Hadronic vacuum polarization of the muon on 2+1+1-flavor HISQ ensembles: an update.

Lattice 2021,

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on behalf of the Fermilab Lattice, HPQCD, and MILC collaborations.

7/26/2021

❖ Fermilab experiment announced first measurement, new exp. average (BNL+FNAL).

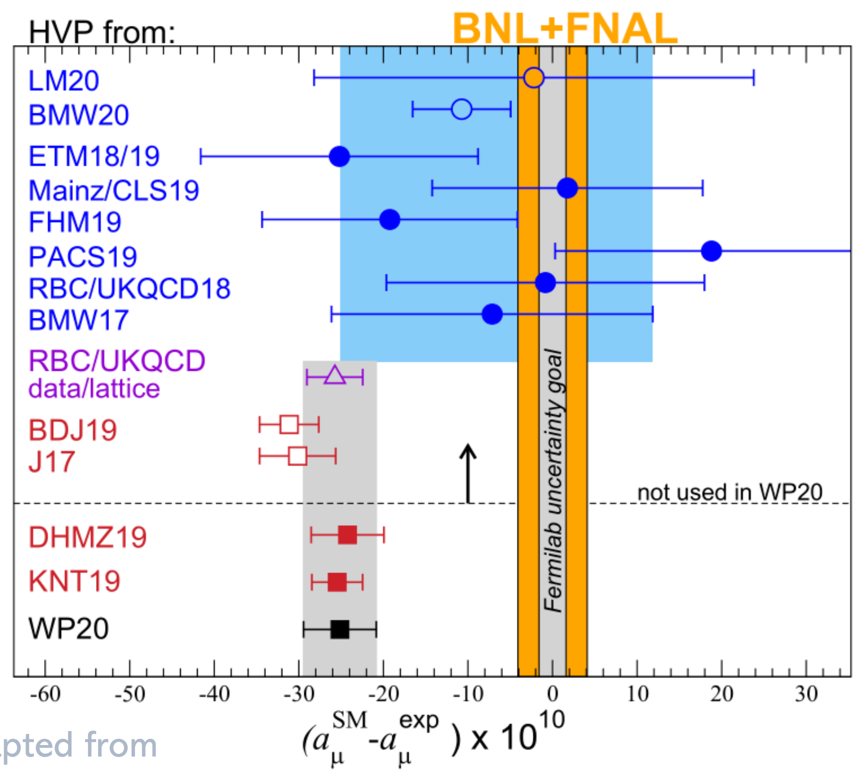
$$a_\mu(\mathrm{Exp}) = 116\,592\,061(41) \times 10^{-11}$$

Anomalous magnetic moment

$$\boldsymbol{\mu} = g\frac{e}{2m_\mu}\mathbf{S} \Rightarrow a_\mu \equiv (g-2)/2$$

g = 2 at tree level

❖ 4.2σ tension with white paper result (WP20).

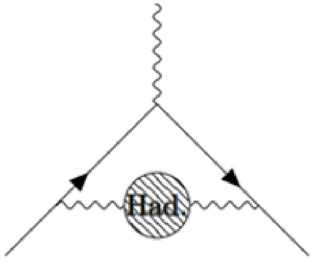


Adapted from 2006.04822

❖ HVP: leading order hadronic vacuum polarization contribution.

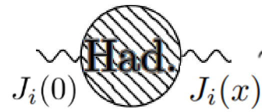
❖ Dominates the Standard Model uncertainty.

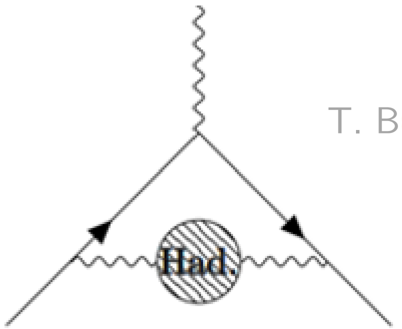
$a_\mu^{\mathrm{HVP,\,LO}}$:



Leading Order Hadronic Vacuum Polarization

- Energy-momentum representation.

FT:  $\sim \widehat{\Pi}_{\text{Had.}}(q^2)$,



T. Blum, Phys. Rev. Lett. 91, 052001

$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dq^2 K(q^2)\, \widehat{\Pi}_{\text{Had.}}(q^2)$$

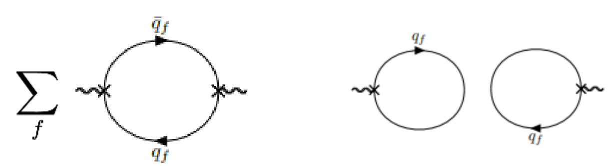
- Time-momentum representation.

Bernecker & Meyer, 2011

$$a_\mu^{\text{HVP,LO}} = 4\alpha^2 \int_0^\infty dt \tilde{K}(t)\, C(t) \quad , \qquad C(t) = \sum_i^3 \int d^3x \, \langle J_i(x) J_i(0) \rangle$$

$$J_i = \sum_{\text{f}} Q_{\text{f}} \bar{q}_{\text{f}} \gamma_i q_{\text{f}}$$

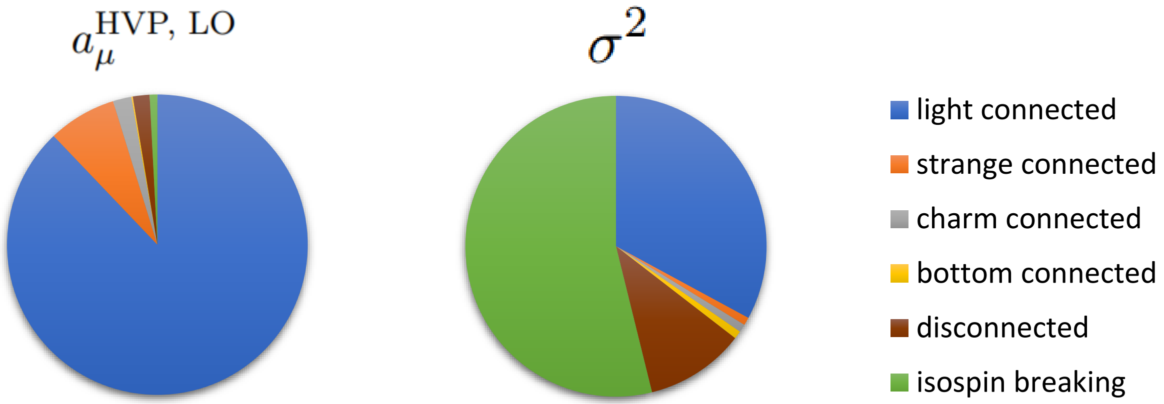
- Contributions separated by
  - connected & disconnected wick contractions
  - quark flavor
  - Isospin symmetric & breaking contributions



LO HVP from Fermilab Lattice, HPQCD and MILC

$$10^{10} a_\mu^{\mathrm{HVP,LO}} = 699(15)_{u,d}(1)_{s,c,b}$$

PhysRevD (2019) 101.034512

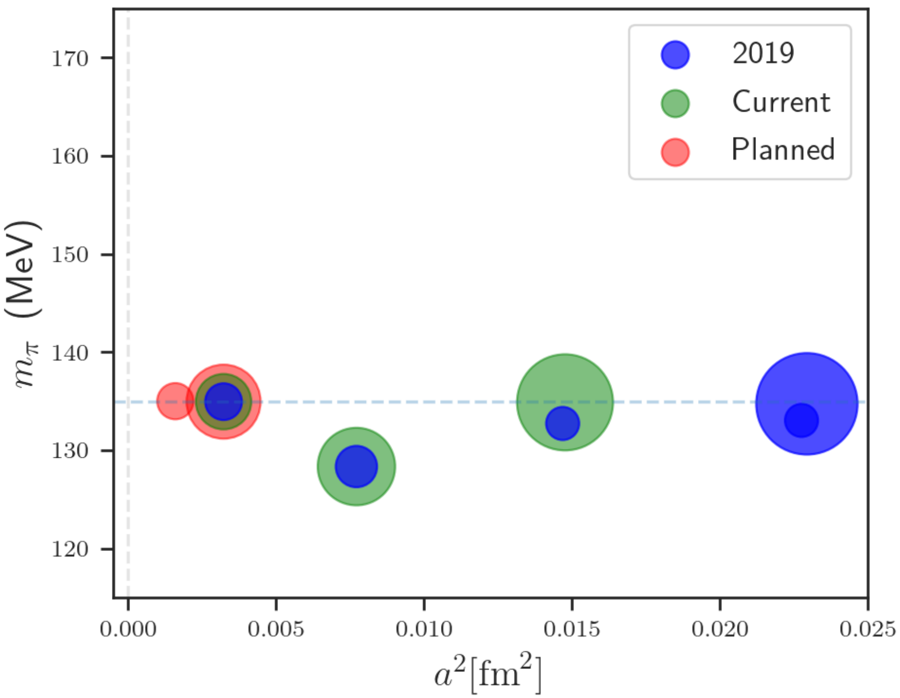


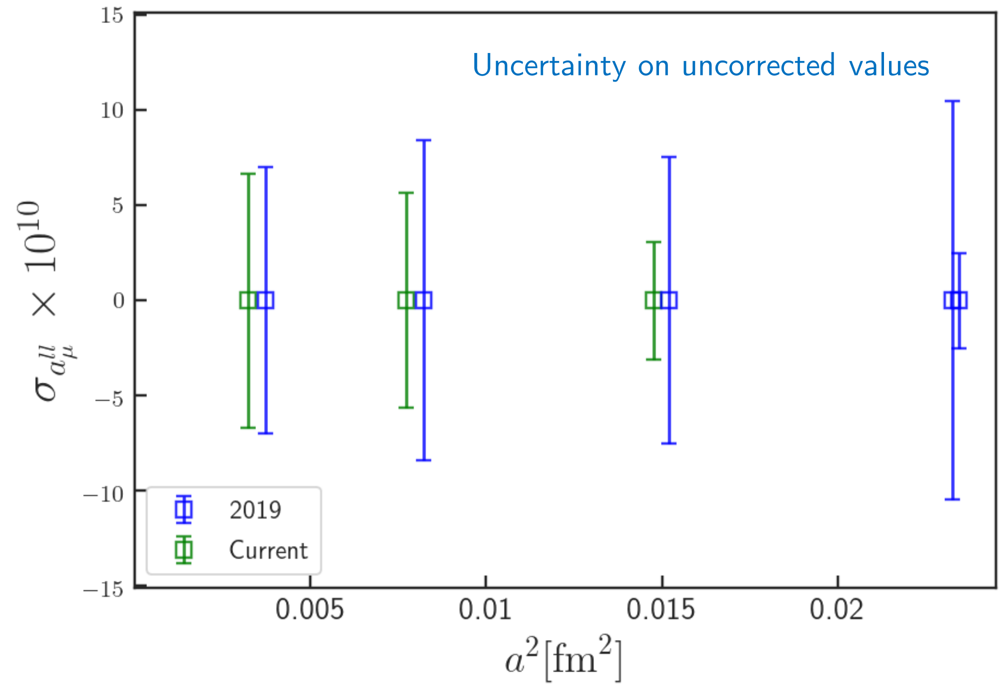
- Light-quark, connected contribution (90% of total). Uncertainties dominated by statistics, continuum extrapolation, scale-setting & finite volume effects.

- This talk will focus on the calculation of this contribution including an exclusive long-distance tail calculation.

- Isospin breaking and disconnected contributions: Poster by Craig McNeile

Simulation Details



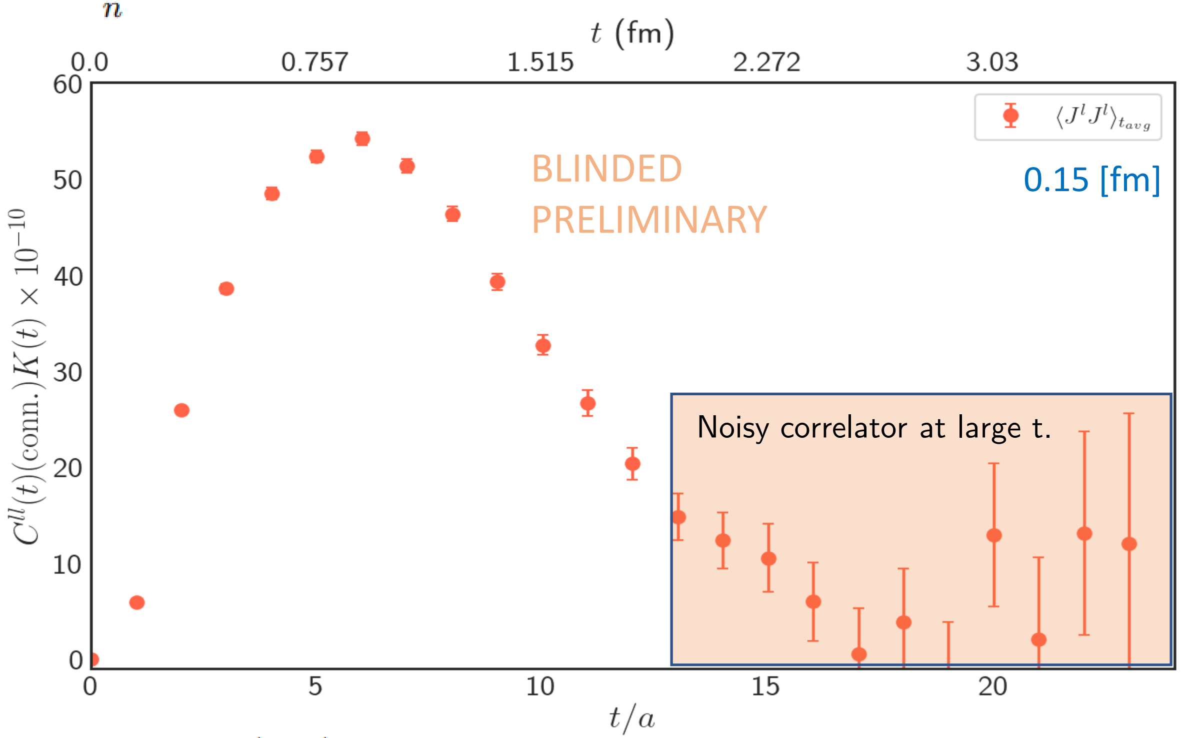


- Nf=2+1+1 HISQ ensembles with physical light-quark masses.
- 4 lattice spacings [0.06-0.15] fm, planning to add a 5th lattice spacing at 0.042 fm.
- All ensembles (except largest spacing) with $m_\pi L > 3.7$.

$$a_\mu^{ll}(\text{conn.}) = 4\alpha^2 \int_0^\infty dt \tilde{K}(t) C(t)^{\text{ll}}_{\text{conn.}} \quad , \quad C(t)^{\text{ll}}_{\text{conn.}} = \sum_i^3 \int d^3x \left\langle J_i^l(x) J_i^l(0) \right\rangle_{\text{conn.}}$$

$$C(t)^{\text{ll}}_{\text{conn.}} = \sum_n \left| \left\langle 0 \left| J_i^l \right| n \right\rangle \right|^2 e^{-E_n t}$$

Lowest energy states are $\pi\pi$.



$\left\langle 0 \left| J_i^l \right| \pi\pi \right\rangle$ small -> signal-to-noise problem.

Perform dedicated spectroscopy study with $\pi\pi$ operators that overlap strongly with these states to precisely determine the energies and amplitudes.

Staggered pion operator

$$\pi^-(\vec{p})_\xi = \sum_x e^{i\vec{p}\cdot\vec{x}}\bar{u}(x)\gamma_5 \otimes \gamma_\xi d(x)$$

Zero momentum taste irreps:

Scalar & Pseudo-scalar (1D)
Vector & Pseudo-vector (3D)
Tensor (3D+3D)

Non-zero momentum taste irreps:

1D irreps are Wilson like.
3D irreps can split (1D+2D) based on momentum & taste direction alignment.

Two-pion operators

$$\mathcal{O}_{\pi\pi}(0) \quad = \quad \sum_{\substack{\xi_1,\xi_2 \\ \vec{p}\in\{p\}^\star}} CG_{\text{stag, iso.}}\ \pi(\vec{p})_{\xi_1}\pi(-\vec{p})_{\xi_2}$$

Clebsch-Gordon's from irreducible representations of staggered lattice symmetry group.

$$SU_I(2) \times \left(T_N^3 \rtimes \{\Xi_\mu, C_0\} \rtimes \left\{\tilde{R}_{ij}, I_S\right\}\right) = SU(2) \times \left(Z_N^3 \rtimes \Gamma_{4,1} \rtimes W_3\right)$$

Wigner's method twice ~ little groups of little groups.

$$CG = \text{ unitarity } : \sum_{g\in G}\left[D^{(\pi_1)}(g) \times D^{(\pi_2)}(g)\right] AD^{(J)}(g)^\dagger$$

-Sakata, J.Math.Phys. 15 (1974) 1702-1709

Taste scalar vector current "one-link" -> taste diagonal two pion states.

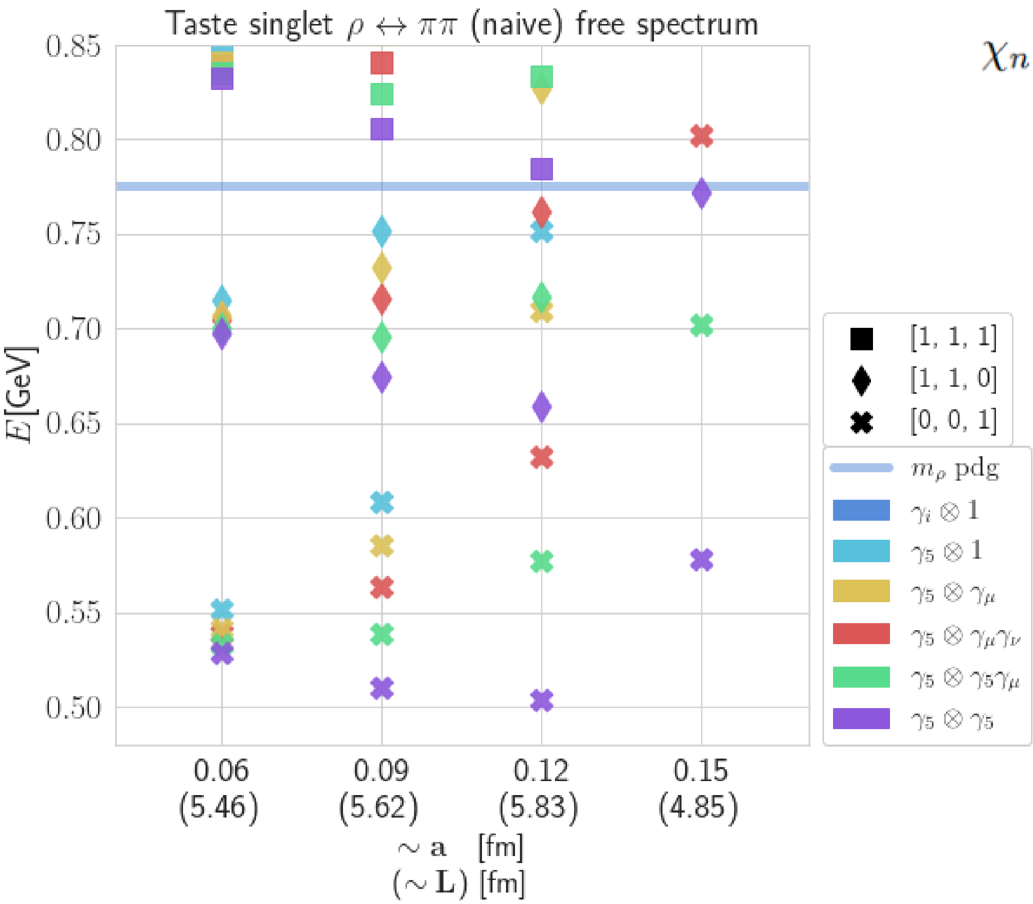
$$\mathbf{C}(t) = \begin{pmatrix} C(t)_{J,\tilde{J}\to J,\tilde{J}} & C(t)_{J,\tilde{J}\to\pi\pi} \\ C(t)_{\pi\pi\to J,\tilde{J}} & C(t)_{\pi\pi\to\pi\pi} \end{pmatrix} \quad \Rightarrow \quad \mathbf{C}(t)v = \lambda \mathbf{C}(t_0)v$$

Solve GEVP:

$\chi_n = (v_n)_i \mathcal{O}_i$ with best overlap with state n.

$$\langle \chi_n \chi_n^\dagger \rangle = \sum_n Z_n^2 e^{-E_n t}$$

$$\langle \chi_n J_l^\dagger \rangle = \sum_n Z_n \langle 0|J_l|n\rangle e^{-E_n t}$$



- Operators for all states up to rho energy.
- Need all taste irrep rows to account for irrep splitting at non-zero mom.

$$E_{\pi_\xi\pi_\xi} = 2\sqrt{m_\xi^2 + p^2} \quad , \quad \vec{p} = \frac{2\pi}{L}[n_x, n_y, n_z]$$

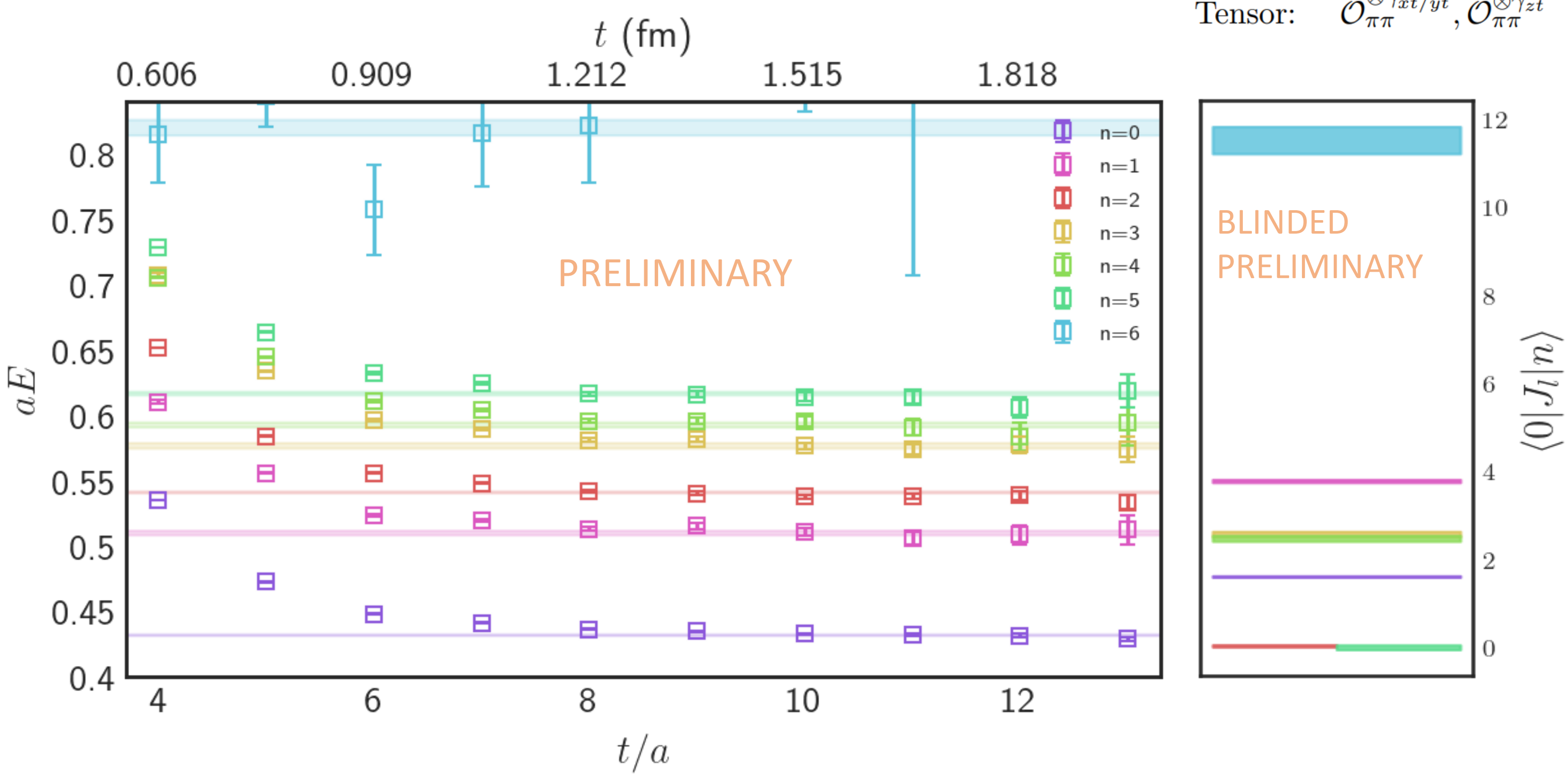
Staggered two-pion states on the lattice: GEVP Results

$$\langle \chi_n \chi_n^\dagger \rangle = \sum_n Z_n^2 e^{-E_n t}, \quad \langle \chi_n J_l^\dagger \rangle = \sum_n Z_n \langle 0 | J_l | n \rangle e^{-E_n t}$$

Energies and Amplitudes from combined fit.

Operator Basis at 0.15 [fm]

Vector current:	$\mathcal{O}_J, \tilde{\mathcal{O}}_J$	
Pseudo-scalar:	$\mathcal{O}_{\pi\pi}^{\otimes\gamma_5}$	$[0,0,1], [1,1,0]$
Pseudo-vector:	$\mathcal{O}_{\pi\pi}^{\otimes\gamma_{5x/y}}, \mathcal{O}_{\pi\pi}^{\otimes\gamma_{5z}}$	$[0,0,1]$
Tensor:	$\mathcal{O}_{\pi\pi}^{\otimes\gamma_{xt/yt}}, \mathcal{O}_{\pi\pi}^{\otimes\gamma_{zt}}$	$[0,0,1]$

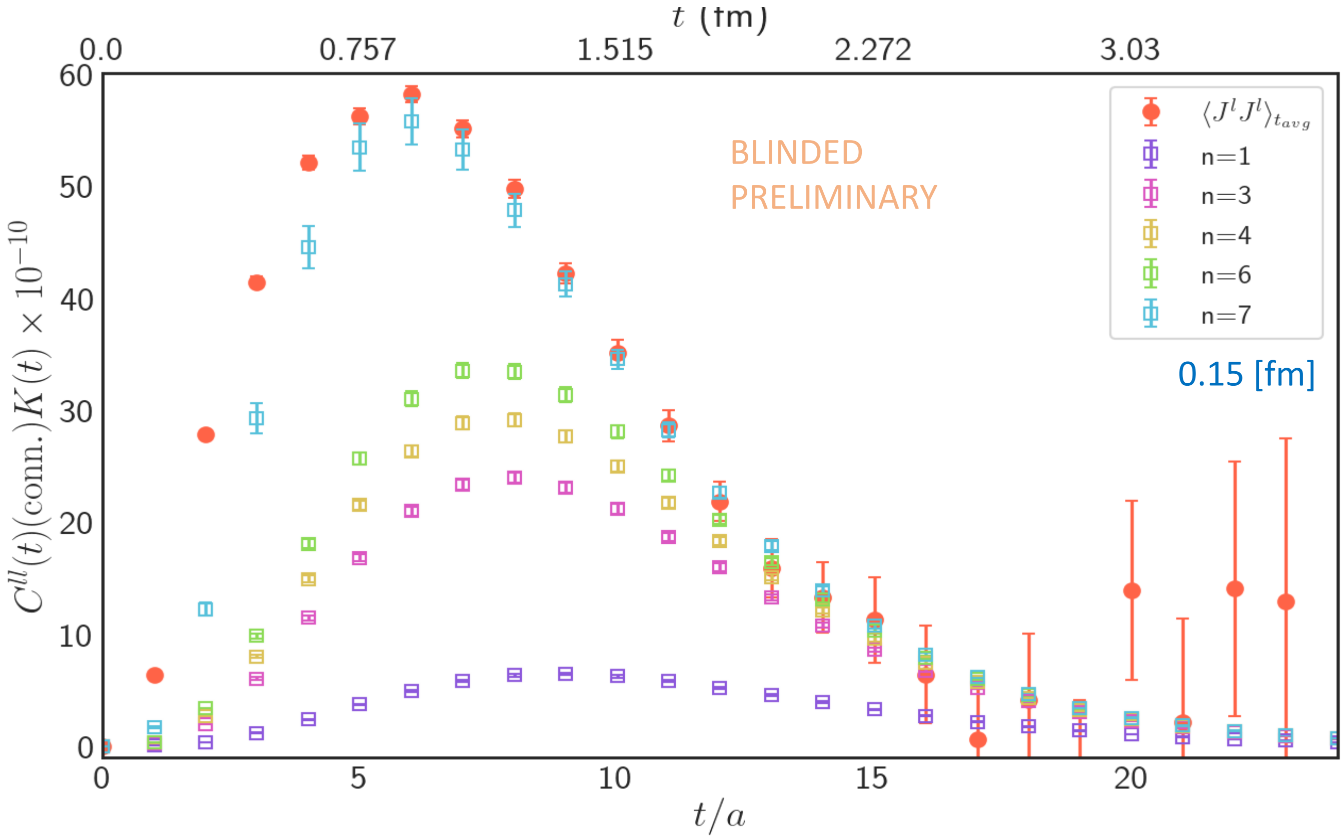


Effective mass plot for diagonal correlation functions, bands are fit results.

- Pseudo-vector & tensor taste irrep splitting's (1&2, 4&5) are non-degenerate.
- Overlap amplitudes heavily suppressed for certain states (2&5).

Correlator Reconstruction

$$C(t)^{\mathrm{ll}}_{\mathrm{conn.}} = \sum_n \left| \left\langle 0 \left| J_i^l \right| n \right\rangle \right|^2 e^{-E_n t}$$



Vector current is time-averaged to suppress oscillating contributions.

$$\frac{\sigma_{a_\mu^{ll}}}{a_\mu^{ll}} \times 100$$

- Raw: 7.8%
- Bounding Method t_c = 2.65 fm: 4.9%
- Reconstruction, n=6, t* = 1.82 fm, two-pion states beneath rho : 1.5%
- Reconstruction, n=7, t* = 1.67 fm, Including the rho: 1.4%

Window Analysis

Restrict integral to some Euclidean time region to emphasize different physics / lattice effects.

$$a_{\mu,\mathrm{win}} = 4\alpha^2 \int_0^\infty \mathrm{d}t C(t)\tilde{K}(t)\Theta(t,t_1,t_2,\Delta)$$

$$\Theta(t,t_1,t_2,\Delta) = \frac{1}{2}\left[\tanh\left(\frac{t-t_1}{\Delta}\right) - \tanh\left(\frac{t-t_2}{\Delta}\right)\right]$$

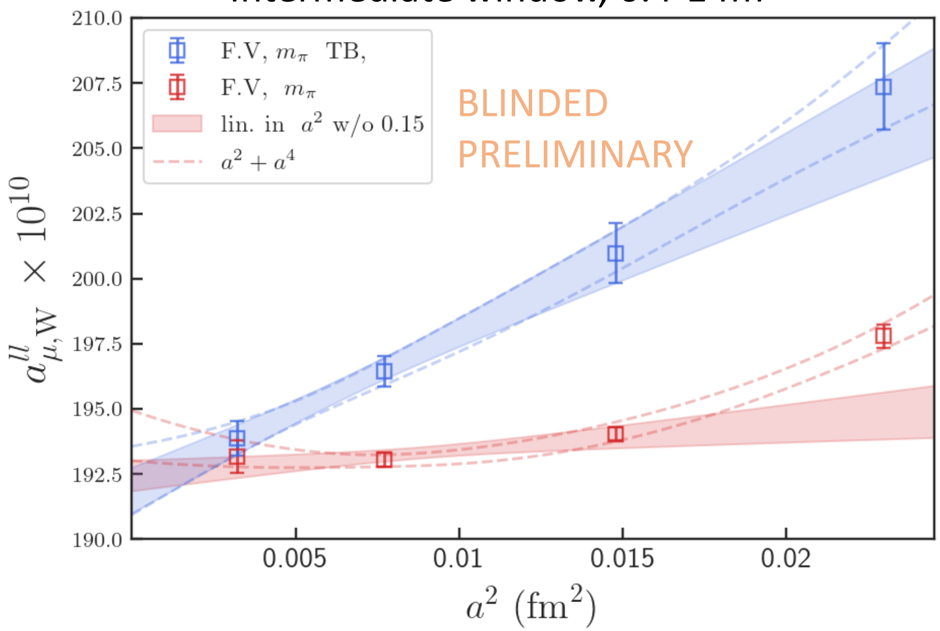
T. Blum et al, arXiv:1801.07224

Window	$[t_1,t_2]$fm	Δ[fm]
SD	$[-\infty, 0.4]$	0.15
W	$[0.4, 1]$	0.15
LD	$[1, \infty]$	0.15

- SD: short-distance, discretization effects prominent.
- LD: long-distance(two-pion states), finite volume effects prominent.
- W: less sensitive to these extremum effects, easier to compute with precision to compare different lattice calculations.

Goal of lattice g-2 community: agreement on these quantities.

Intermediate window, 0.4-1 fm



- Corrections from $\rho - \gamma - \pi\pi$ model (leading order).
- Good consistency between extrapolations of data with(out) discretization effect corrections.

Conclusions

Goal of the Fermilab Lattice, HPQCD & MILC collaboration:

Compute the HVP contribution to g-2 using lattice QCD with an uncertainty of $< 0.5\%$.

To achieve this, the light-quark, connected contribution must be known with commensurate precision as it accounts for ~90% of the total value. To achieve this, we aim to

- Increase statistics at finest lattice spacings.
- Compute two-pion tail exclusively.
- Add a 5^{th} lattice spacing (0.042 fm) to analysis.
- Reducing scale setting uncertainty through absolute & relative scale calculations. 1912.00028.
- Refining estimates on FV corrections & uncertainties.

In parallel to the work presented here, calculations for

- Disconnected contribution: goal $<10\%$ total.
- IB effects: goal is explicit calculation of all leading-order effects.
  - SIB effects on connected & disconnected contributions.
  - Leading QED effects.

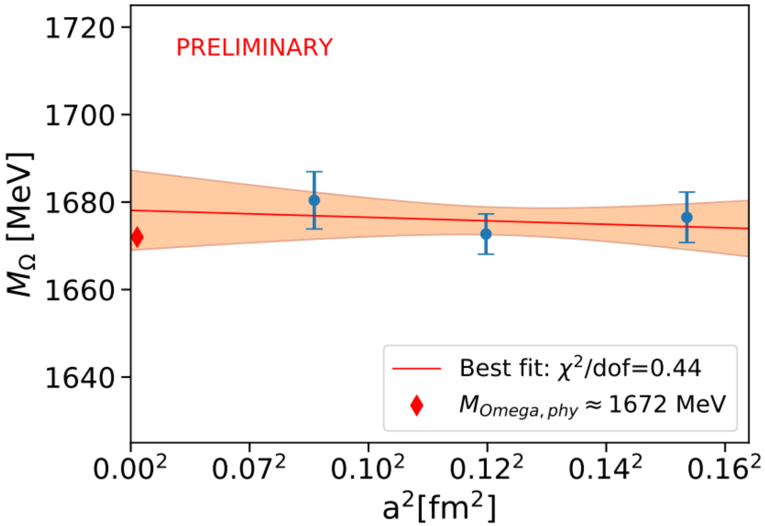
Poster from Craig McNeile for disconnected & IB results.

Thank you

Scale-setting – M_Ω & w_0/a

Absolute scale parameter M_Ω

- Insensitive to isospin breaking effects compared with f_π.
- M_Ω at 0.15, 0.12 and 0.09 fm, C Hughes et al., 1912.00028.
- Increasing statistics & extending to 0.06 fm.



Relative scale parameter w_0/a

- Ongoing high statistics gradient flow study on all HISQ ensembles.
- 6 combinations of ensemble/flow/observable action discretization to study & quantify cut-off effects.

Windows Consistency

- Self-consistency check:

$$a_\mu^{ll}(\text{conn.}) = a_{\mu,\text{SD}}^{ll} + a_{\mu,\text{W}}^{ll} + a_{\mu,\text{LD}}^{ll}$$