CONTINUUM EXTRAPOLATION OF THE HVP

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Borsanyi,Fodor,Guenther,Hoelbling,Katz,Lellouch,Lippert Miura,
Parato, Stokes, Toth, Torok, Varnhorst

[2002.12347,Nature (2021)] Leading-order hadronic vacuum polariz ...

Lattice ensembles

β	a[fm]	#conf
3.7000	0.1315	900
3.7500	0.1191	2100
3.7753	0.1116	1900
3.8400	0.0952	3100
3.9200	0.0787	4300
4.0126	0.0640	7000

- six lattice spacings
- (Goldstone) pion/kaon around the physical point
- statistical precision a_μ^{light} ca. 0.2% for each lattice spacing $\rightarrow$ increase statistics for finer lattices

- (one type of) cont. extrapolation systematic error by skipping 0,1,2,3 of the coarsest lattices

Lattice spacing dependence

staggered naive scaling is a^2, can be modified by logarithms
$\rightarrow a^2/\log(a)^\Gamma$ What is the value of Γ?

- For $O(N)$ model $\Gamma < 0$. For pure YM with Wilson action $\Gamma > 0$, probably also for full QCD. [Husung et al '19]
- Major staggered artefact (taste violation) scales naively $a^2\alpha_s(\frac{1}{a})$, ie. $\Gamma = 1$. We observe approximately $a^2\alpha_s^3(\frac{1}{a})$, ie. $\Gamma \approx 3$
- Note, there can be $\Gamma < 0$ exponent in short-distance part of a_μ, probably relevant only for charm. [Ce et al '21]

Use two types of power series:

1. $A_0 + A_1\left[a^2\right] + A_2\left[a^2\right]^2$
2. $A_0 + A_1\left[a^2\alpha_s^3(\frac{1}{a})\right] + A_2\left[a^2\alpha_s^3(\frac{1}{a})\right]^2$

Difference is (another type of) systematic error of cont. extrapolation.

Lattice artefacts

%= Interpolate lattice data to a lattice spacing of $a = 0.10$ fm and compute its deviation from the quoted continuum limit.

- strange and window observable around $2 - 3\%$
- light observable much “larger” 11%, due to taste violation

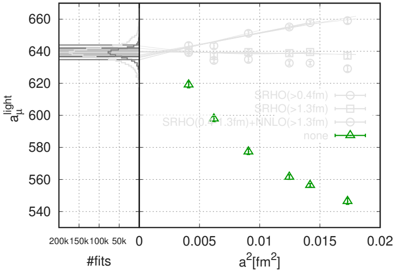
Comparison with other discretizations:

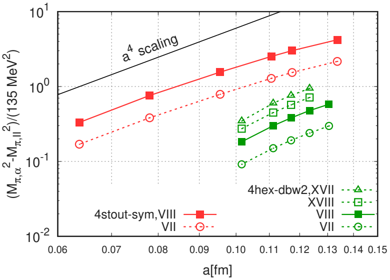
	total	window
twisted mass (ETM/Rome)	28%	-
Wilson (Mainz)	18%	14%
HISQ (Aubin et al)	9%	4%
HISQ (FHM)	3%	0%
domain wall (RBC/UKQCD)	-	3.5%

Taste violation

a_μ^{light}

- not only “larger”, but deviates from a^2-linear behavior
- need high precision data to see the curvature

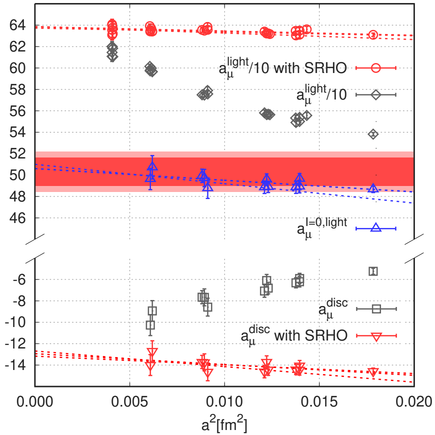




Pion splitting

- decreases faster than naive a^2 - more like $\approx a^2 \alpha_s^3(\frac{1}{a}) \approx a^4$
- simple a^2 fit can mislead

Isospin decomposition



- "large" lattice artefacts both in connected a_μ^{light} and disconnected a_μ^{disc}

- isospin combinations

$$a_\mu^{\mathrm{light}} = \frac{10}{9} a_\mu^{\mathrm{I=1}}$$

$$a_\mu^{\mathrm{disc}} = a_\mu^{\mathrm{I=0}} - \frac{1}{9} a_\mu^{\mathrm{I=1}}$$

- isoscalar $I = 0$ continuum extrapolation is much easier - starts with three pions and contains narrow ω resonance

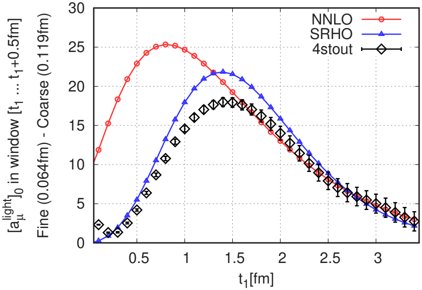
- taste violation comes mainly from isovector $I = 1$ channel - two pions and ρ resonance

Taste improvement I

Describe taste violation:

1. field theory model of pion/rho/photon (SRHO) $\rightarrow$ depends on rho parameters [Jegerlehner,Szafron'11;HPQCD'16]
2. staggered chiral perturbation theory (NNLO) $\rightarrow$ depends only on 1 LEC (l_6) [Lee,Sharpe,VandeWater,Bailey $\geq$'98]

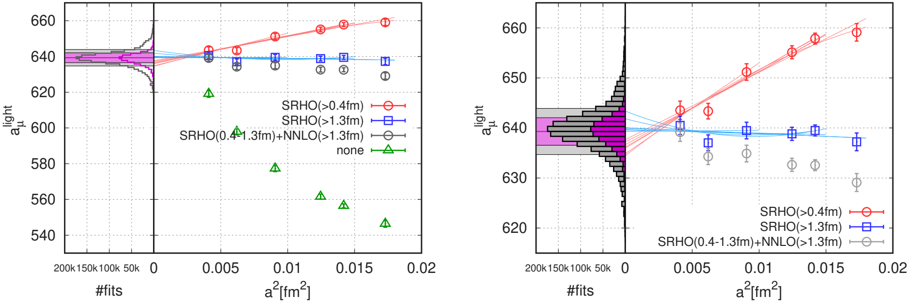
Compare it to lattice data:



- for $t \gtrsim 2.0$ fm all reproduce lattice artefacts well (not fits)
- SXPT breaks down below ca. 1.3 fm - resonance contrib. is missing.
- SRHO seems to work well over the whole range

Taste improvement II

- $a_\mu(a) \to a_\mu(a) - a_\mu^{\mathrm{SRHO}}(a) + a_\mu^{\mathrm{RHO}}$
- reduces lattice artefact, also makes a^2 dependence linear

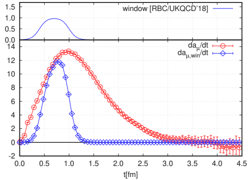


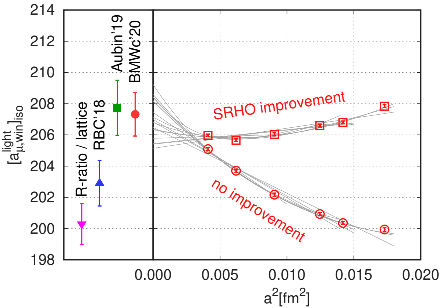
SRHO improvement gives central value. Systematic errors by:

1. change starting point of improvement $t = 0.4 \to 1.3$ fm
2. skip coarse lattices
3. change $\Gamma = 0$ and $\Gamma = 3$
4. replace SRHO by NNLO SXPT above 1.3 fm

Window observable

- restrict correlator to window 0.4 – 1.0 fm [RBC/UKQCD'18]
- fewer difficulties



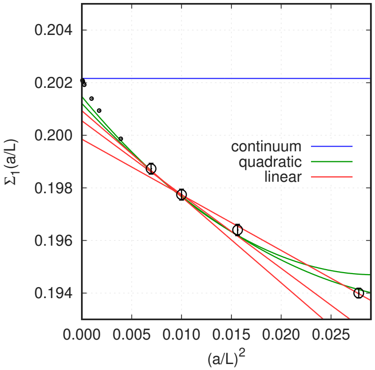


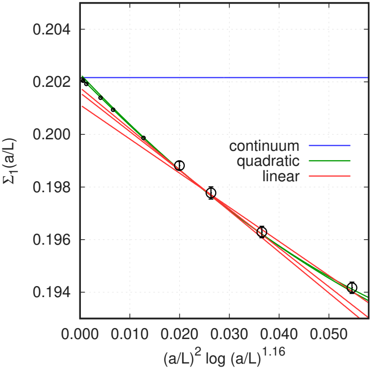
Cont. extrap. systematics:

1. SRHO vs no improvement
2. $\Gamma = 0$ or 3
3. linear, quadratic or cubic
4. skip coarse lattices

Lattice spacing dependence

$O(N)$ nonlinear sigma model $\Gamma = -1.16$ [hep-lat/0506010]

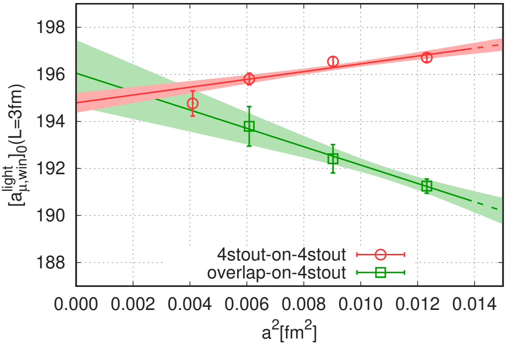




$A_0 + A_1 a^2 + A_2 a^4$

$A_0 + A_1[a^2/\log(a)^\Gamma] + A_2[a^2/\log(a)^\Gamma]^2$

Crosscheck - overlap



- compute $a_{\mu,\mathrm{win}}$ with overlap valence
- local current instead of conserved $\rightarrow$ had to compute Z_V
- cont.limit in $L = 3$ fm box consistent w/ staggered valence