde{\Pi}_{\mu\nu\sigma\lambda}(x,y,z) \equiv \langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0)\rangle_{\mathrm{QCD}} \,,$$

where j_μ's are electromagnetic currents. In $N_f = 3$,

$$j_\mu(x) = \frac{2}{3}(\bar{u}\gamma_\mu u)(x) - \frac{1}{3}(\bar{d}\gamma_\mu d)(x) - \frac{1}{3}(\bar{s}\gamma_\mu s)(x) \,.$$

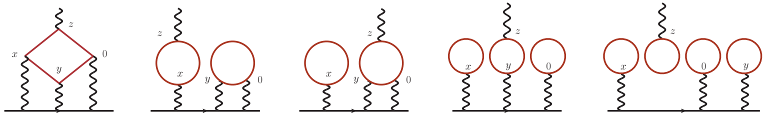
Theory background

- O(4)-symmetry restoration of the QED kernel allows to write

$$a_\mu^{\rm hlbl} = \lim_{|y|_{\rm max}\to\infty} a_\mu^{\rm hlbl}(|y|_{\rm max})\,, \quad a_\mu^{\rm hlbl}(|y|) = \int_0^{|y|_{\rm max}} d|y| f(|y|)\,.$$

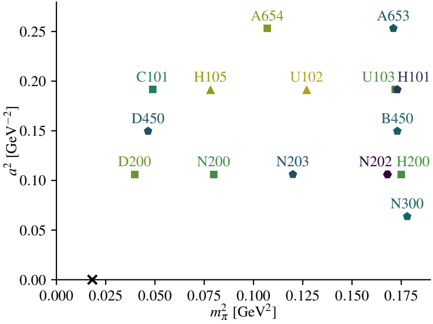
  $\Rightarrow$ compute the integrand $f(|y|)$ for each $|y|$ and get the $|y|$-integral using trapezoidal rule.
- Terminology:
  - **Leading topologies**: fully-connected, (2+2)
  - **Subleading topologies**: (3+1), (2+1+1), (1+1+1+1)

  Motivated by light pseudoscalar (PS) meson contributions and large-N_c arguments.
- Translational invariance + change of variables $\Rightarrow$ compute $a_\mu^{\rm hlbl}$ for each topology from only a subset of "easy" diagrams. [E.-H. Chao *et al*, EPJC '20]
- Focus of this talk: the leading topologies with purely light quarks and the (3+1) with a light quark "triangle".



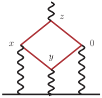
Numerical setup

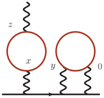
- $N_f = 2 + 1$ ensembles generated by the CLS consortium.
- Noise reduction: point-source solve with self-averaging on $f(|y|)$
- Single-propagator trace available in position-space via the One-End-Trick
  [K. Jansen, C. Michael & C. Urbach, EPJC '08; L. Giusti *et al*, EPJC '19]
- The meson 2-point functions saved in position space for the (2+1+1+1).



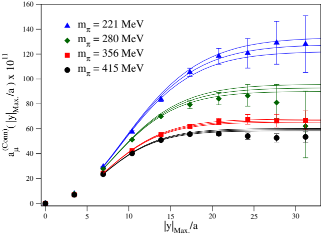
Leading topologies with purely light quarks

Data analysis

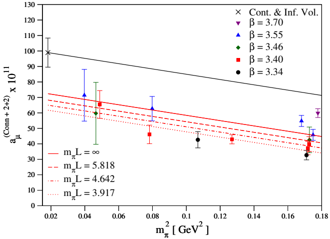




- Tail reconstruction of the leading topology data with a functional ansatz inspired by the π^0-exchange.
- Sizeable cancellation between the leading topologies from π^0-exchange $\Rightarrow$ a flatter chiral extrapolation with the combined data.
- A simultaneous chiral, continuum and infinite-volume extrapolation is applied to get a_μ^{hlbl} at the physical point.



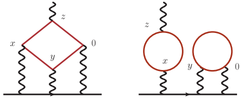
Tail reconstruction for the fully-conn.



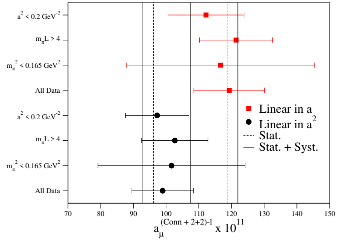
Global fit linear in m_π^2 and in a^2.

Leading topologies with purely light quarks

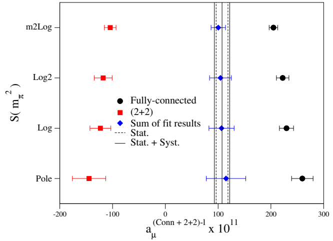
Systematic error estimation



- Continuum extrapolation: a v.s. a^2
  $\Rightarrow$ fit to a constant and assign the r.m.s deviation of the central values from different cut datasets for the systematic error.
- No clear curvature in m_π^2 in the combined data
  $\Rightarrow$ consistency check with seperately fitted data with various curvatures in m_π^2.
- New after review: strong constraint from our SU(3)$_f$-symmetric data
  $\Rightarrow$ replacing m_π^2 by $\log(m_\pi^2)$, inspired by the light PS meson prediction

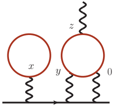


a v.s. in a^2, with different cuts in the data.

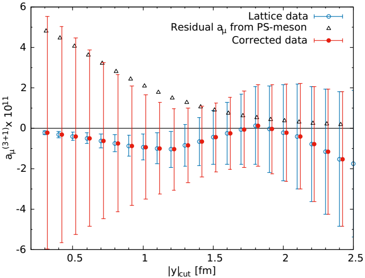


Combined v.s. summed seperate fit results.

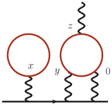
Subleading topology: $(3+1)_{\text{light}}$



- Light quark triangle + $(l-s)$-disc. loop:
  $\propto \pi^{\pm}$-loop $-$ $K^{\pm}$-loop according to Partially Quenched ChPT.
- Noisy data consistent with 0 $\Rightarrow$ truncation criteria of the integrand needed.
- Procedure:
  1. Compute the residual contribution from the PS-loop δa_μ^{PS} down to $|y| = |y|_{\text{cut}}$.
  2. Estimate the systematic error of the truncation with $w_{\text{sys.}} \times \delta a_\mu^{\text{PS}}$.
  3. The central value is taken from the lattice data and the total error is the stat. + syst. added in quarature.
  4. Choose $|y|_{\text{cut}}$ by minimizing the total error.



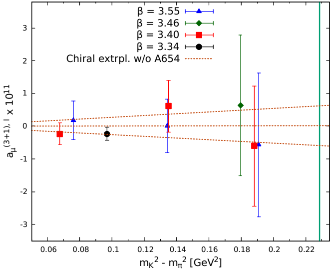
Subleading topology: $(3+1)_{\text{light}}$



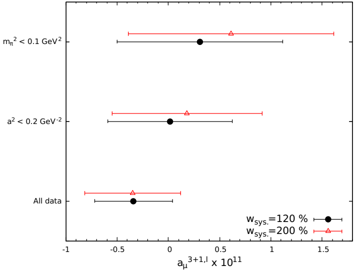
- Noisy data, no clear diverging trend in m_π^2
  $\Rightarrow$ to avoid over-fitting, use only the simplest ansatz with a constraint from the $\mathrm{SU}(3)_f$:

$$a_\mu^{(3+1)_{\text{light}}} = A(m_K^2 - m_\pi^2)\,.$$

- Very conservative choice for $w_{\text{sys.}} = 120\%$, consistency between different cut data sets and for other more conservative choices of $w_{\text{sys.}}$.
- Final result quoted from the fit with the coarsest lattice spacing dropped.



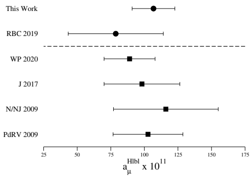
Chiral extrapolation.



Consistency check.

Final result, summary and outlook

- Our estimate for a_μ^{hlbl} at the physical point: $a_\mu^{\mathrm{hlbl}} = 106.8(15.9) \times 10^{-11}$.
- We provide the first lattice determination of a_μ^{hlbl} at the physical point including all 5 Wick-contraction topologies from the 4-pt function, with an overall precision of 15%.
- The required precision for a_μ^{hlbl} is met and the HLbL does not seem to be able to explain the discrepancy between theory and experiment.
- Outlook:
  - A better understanding of the hadronic composition of HLbL by describing the lattice integrand.
  - Identification of the charged PS-loop contribution.
  - Further investigation of the chiral behavior of a_μ^{hlbl}.



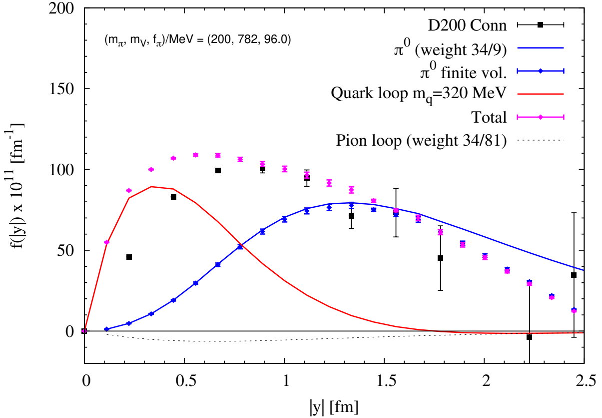
Contribution	Value $\times 10^{11}$
Light-quark fully-conn. and $(2+2)$	$107.4(11.3)_{\mathrm{stat.}}(9.2)_{\mathrm{syst.}}(6.0)_{\mathrm{chiral}}$
Strange-quark fully-conn. and $(2+2)$	$-0.6(2.0)$
$(3+1)$	$0.0(0.6)$
$(2+1+1)$	$0.0(0.3)$
$(1+1+1+1)$	$0.0(0.1)$
Total	$106.8(15.9)$

A breakdown of our result for a_μ^{hlbl}.

Back-up slides

Attempt of modelling the integrand

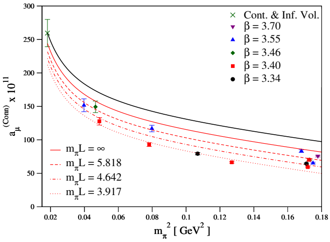
- Modelling of the fully-conn. integrand with π^0-exchange, $\pi^\pm$-loop and a constituent quark for D200 ($m_\pi \approx 200$ MeV).



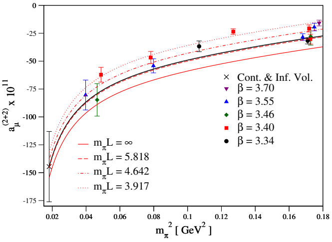
Individual leading topologies with purely light quarks

- Leading topologies fitted individually to the ansatz

$$a_\mu(m_\pi^2, m_\pi L, a^2) = Ae^{-m_\pi L/2} + Ba^2 + C\frac{1}{m_\pi^2} + D + Em_\pi^2 \, .$$



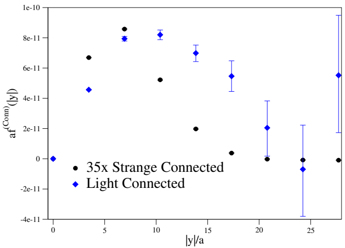
Fully-connected.



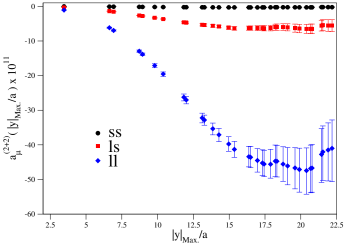
(2+2).

Strange quark: leading topologies

- Comparisons between light- and strange-quark contributions for the leading topologies on the ensemble C101 ($m_\pi \approx 220$ MeV, $m_K \approx 460$ MeV).



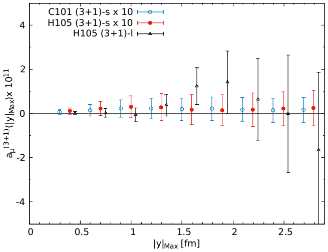
Light v.s. strange fully-conn.



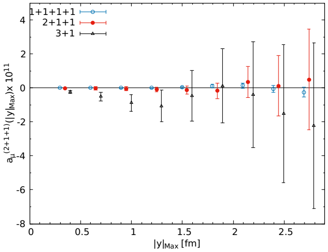
The (2+2) with different quark-loop combinations.

Other subleading contributions

- Comparison between the $(3{+}1)_{\rm light}$ and $(3{+}1)_{\rm strange}$: small, no strong PS meson dependence.
  H105: $(m_\pi, m_K) \approx (280, 470)$ MeV; C101: $(m_\pi, m_K) \approx (220, 460)$ MeV.
- The (2+1+1) with light quarks in the 2-point meson loop and the (1+1+1+1) are numerically negligible compared to the $(3{+}1)_{\rm light}$. Neither π^0-exchange nor charged-PS loop contribute at leading order in Partially Quenched ChPT.



Light v.s. strange (3+1), H105 and C101.



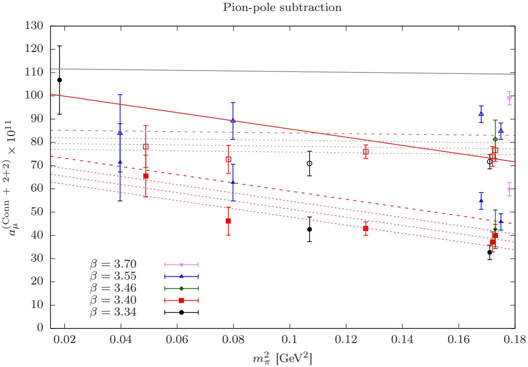
Comparison between different subleading topologies, C101.

An alternative chiral extrapolation

- Subtract the π^0-exchange prior to the chiral extrapolation:

$$a_\mu^{\mathrm{hlbl}}(m_\pi) - a_\mu^{\mathrm{hlbl},\pi^0}(m_\pi) + a_\mu^{\mathrm{hlbl},\pi^0}(m_\pi^{\mathrm{phys.}})\,.$$

- Flatter extrapolation with an ansatz linear in m_π^2 and in a^2.



Black lines: π^0-exchange subtracted. Red lines: original tail-reconstructed data.