$B \to D^{(*)} \ell \nu$ semileptonic decays in lattice QCD with domain-wall heavy quarks

JLQCD Collaboration

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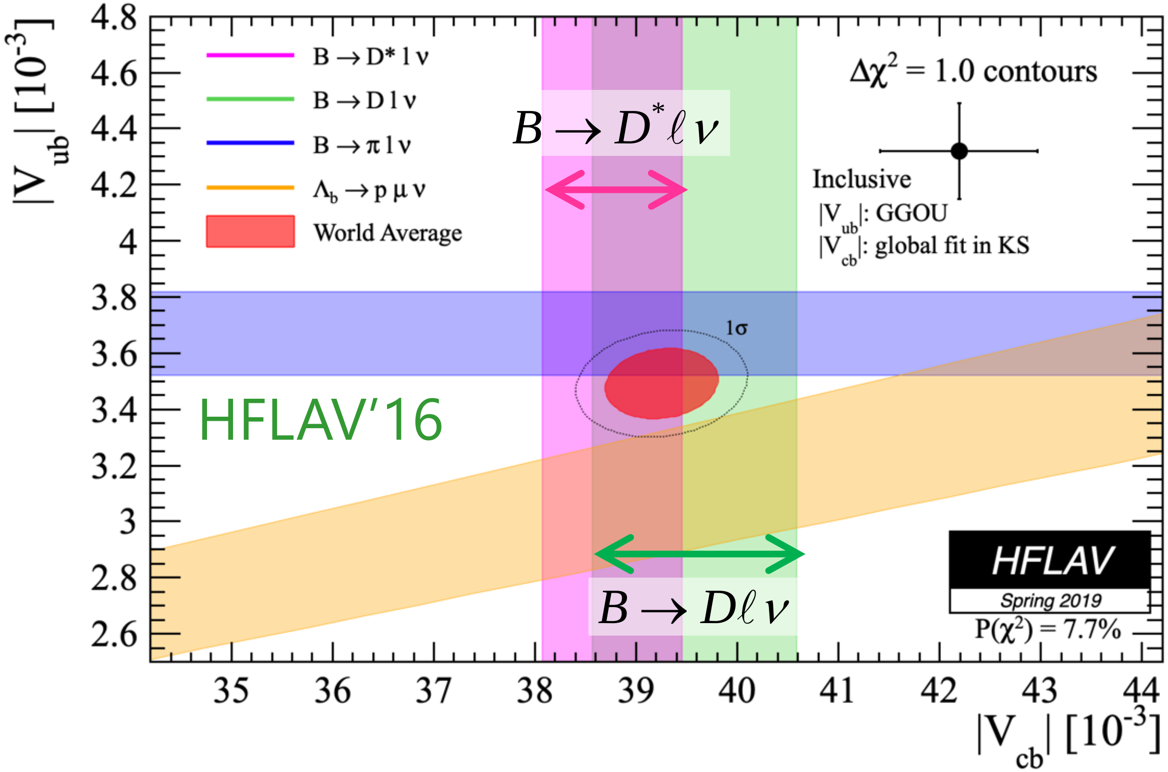
The 38th International Symposium on Lattice Field Theory, Lattice 2021
July 27th, 2021, Zoom+Gather@MIT

the $|V_{cb}|$ tension



vs inclusive $B \longrightarrow X_c \ell \nu$

- $B \to D \ell \nu$, $\Delta|V_{cb}|$ ~ 6%, 3σ
- $B \to D^* \ell \nu$, $\Delta|V_{cb}|$ ~ 8%, 4σ

new physics?

Crivellin-Pokorski '18

$$d_L^{qb} \partial^\nu \left(\bar{q} \sigma_{\mu\nu} P_L b \right) \Leftrightarrow \Gamma\left(Z \to b\bar{b}\right)$$

tension also for $|V_{ub}|$

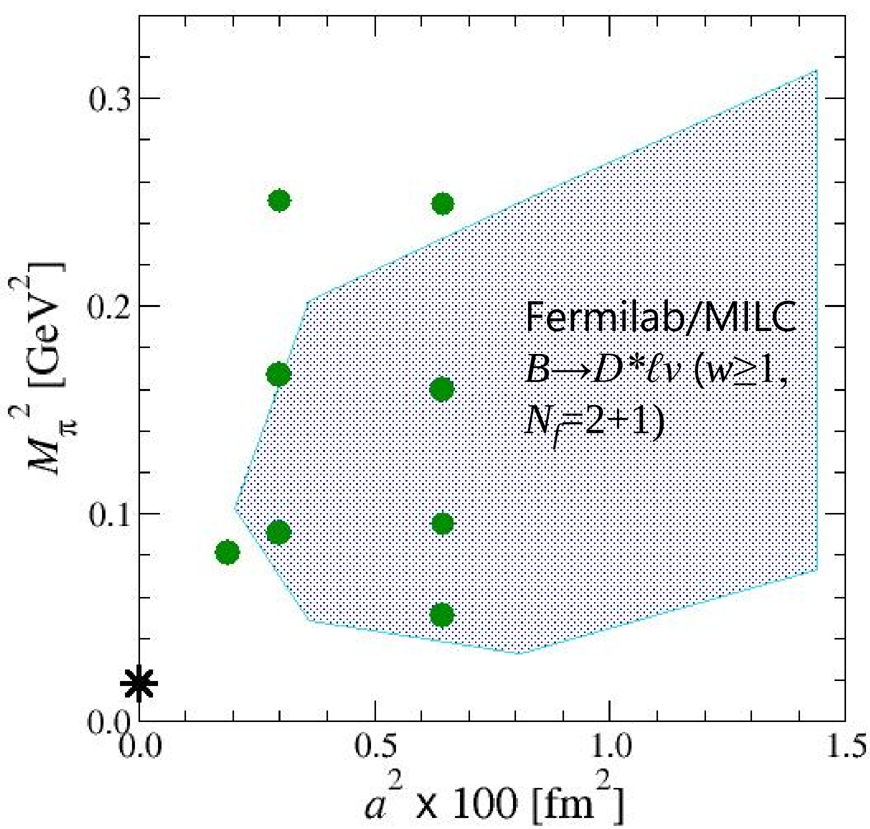
need deeper understanding of th. and/or exp't uncertainties

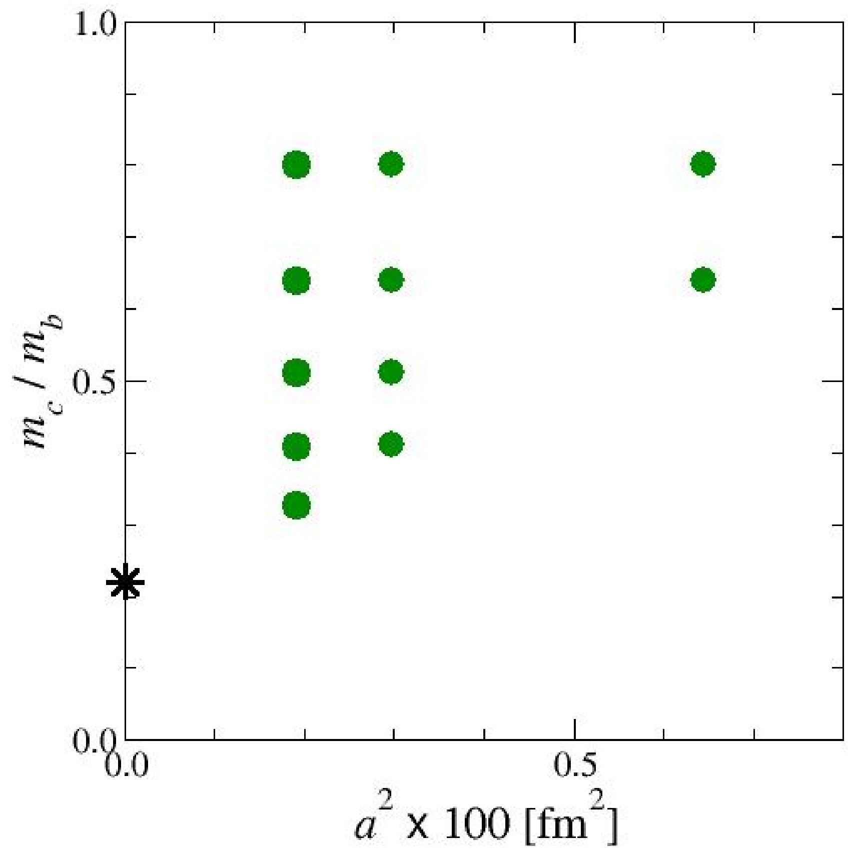
- $B \to D \ell \nu$ FFs : Fermilb/MILC '15, HPQCD '15
- $B \to D^* \ell \nu$ FFs @ non-zero recoils : Fermilab/MILC '21

this talk : JLQCD's calculation of $B \to D^{(*)} \ell \nu$ form factors

simulation method

relativistic approach w/ Möbius domain-wall heavy quarks





- $a^{-1} \leq$ 4.5 GeV ⊕ no $O(a)$ errors
- $M_\pi \geq$ 230 MeV ⊕ HMChPT
- $M_\pi L \geq 4$; direct check w 2 L's

- $m_{c,\mathrm{phys}} \leftarrow M_{\eta c, J/\psi}$
- 3 – 6 m_b's < $0.7a^{-1} \Rightarrow$ $m_{b,\mathrm{phys}}$
- Fermilab/MILC w/ Fermilab interpre.

ratio method (Hashimoto *et al.* '99)

$$\left\langle D^*(p',\varepsilon')\middle|V_\mu\middle|B(p)\right\rangle = i\varepsilon_{\mu\nu\rho\sigma}\varepsilon'^{*\nu}v'^{\rho}v^{\sigma}h_V(w)$$

$$\left\langle D^*(\varepsilon,p')\middle|A_\mu\middle|B(p)\right\rangle = \varepsilon^*_\mu(1+w)h_{A_1}(w) - \varepsilon^* v\left\{v_\mu h_{A_2}(w) + v_\mu h_{A_3}(w)\right\}$$

$v = p/M_B$, $v' = p'/M_{D^{(*)}}$, $w = vv' \geq 1$ (zero recoil)

$$\frac{\left\langle O_{D^*}(\varepsilon,\mathbf{0})A_1^{(\mathrm{lat})}O_B^\dagger(\mathbf{0})\right\rangle\left\langle O_B(\mathbf{0})A_1^{(\mathrm{lat})}O_{D^*}^\dagger(\varepsilon,\mathbf{0})\right\rangle}{\left\langle O_{D^*}(\varepsilon,\mathbf{0})V_4^{(\mathrm{lat})}O_{D^*}^\dagger(\varepsilon,\mathbf{0})\right\rangle\left\langle O_B(\mathbf{0})V_4^{(\mathrm{lat})}O_B^\dagger(\mathbf{0})\right\rangle} \Rightarrow h_{A_1}(1)$$

$$\frac{\left\langle O_{D^*}(\varepsilon,\mathbf{p}_\perp)A_1^{(\mathrm{lat})}O_B^\dagger(\mathbf{0})\right\rangle\left\langle O_{D^*}(\varepsilon,\mathbf{0})O_{D^*}^\dagger(\varepsilon,\mathbf{0})\right\rangle}{\left\langle O_{D^*}(\varepsilon,\mathbf{0})A_1^{(\mathrm{lat})}O_B^\dagger(\mathbf{0})\right\rangle\left\langle O_{D^*}(\varepsilon,\mathbf{p}_\perp)O_{D^*}^\dagger(\varepsilon,\mathbf{p}_\perp)\right\rangle} \Rightarrow \frac{h_{A_1}(w)}{h_{A_1}(1)}$$

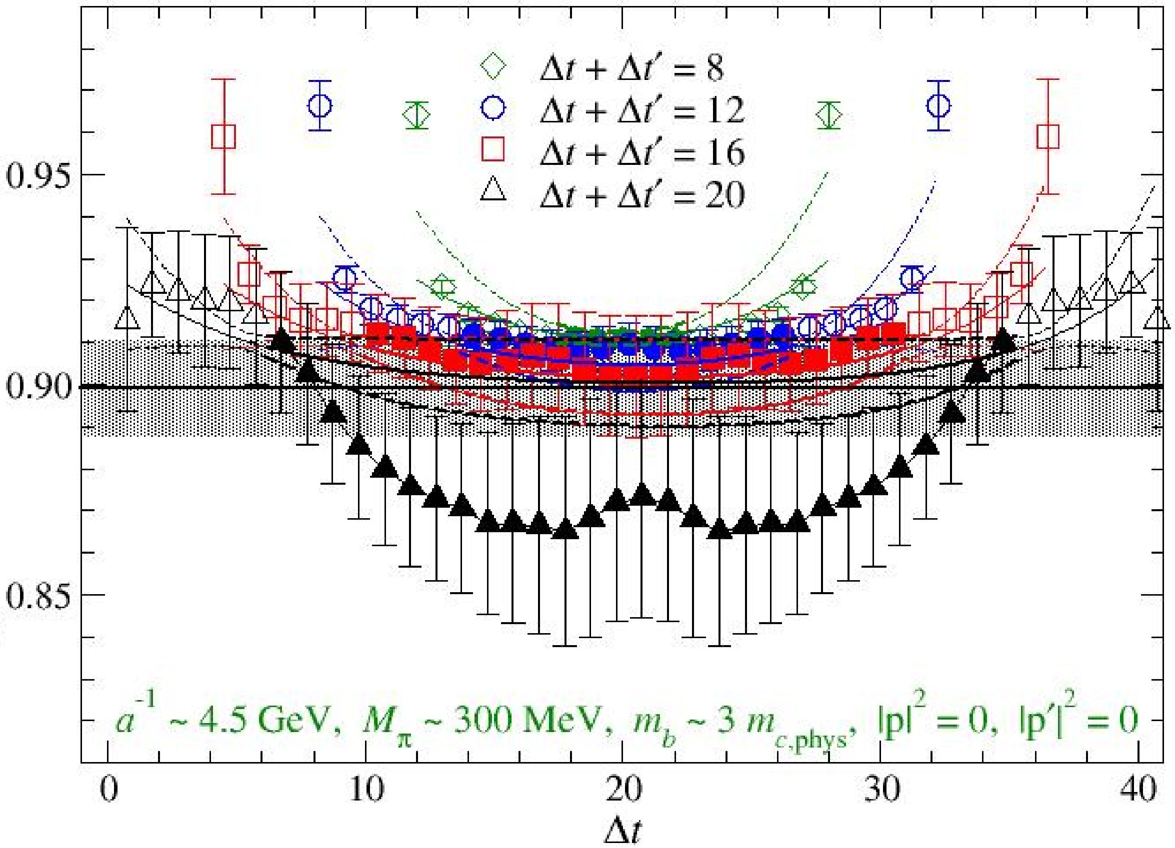
$$\frac{\left\langle O_{D^*}(\varepsilon,\mathbf{p}_\perp)V_1^{(\mathrm{lat})}O_B^\dagger(\mathbf{0})\right\rangle}{\left\langle O_{D^*}(\varepsilon,\mathbf{p}_\perp)A_1^{(\mathrm{lat})}O_B^\dagger(\mathbf{0})\right\rangle} \Rightarrow \frac{h_V(w)}{h_{A_1}(w)} \qquad \frac{\left\langle O_{D^*}(\varepsilon,\mathbf{p}_{\not\perp})A_{1(4)}^{(\mathrm{lat})}O_B^\dagger(\mathbf{0})\right\rangle}{\left\langle O_{D^*}(\varepsilon,\mathbf{p}_\perp)A_1^{(\mathrm{lat})}O_B^\dagger(\mathbf{0})\right\rangle} \Rightarrow \frac{h_{A_{3(2)}}(w)}{h_{A_1}(w)}$$

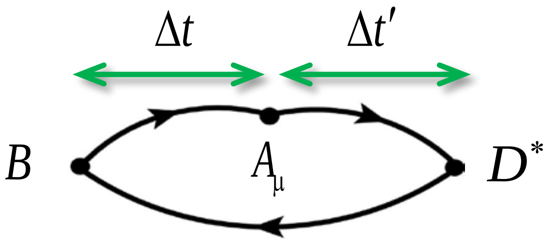
- B at rest
- $|\mathbf{p}|^2 = 0,1,2,3,4$
- $\boldsymbol{\varepsilon}\mathbf{p}_\perp = 0$
- $\boldsymbol{\varepsilon}\mathbf{p}_{\not\perp} \neq 0$

do not need explicit renormalization for SM FFs

extracting FFs

$$\langle D^*|A_1|B\rangle\langle B|A_1|D^*\rangle / \langle D^*|V_4|D^*\rangle\langle B|V_4|B\rangle \rightarrow h_{A1}(1)$$





- 4 values of source-sink separation $\Delta t+\Delta t'$
- multi-exponential fit

$$\left|\langle D^*|A|B\rangle\right|^2 \left\{1+c\,e^{-\Delta E_B \Delta t}+c' e^{-\Delta E_{D^*}\Delta t'}\right\}$$

- large $\Delta t+\Delta t'$ $\Rightarrow$ ground state saturation
- small $\Delta t+\Delta t'$ $\Rightarrow$ statistical accuracy: e.g. 1 - 2% for $h_{A1}(w)$

continuum + chiral extrapolation

NLO HMChPT (Randall-Wise '92, Savage '01) + polynomial corrections

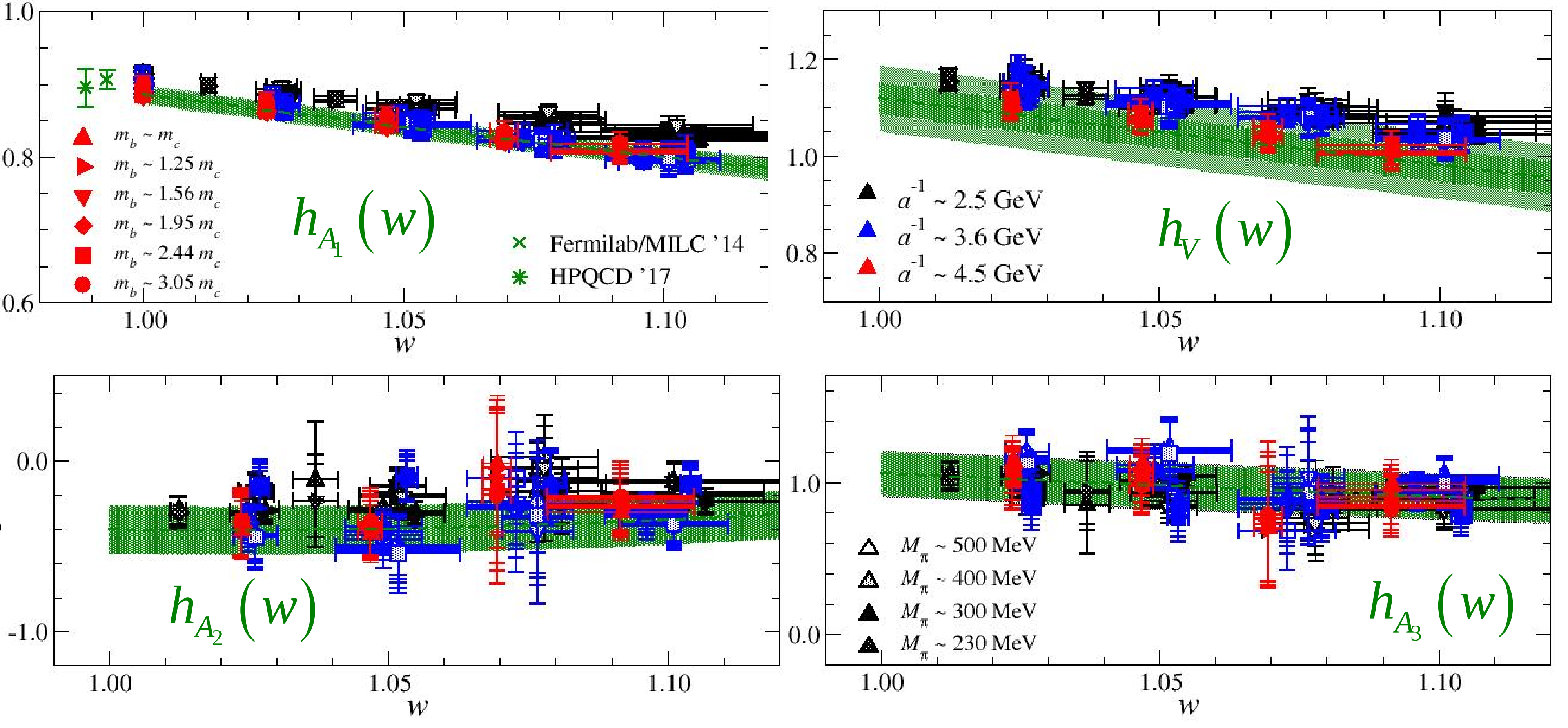
$$\frac{h_{A1}(w)}{\eta_{A_1}} = c + \frac{g_{D^*D\pi}^2}{16\pi^2 f_\pi^2} \Delta_c^2 b_{\log} \bar{F}_{\log}\left(M_\pi, \Delta_c, \Lambda_\chi\right)$$

$$+c_w\left(w-1\right) + c_b\left(w-1\right)\varepsilon_b + c_\pi \xi_\pi + c_{\eta s}\xi_{\eta s} + c_a \xi_a + c_{am_b}\xi_{amb} + d_w\left(w-1\right)^2$$

$$\varepsilon_b = \frac{\bar{\Lambda}}{2m_b}, \quad \xi_\pi = \frac{M_\pi^2}{\left(4\pi f_\pi\right)^2}, \quad \xi_{\eta s} = \frac{M_{\eta s}^2}{\left(4\pi f_\pi\right)^2}, \quad \xi_a = \left(a\Lambda_{\mathrm{QCD}}\right)^2, \quad \xi_a = \left(am_b\right)^2$$

- one-loop radiative correction η_X is taken into account (Neubert '92)
- $g_{D^*D\pi}$ = 0.53(8) (Fermilab/MILC '14) $\Rightarrow$ small systematic error
- ξ - expansion : better convergence for light quark obs. (JLQCD '08)
- $O((w\text{-}1)/m_b)$ for h_{A1}, h_+ $\Leftrightarrow$ Luke's theorem '90 ; include $O(1/m_b^2)$

$B \to D^* \ell \nu$ form factors

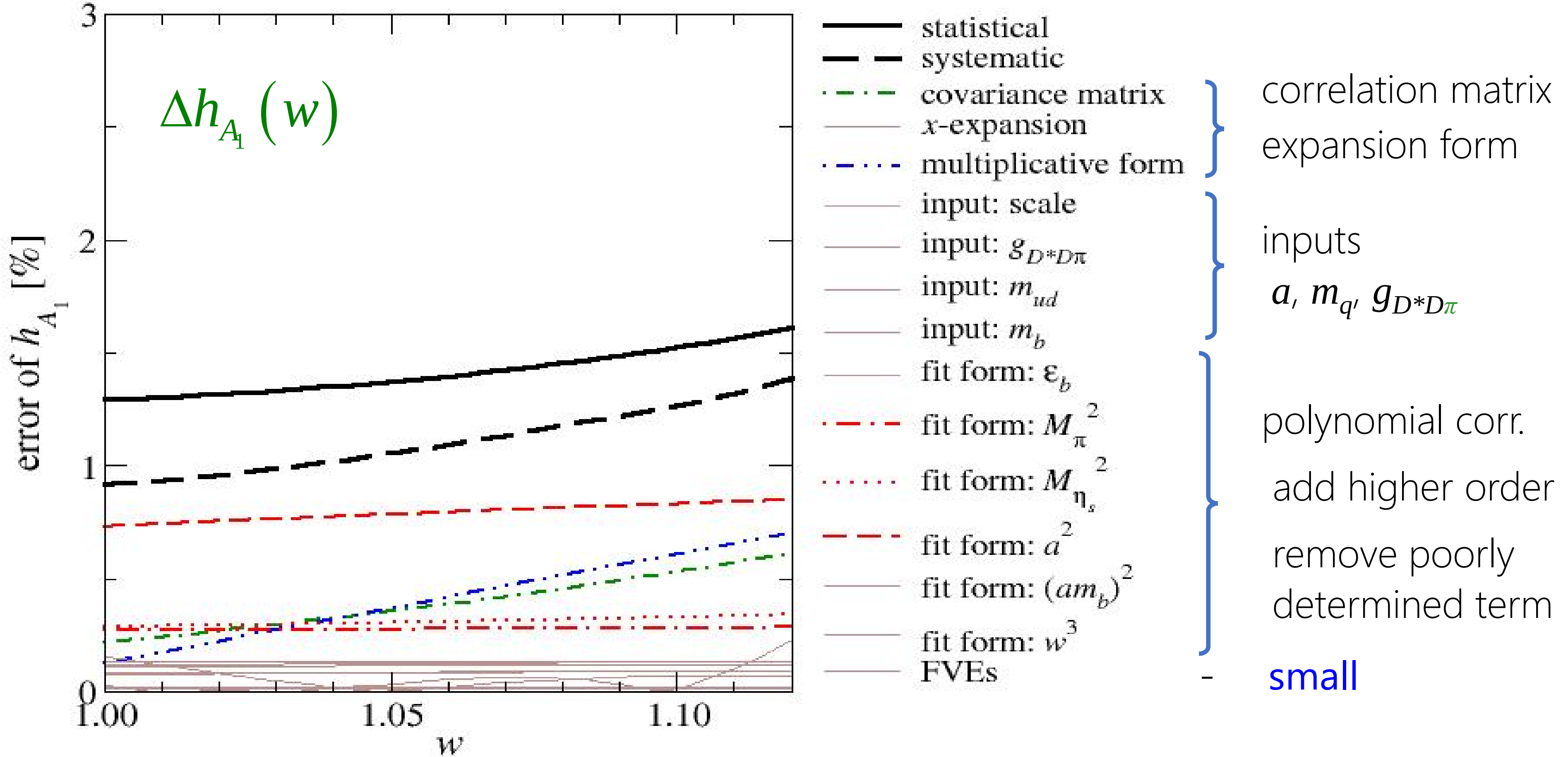


- mild dependence on a, M_π, m_s, m_b

  $\Rightarrow$ all coefficients $c_X \le O(1)$; $\ge$ 50% error for c_π, $c_{\eta s}$, c_{a2} [except h_{A1}]

- extrapolation : reasonably controlled w/ χ^2/d.o.f. $\le$ 1

systematic uncertainties



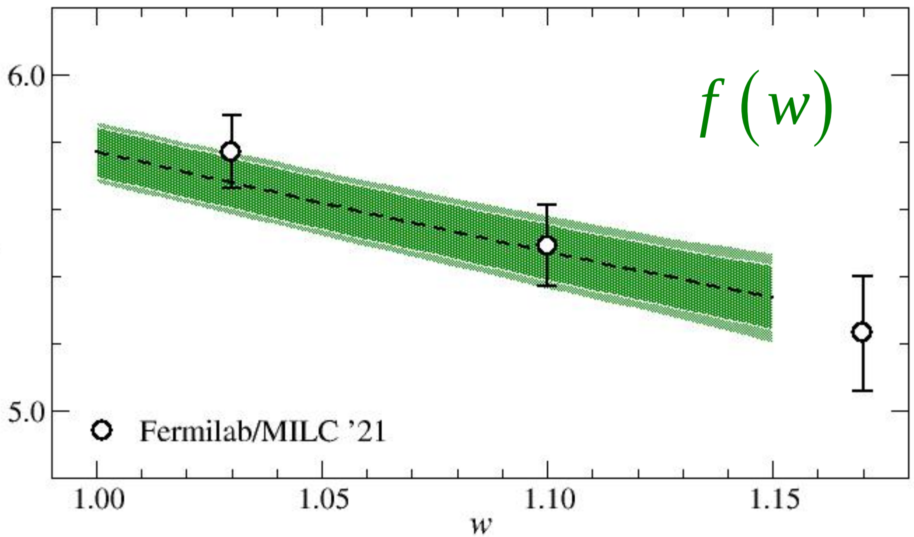
- h_{A1} : largest uncertainties – statistics and discretization – but 1-2 %
- other FFs : larger and more dominant statistical error

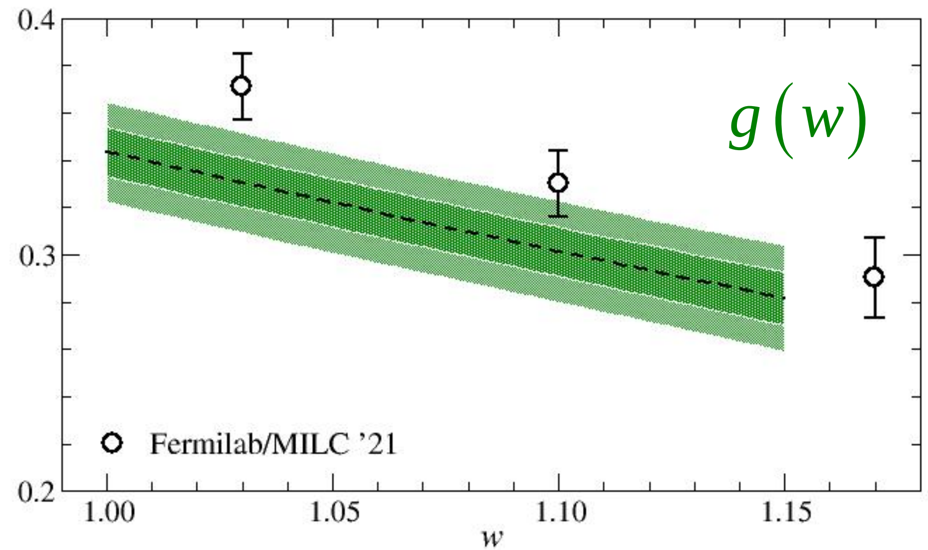
in relativistic convention

$$\langle D^*(p',\varepsilon')|V_\mu|B(p)\rangle = i\varepsilon_{\mu\nu\rho\sigma}\varepsilon'^{*\nu}p'^\rho p^\sigma g(q^2)$$

$$\langle D^*(\varepsilon,p')|A_\mu|B(p)\rangle = \varepsilon^*_\mu f(q^2) - \varepsilon^* v\{(p+p')_\mu a_+(q^2) + (p-p')_\mu a_-(q^2)\}$$

$$q^2 = (p-p')^2$$

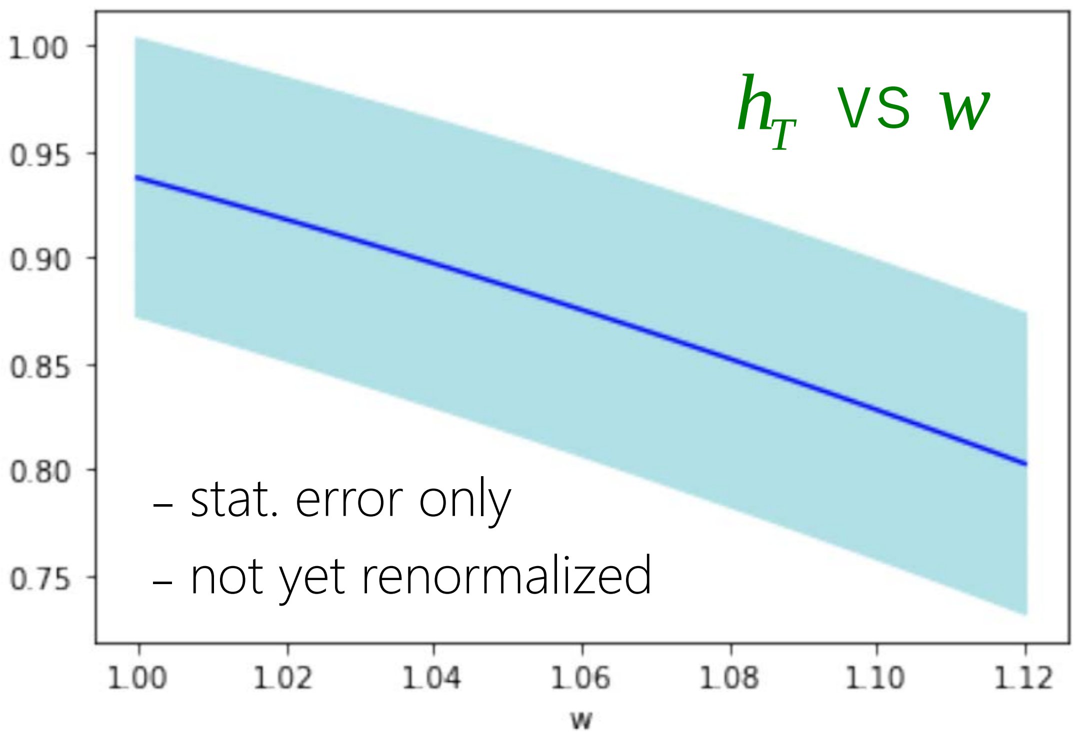




- Fermilab/MILC '21 : reasonably consistent
  - different systematics: EFT ⊗ direct $m_{b,\mathrm{phys}}$ ⇔ rel. QCD ⊗ extrap. to $m_{b,\mathrm{phs}}$
  - slightly smaller slope for g?

beyond SM

$B \longrightarrow D\ell\nu$ tensor FF (M. Faur [École normale supérieure])



$$\langle D(p')|T_{\mu\nu}|B(p)\rangle = i\left(v'^{\mu}v^{\nu} - v'^{\nu}v^{\mu}\right) h_T(w)$$

- similar setup
  - Möbius DWF ensembles
  - Möbius DWF c and b
  - polynomial cont+chiral extrap

- analysis in progress
  - 8% statistical uncertainty ; comparable systematic uncertainties
  - NPR $\Rightarrow$ talk by T. Ishikawa
- important input for new physics interpretation

Summary

$B \to D^{(*)} \ell \nu$ form factors from relativistic QCD

- all relevant SM form factors @ zero and non-zero recoils
  - multiple source-sink separations / average over source point
  - no need for renormalization / mild a, m_q dependence
    $\Rightarrow$ *e.g.* 1-2% accuracy for h_{A1}
  - Fermilab/MILC '21: reasonably consistent
  - impact on $|V_{cb}|$ determination to be studied
  - *cf.* 1-2 % experiment accuracy, systematic limited
- on-going
  - BSM FFs $\Rightarrow$ need NPR : talk by T. Ishikawa 7/28th 22:15- EDT
  - $B \to \pi \ell \nu$ $\Rightarrow$ talk by J. Koponen 7/27th 9:00- EDT, and Fugaku