Pseudoscalar Transition Form Factors and the Hadronic Light-by-Light Contribution to a_μ

η AND η' MESONS

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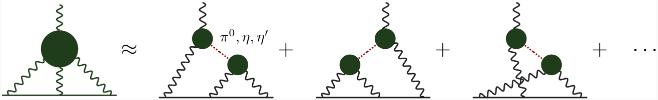
Contribution to LATTICE2021

Motivation

Aim of the project:

1. **Calculate $\eta^{(\prime)}$ meson masses and mixing angles** on the lattice with staggered fermions:
    - Test of QCD (related to $U(1)_A$ anomaly).
    - Two recent analyses performed by ETM (Ottnad and Urbach, 2018), with twisted-mass quarks and CLS (Bali et al., 2021), with Wilson-Clover quarks.
    - We use a completely different action: staggered quarks.
2. **Calculate the η and η' transition form factors** (TFF), that play a dominant role in the HLbL determination of the a_μ.
    - Useful input for the dispersive and lattice approach to HLbL.
    - Difficult to assess in the low-Q^2 doubly virtual regime for experiments.
    - The pion TFF has already been calculated by Mainz (Gérardin et al., 2016, 2019).
    - This work: focus on the $\eta^{(\prime)}$ mesons.



Ensembles

$2+1+1$ dynamical staggered fermions with 4 steps of stout smearing (details in (Borsanyi et al., 2021))

- Gauge ensembles at (nearly) physical pion mass.
- Exploit five different lattice spacings.
- Consider boxes of $\sim 3, 4$ and 6 fm for finite-size effect studies.
- Ensembles in isosymmetric limit ($\rightarrow$ no mixing between π^0 and $\eta^{(\prime)}$).

In this talk I focus on 3 and 4 fm boxes with

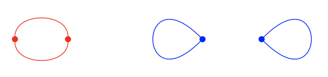
	β	a[fm]	$L/a \times T/a$	# conf
3-pt	3.7000	0.1315	32 × 64	900
2-pt	3.8400	0.0952	32 × 64	1100

Extraction $\eta^{(\prime)}$ masses

From the quark model, the SU(3) octet η_8 and singlet η_0 states are

$$O_8 = \frac{1}{\sqrt{6}}\left(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d - 2\bar{s}\gamma_5 s\right),$$
$$O_0 = \frac{1}{\sqrt{3}}\left(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s\right).$$

Consider matrix of correlators



$$C(t) = \begin{pmatrix} \langle O_8(t)O_8^\dagger(0)\rangle & \langle O_8(t)O_0^\dagger(0)\rangle \\ \langle O_0(t)O_8^\dagger(0)\rangle & \langle O_0(t)O_0^\dagger(0)\rangle \end{pmatrix}$$
$$= \begin{pmatrix} \frac{1}{3}\left(C_\ell + 2C_s + 4D_{\ell s} - 2D_{\ell\ell} - 2D_{ss}\right) & \frac{\sqrt{2}}{3}\left(C_\ell + D_{\ell s} + D_{ss} - C_s - 2D_{\ell\ell}\right) \\ \frac{\sqrt{2}}{3}\left(C_\ell + D_{\ell s} + D_{ss} - C_s - 2D_{\ell\ell}\right) & \frac{1}{3}\left(2C_\ell + C_s - 4D_{\ell\ell} - 4D_{\ell s} - D_{ss}\right) \end{pmatrix}.$$

Masses of physical $\eta^{(\prime)}$ mesons can be obtained by solving a GEVP

$$C(t)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0),$$

where the eigenvalues λ_n are related to the meson mass through

$$m_n^{\text{eff}} = \log\left(\frac{\lambda_n(t,t_0)}{\lambda_n(t+1,t_0)}\right).$$

Staggered Mesonic Operators

1. Classification of the staggered mesonic operator by Golterman (Golterman, 1986).
2. Two (taste-singlet) operators couple to the $\eta^{(\prime)}$ mesons:

- **3-link operator** $\mathcal{O}_3$ (couples to spin $\otimes$ taste $= \gamma_4\gamma_5 \otimes 1$ and $1 \otimes \gamma_4\gamma_5$), defined as (Altmeyer et al., 1993)

$$\mathcal{O}_3(x) = \frac{1}{6}\sum_{ijk} \epsilon_{ijk}\overline{\chi}(x)[\eta_i\Delta_i[\eta_j\Delta_j[\eta_k\Delta_k]]]\chi(x) \equiv \overline{\chi}(x)\hat{O}_3\chi(x),$$

$$\text{Symmetric shift} \quad \Delta_\mu\chi(x) = \frac{1}{2}\left[U_\mu(x)\chi(x+\hat{\mu}) + U_\mu^\dagger(x-\hat{\mu})\chi(x-\hat{\mu})\right].$$

  - Con: Oscillating parity partner state (scalar).

- **4-link operator** $\mathcal{O}_4$ (couples to $\gamma_5 \otimes 1$), defined as

$$\xrightarrow{\text{Used in analysis}} \boxed{\mathcal{O}_4(x) = \frac{1}{2}\eta_4(x)\left[\overline{\chi}(x)\hat{O}_3\chi_+(x) + \overline{\chi}_+(x)\hat{O}_3\chi(x)\right]},$$

$$\chi_+(x) = U_0(x)\chi(x+\hat{0}).$$

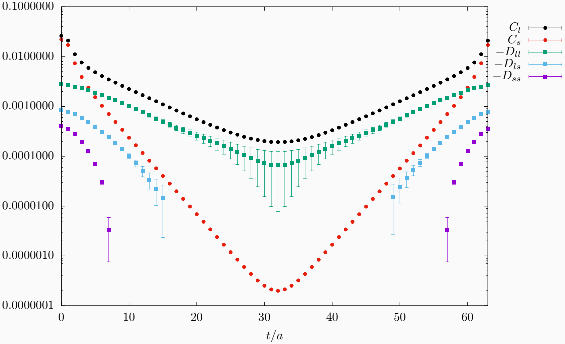
  - Con: Non-local in time.
  - Pro: Parity partner state with exotic quantum number (no contribution).

First Look at Two-point Functions

Noise-reduction techniques to compute the correlation functions:

- Low-mode averaging (LMA) with n = 300 modes (4fm box) (Giusti et al., 2004; DeGrand and Schaefer, 2005).
- All-mode averaging (AMA) for the stochastic part of the estimator (Bali et al., 2010; Blum et al., 2013).
- Venkataraman-Kilcup variance reduction trick for the one-point function (Venkataraman and Kilcup, 1997).

→ We reach the gauge noise of the disconnected contributions.



Analysis (Effective Mass $\eta^{(\prime)}$)

Remove excited states in *connected* correlation function $C_{\ell,s}$ by fitting it to a one-exponential fit (Michael et al., 2013)

$$C_{\ell,s}(t) = A_{\ell,s}\left(\exp(-E_{\ell,s}t) + \exp(-E_{\ell,s}(T-t))\right),$$
$$D_{ii}(t) = \text{unchanged}, \qquad i = \ell, s,$$

in region where excited states are highly suppressed $\longrightarrow$ Replace $C_{\ell,s}$ by fit result in the GEVP (Neff et al., 2001).

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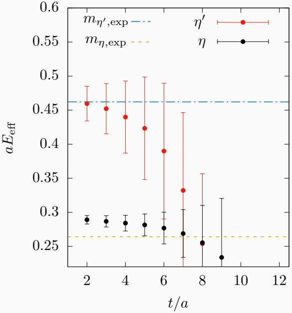
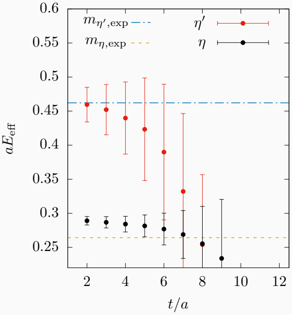


Analysis (Effective Mass $\eta^{(\prime)}$)

For $|\vec{p}| = 0$, disconnected correlation functions in finite volume, at large-time do not tend to zero (Aoki et al., 2007). Instead of $C(t)$ we consider $\boxed{C'(t) \equiv C(t) - C(t + \Delta t)}$ (here: $\Delta t/a = 1$) in the GEVP (Ottnad and Urbach, 2018)

$\rightarrow$ Removes correlations between time-slices and improves point error.

$\rightarrow$ Removes bias in disconnected correlators due to incorrect sampling topological charge.



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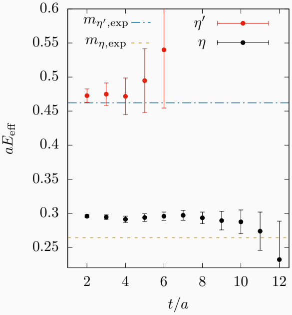
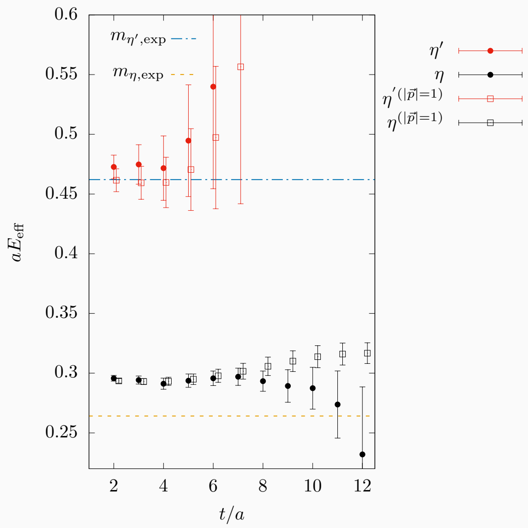


Adding $|\vec{p}| = 1$.



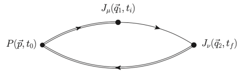
Transition Form Factor

The Transition Form Factor (TFF) for a pseudoscalar meson is extracted from matrix elements $M_{\mu\nu}$ (Ji and Jung, 2001)

$$\begin{aligned} M_{\mu\nu}(p, q_1) &= i \int d^4x \, e^{iq_1 \cdot x} \langle \Omega | \, T\{J_\mu(x) J_\nu(0)\} \, | PS(p) \rangle \\ &= \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \mathcal{F}_{PS\gamma^*\gamma^*}(q_1^2, q_2^2). \end{aligned}$$

(Euclidean) Matrix elements are related to 3-point correlation function $C^{(3)}_{\mu\nu}$ on lattice through

$$M^E_{\mu\nu} = \frac{2E_{PS}}{Z_{PS}} \int_{-\infty}^{\infty} d\tau \, e^{\omega_1 \tau} \tilde{A}_{\mu\nu}(\tau),$$



where

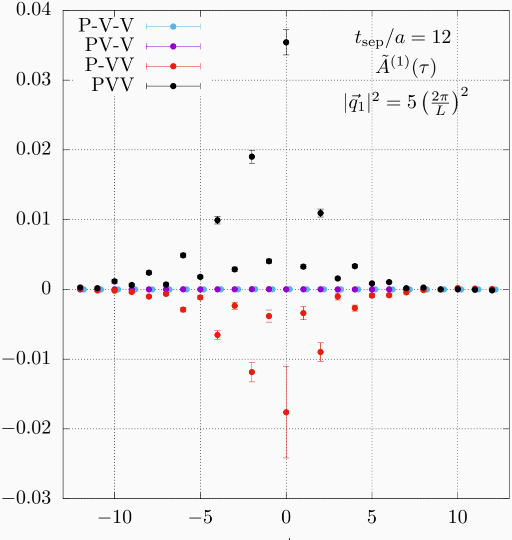
and $\tilde{A}_{\mu\nu}(\tau) \sim C^{(3)}_{\mu\nu}(\tau, t_{PS}) = a^6 \sum_{\vec{x}, \vec{z}} \langle J_\mu(\vec{z}, t_i) J_\nu(\vec{0}, t_f) P^\dagger(\vec{x}, t_0) \rangle e^{i\vec{p}\cdot\vec{x}} e^{-i\vec{q}_1 \cdot \vec{z}}$.

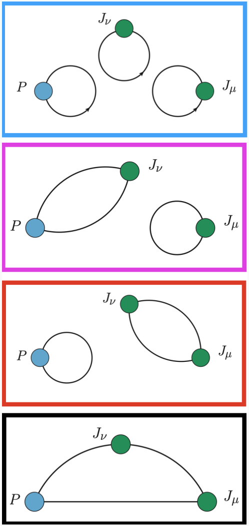
- $\tau = t_i - t_f$.
- $t_{\text{sep}} = t_f - t_0$.

$\tilde{A}_{\mu\nu}$ can be decomposed into scalar functions $\tilde{A}^{(1)}$ and $\tilde{A}^{(2)}$, that form the integrand of the TFF (technical details in (Gérardin et al., 2019)).

First Look at the Integrands η_8

- We are still accumulating data for the disconnected contributions.





Conclusion

Summary:

- Succesfully implemented GEVP, obtaining signal for $\eta^{(\prime)}$ in staggered quark formalism in two kinematic frames.
- Signal measured for all connected and disconnected contributions to integrand TFFs $(\eta + \eta')$.
- Shown marginal effect of PV-V and P-V-V topologies for TFF.

Outlook

- Implement η, η' mixing in TFF.
- Improve statistics on different ensembles.
- Do the continuum extrapolation.

References

Altmeyer, R., Born, K. D., Gockeler, M., Horsley, R., Laermann, E., and Schierholz, G. (1993). The Hadron spectrum in QCD with dynamical staggered fermions. *Nucl. Phys. B*, 389:445–512.

Aoki, S., Fukaya, H., Hashimoto, S., and Onogi, T. (2007). Finite volume qcd at fixed topological charge. *Physical Review D*, 76(5).

Bali, G. S., Braun, V., Collins, S., Schäfer, A., and Simeth, J. (2021). Masses and decay constants of the η and η' mesons from lattice qcd.

Bali, G. S., Collins, S., and Schäfer, A. (2010). Effective noise reduction techniques for disconnected loops in lattice qcd. *Computer Physics Communications*, 181(9):1570–1583.

Blum, T., Izubuchi, T., and Shintani, E. (2013). New class of variance-reduction techniques using lattice symmetries. *Physical Review D*, 88(9).

Borsanyi, S. et al. (2021). Leading hadronic contribution to the muon magnetic moment from lattice QCD. *Nature*, 593(7857):51–55.

DeGrand, T. and Schaefer, S. (2005). Improving meson two-point functions by low-mode averaging. *Nuclear Physics B - Proceedings Supplements*, 140:296–298.

Gérardin, A., Meyer, H. B., and Nyffeler, A. (2016). Lattice calculation of the pion transition form factor $\pi^0 \to \gamma^*\gamma^*$. *Phys. Rev. D*, 94(7):074507.

Gérardin, A., Meyer, H. B., and Nyffeler, A. (2019). Lattice calculation of the pion transition form factor with $N_f = 2 + 1$ Wilson quarks. *Phys. Rev. D*, 100(3):034520.

Giusti, L., Hernandez, P., Laine, M., Weisz, P., and Wittig, H. (2004). Low-energy couplings of qcd from current correlators near the chiral limit. *Journal of High Energy Physics*, 2004(04):013–013.

Golterman, M. F. L. (1986). STAGGERED MESONS. *Nucl. Phys. B*, 273:663–676.

Ji, X. and Jung, C. (2001). Studying hadronic structure of the photon in lattice qcd. *Physical Review Letters*, 86(2):208–211.

Michael, C., Ottnad, K., and Urbach, C. (2013). η and η' masses and decay constants from lattice qcd with 2+1+1 quark flavours.

Neff, H., Eicker, N., Lippert, T., Negele, J. W., and Schilling, K. (2001). Low fermionic eigenmode dominance in qcd on the lattice. *Physical Review D*, 64(11).

Ottnad, K. and Urbach, C. (2018). Flavor-singlet meson decay constants from $N_f = 2 + 1 + 1$ twisted mass lattice QCD. *Phys. Rev. D*, 97(5):054508.

Venkataraman, L. and Kilcup, G. (1997). The eta-prime meson with staggered fermions.