THE UNIVERSITY *of* EDINBURGH

Calculating the $K \to \pi\ell^+\ell^-$ Rare Kaon Decay Amplitude at the Physical Point

Fionn Ó hÓgáin Lattice 2021 MIT

The RBC & UKQCD collaborations

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UC Berkeley/LBNL
Aaron Meyer

BNL and BNL/RBRC
Yasumichi Aoki (KEK)
Peter Boyle (Edinburgh)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Christopher Kelly
Meifeng Lin
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

CERN
Andreas Jüttner*
(Southampton)

Columbia University
Norman Christ*
Duo Guo
Yikai Huo
Yong-Chull Jang
Joseph Karpie
Bob Mawhinney
Ahmed Sheta
Bigeng Wang
Tianle Wang
Yidi Zhao

University of Connecticut
Tom Blum
Luchang Jin (RBRC)
Michael Riberdy
Masaaki Tomii

Edinburgh University
Matteo Di Carlo
Luigi Del Debbio
Felix Erben
Vera Gülpers
Tim Harris
Raoul Hodgson
Nelson Lachini
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli*
James Richings
Azusa Yamaguchi
Andrew Z.N. Yong

KEK
Julien Frison

University of Liverpool
Nicolas Garron

Michigan State University
Dan Hoying

Milano Bicocca
Mattia Bruno

Peking University
Xu Feng*

University of Regensburg
Davide Giusti
Christoph Lehner (BNL)

University of Siegen
Matthew Black
Oliver Witzel

University of Southampton
Nils Asmussen
Alessandro Barone
Jonathan Flynn
Ryan Hill
Rajnandini Mukherjee
Chris Sachrajda*

University of Southern Denmark
Tobias Tsang

Stony Brook University
Jun-Sik Yoo
Sergey Syritsyn (RBRC)

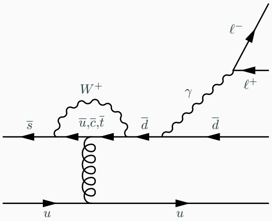
*[arXiv:1507.03094v2, arXiv:1806.11520v1]

$K \to \pi \ell^+ \ell^-$

Motivations

- $K \to \pi \ell^+ \ell^-$ are FCNC processes
- Forbidden at tree level

  $\implies$ Sensitive to new physics



- Long distance effects dominate: use lattice QCD
- NA62 has collected $28,000$ $K \to \pi \mu^+ \mu^-$ samples in their 2016-18 dataset
  - 2021-2024 runs set to start July 2021
  - More $\mu^+ \mu^-$ samples and potential for $e^+ e^-$ samples [C. Parkinson: RKF 2019]

Long-Distance Amplitude: Phenomenology

In terms of EM transition form factor $V_j(z) \quad (j=+,S)$
[G. D'Ambrosio et al. arXiv:hep-ph/9808289]

- $\mathscr{A}^j_\mu(q^2) = -iG_F\frac{V_j(z)}{(4\pi)^2}\left(q^2(k+p)_\mu - \left(M_K^2 - M_\pi^2\right)q_\mu\right)$

$V_j(z) = a_j + b_j z + V_j^{\pi\pi}(z)$, $\qquad z = q^2/M_K^2$, $\qquad q \equiv k - p$

Phenomenological predictions:
[V. Cirigliano et al. arXiv:1107.6001] [L. Bician et al. ICHEP2020]

e	$\lvert a_S\rvert = 1.06^{+0.26}_{-0.21}$	$a_+ = -0.578 \pm 0.016$	$b_+ = -0.779 \pm 0.066$
μ	$\lvert a_S\rvert = 1.54^{+0.40}_{-0.32}$	$a_+ = -0.592 \pm 0.015$	$b_+ = -0.699 \pm 0.058$

Long-Distance Amplitude: Theory Prediction

e	$\lvert a_S \rvert = 1.06^{+0.26}_{-0.21}$	$a_+ = -0.578 \pm 0.016$	$b_+ = -0.779 \pm 0.066$
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[G. D'Ambrosio et al. arXiv:1906.03046]

Theoretical prediction given by long and short distances matching at $\mathcal{O}(\alpha_s)$

- 3 flavor QCD form factor
- Low energy expansion
  - Two-loop representation of the form factor
- Phenomenological determination of unknown quantities at vanishing momentum transfer

ℓ	$a_+ = -1.59 \pm 0.08$	$b_+ = -0.82 \pm 0.06$

Long-Distance Amplitude: Lattice

Long-distance Minkowski amplitude:
[G. Isidori et al. arXiv:hep-lat/0506026]

- $\mathscr{A}_\mu^j(q^2) = \int \mathsf{d}^4x \, \left\langle \pi^j(\mathbf{p}) \middle| \mathsf{T}\left[J_\mu(0)H_W(x)\right] \middle| K^j(\mathbf{k}) \right\rangle$

$\Delta S = 1$ effective weak Hamiltonian:

- $H_W(x) = \frac{G_F}{\sqrt{2}} V_{us}^* V_{ud} \left(C_1(Q_1^u - Q_1^c) + C_2(Q_2^u - Q_2^c)\right)$

Wilson coefficients C_1 and C_2 are much larger than $C_{3,\ldots,8}$

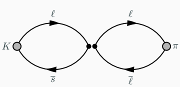
- $Q_1^q = (\bar{s}_a \gamma_\mu^L d_a)(\bar{q}_b \gamma^{L\,\mu} q_b)$ and $Q_2^q = (\bar{s}_a \gamma_\mu^L q_a)(\bar{q}_b \gamma^{L\,\mu} d_b)$

where $\gamma_\mu^L = \gamma_\mu(1 - \gamma_5)$.

Current can be either the local or conserved lattice vector current.

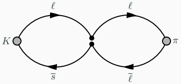
Wick Contractions

- The Wick contractions for $K \to \pi\, H_W$ 3-pt functions gives:



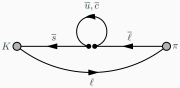
W

(Wing)



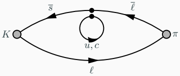
C

(Connected)



S

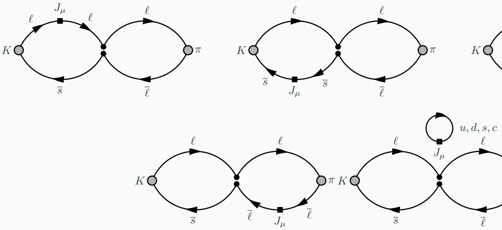
(Saucer)



E

(Eye)

- Including the current insertions:



Physical Point Runs

Physical Point Gauge

$48^3 \times 96$ gauge configuration [T. Blum et al. arXiv:1411.7017]

- $M_\pi \approx 140$ MeV, $M_K \approx 500$ MeV
- $a^{-1} = 1.73$ GeV
- $V = (5.476(12) fm)^3$
- $m_\pi L = 3.86$
- $2 + 1$ flavor
- Möbius DWF action for light quarks
- Scaled DWF action for strange quarks

Physical Point Setup

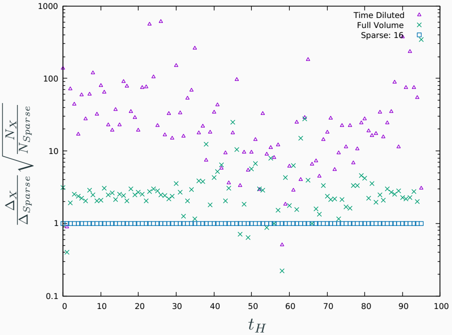
Analysis performed using C++ library “Grid,” and the framework based on Grid, “Hadrons.”

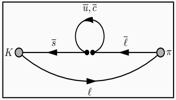
- Coulomb gauge fixed wall sources
- 2000 ZMöbius light quark low modes $\rightarrow$ deflated solves
- Sparse noises for the loop
    - Multiple hits: inexact solves and an AMA correction step
- Conserved currents
    - Möbius conserved current recently implemented in Grid
- 3 unphysical ZMöbius DWF charm quarks
- $\mathbf{p}_K = (0,0,0)$, $\mathbf{p}_\pi = \frac{2\pi}{L}(1,0,0) \rightarrow z = \frac{q^2}{M_K^2} \approx 0.009$

Sparse Noise

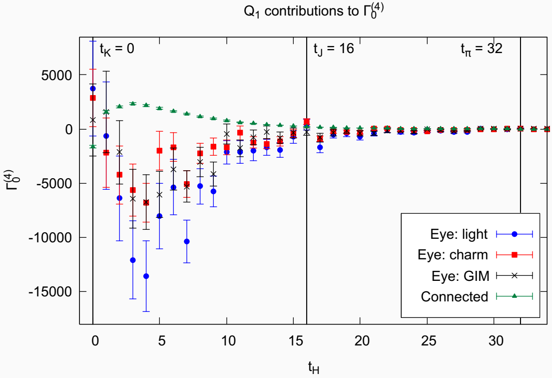
$$\eta_{sparse}(x) = \begin{cases} \eta_{z_2}(x) : x_\mu \bmod n = 0 \\ 0 : o/w \end{cases}$$

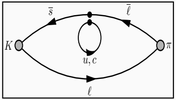
- Shift in each dimension to cover full volume
- n=2: 16 sources
- Cost benefit:
  - $\frac{\Delta_X}{\Delta_{Sparse}}\sqrt{\frac{N_X}{N_{Sparse}}}$
- Compared to 20 full volume and time-diluted A2A

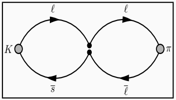




Preliminary Results: 4-point Q1 Correlators



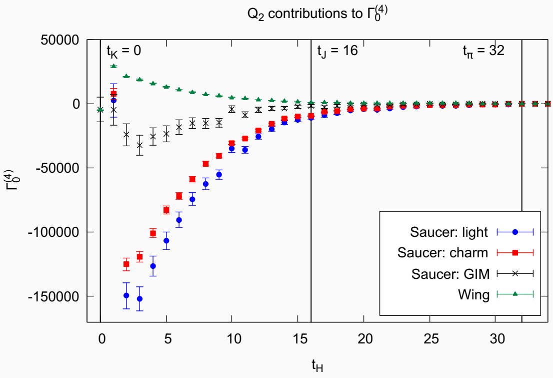


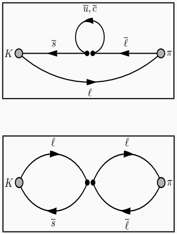


- 87 configurations
- 6 time translations each
- 1 exact hit and 10 inexact hits for the loop

	light	strange	charm
Exact	148	12	816
Inexact	1120	0	3360
Total	1268	12	4176

Preliminary Results: 4-point Q2 Correlators





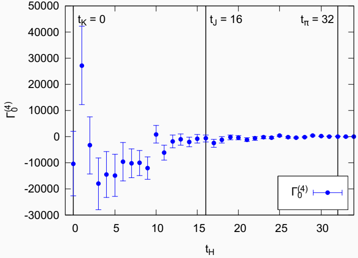
Amplitude is found by integrating the 4-point correlators over Weak Hamiltonian time, t_H

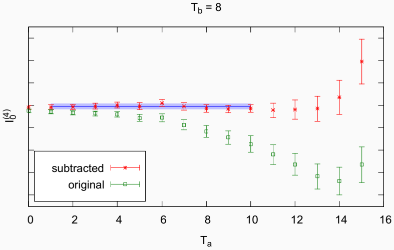
- requires the removal of intermediate states

Preliminary Results: 2-/3-point Fits

	$\mathbf{p}$	
	$(0,0,0)$	$\frac{2\pi}{L}(1,0,0)$
$E_K(\mathbf{p})$	0.28889(35)(25)	0.31711(48)(49)
$E_\pi(\mathbf{p})$	0.08008(25)(21)	0.15467(93)(68)
$Z_K(\mathbf{p})$	30743(97)(100)	28787(133)(172)
$Z_\pi(\mathbf{p})$	28471(103)(77)	26455(263)(236)
$\mathcal{M}_H(\mathbf{p})$	0.000495(49)(125)	0.00053(39)(12)
$\mathcal{M}^{J,K}_{\mu=3}(\mathbf{0},\mathbf{p})$	-	0.5681(55)(58)
$\mathcal{M}^{J,\pi}_{\mu=3}(\mathbf{0},\mathbf{p})$	-	0.2257(33)(56)

- 2-, 3-point fits can be used to remove intermediate states
- Independent analyses being performed in Edinburgh (myself) and Southampton (Ryan Hill)
- Same procedure for 4-point fits next





$$\mathcal{A}_\mu \propto \lim_{T_a, T_b \to \infty} \int_{t_J - T_a}^{t_J + T_b} dt_H \; \Gamma_\mu^{(4)}\left(t_H, t_J, \mathbf{k}, \mathbf{p}\right)$$

Qualitatively:

- $\mathcal{A}_\mu$ is in the right ballpark
- Suffers from $\sim 100\%$ errors

Future Outlook

Future Outlook

To extract the amplitude

- Independent 4-point analysis
- Möbius/ZMöbius bias corrections (MADWF solver)
- Charm quark extrapolation
- More loop hits

Beyond that (prospects, not promises)

- More kinematics?

$$V_j(z) = a_j + b_j z + V_j^{\pi\pi}(z)$$
$$\mathbf{p}_K = (0,0,0), \quad \mathbf{p}_\pi = \frac{2\pi}{L}(1,0,0): \quad z = \frac{q^2}{M_K^2} \approx 0.009$$

- Different approach to analysis? Equivalent to baryonic

$\mathcal{A}_\mu \propto \int dt_H \to \int dt_H dt_J$ Raoul Hodgson: 10:45 AM - 28/07

Summary

- Rare kaon decays are an potential insight into NP
  - With recent activity in experiment and theory
- Physical point calculation is underway
- Preliminary analysis is being performed
- Rare kaon amplitude is in the right ballpark
- More statistics need to be gathered

Summary

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Thank you. Any questions?

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- C. Parkinson: $K \to \pi\mu^+\mu^-$ at NA62 - The 2nd Forum on Rare Kaon Decays 2019
- G. D'Ambrosio et al.: The Decays $K \to \pi\ell^+\ell^-$ beyond Leading Order in the Chiral Expansion - arXiv:hep-ph/9808289
- V. Cirigliano et al.: Kaon Decays in the Standard Model - arXiv:1107.6001
- L. Bician et al.:New measurement of the $K^+ \to \pi^+\mu^+\mu^-$ decay at NA62 - https://doi.org/10.22323/1.390.0364
- G. D'Ambrosio et al.: Matching long and short distances at order $\mathcal{O}(\alpha_s)$ in the form factors for $K \to \pi\ell^+\ell^-$ - arXiv:1906.03046
- Gino Isidori et al,: Rare Kaon Decays on the Lattice - arXiv:hep-lat/0506026
- T. Blum et al.: Domain wall QCD with physical quark masses - arXiv:1411.7017