$K \to \pi\pi$ decay matrix elements at physical points with periodic BCs

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LATTICE 2021
July 26–30, 2021

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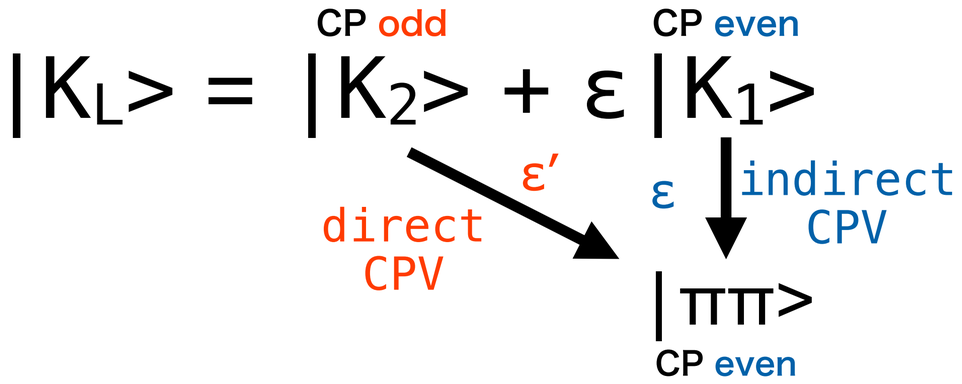
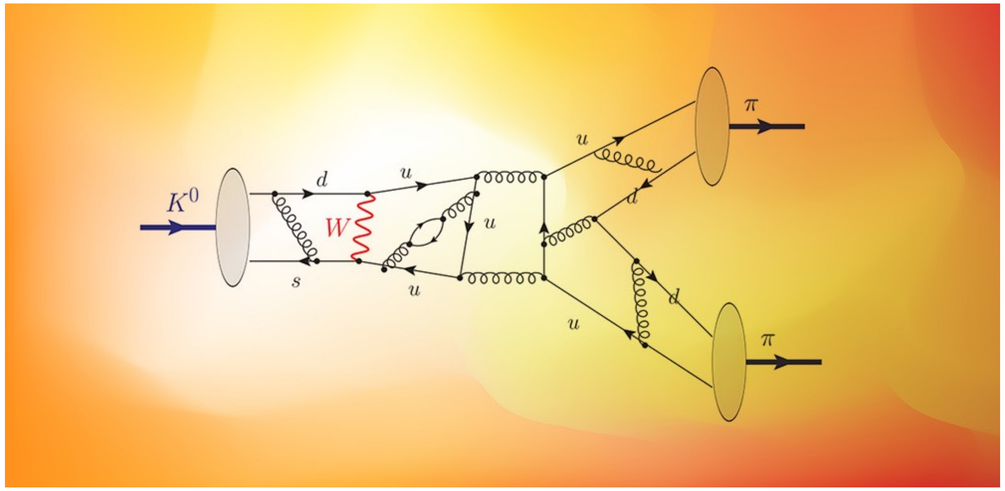
K → ππ & Direct CPV



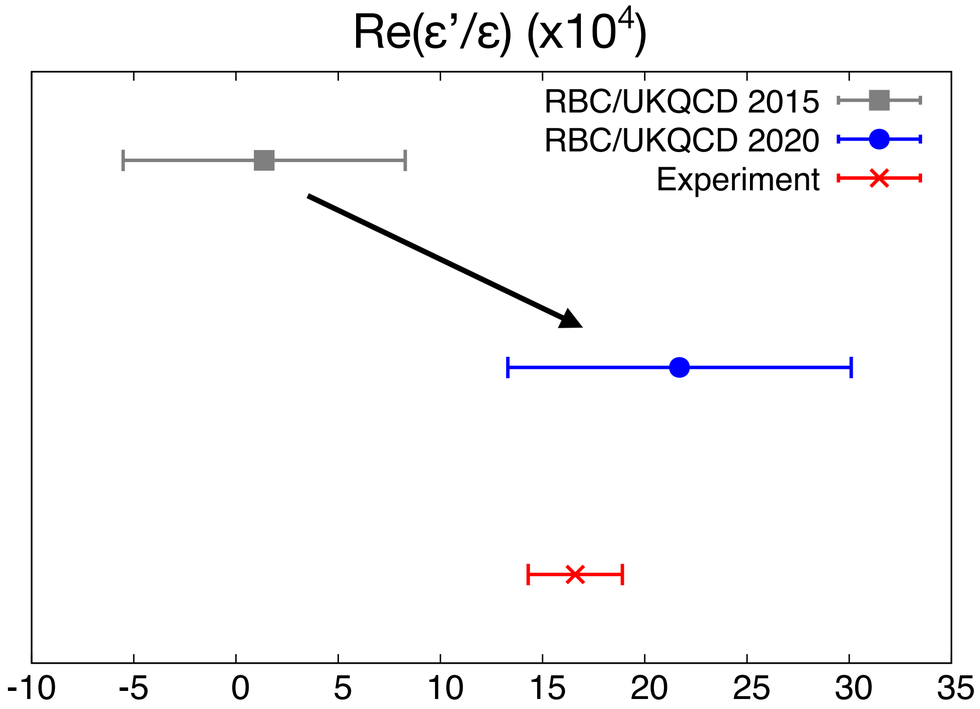


- ε' vs ε
  - Re $(\varepsilon'/\varepsilon)_{exp}$ = 16.6(2.3) x 10^{-4} (circa 2000)
  - Explained by SM?
- Key to understanding the nature of matter/anti-matter asymmetry

G-parity BC calculation done

- $E_{\pi\pi} = 2m_\pi \approx 280$ MeV state in Euclidean correlators forbidden
- Useful to extract $E_{\pi\pi} = m_K$ state at large time separations

- **C. Kelly's plenary talk (Tuesday 9AM EDT)**
- **PRD 102,054509**



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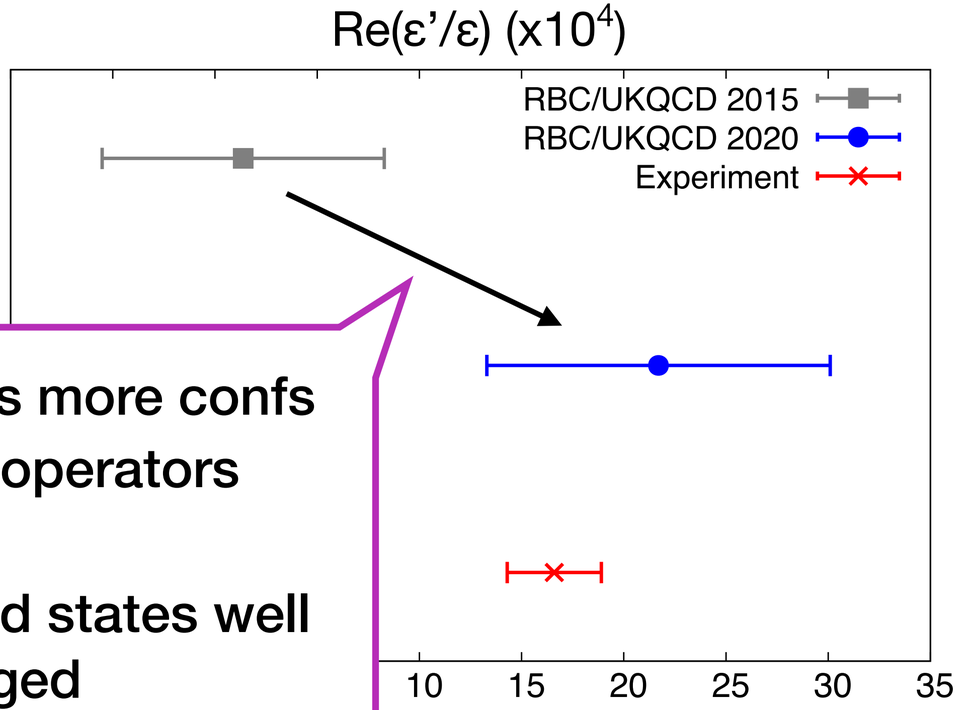

- 3+ times more confs
- # of ππ operators
  - 1 → 3
  - excited states well managed
- Step scaling in NPR

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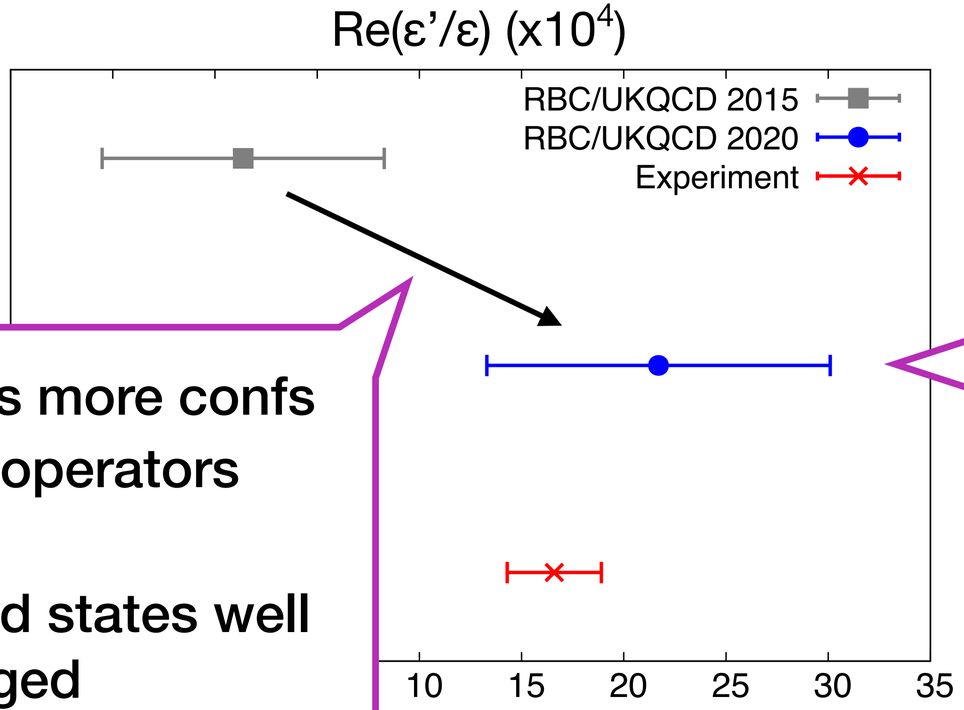

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- Step scaling in NPR

$$21.7(2.6)_{\text{stat}}(6.2)_{\text{sys}}(5.0)_{\text{EM/IB}} \times 10^{-4}$$

- Independent calculations desired
- Systematic errors
  - Isospin breaking effects
  - Finite lattice cutoff

Why periodic BCs?

- Already have lattice ensembles with physical pion mass
  - 1 GeV, 24^3 x 64, 1.4 GeV, 32^3x64 and …
  - Continuum limit possible

- Hope to introduce QED/IB effects near future
  - Difficult with G-parity BCs
  - Periodic BCs study valuable

- Presence of $E_{\pi\pi} = 2m_\pi$ state challenging
  - S/N ratio of $E_{\pi\pi} = m_K$ state should be the same as in G-parity BCs

What to calculate

ππ phase shifts

$$\mathrm{Re}\left(\frac{\epsilon'}{\epsilon}\right) = \mathrm{Re}\left\{\frac{i\omega e^{i(\delta_2-\delta_0)}}{\sqrt{2}\epsilon}\left[\frac{\mathrm{Im}\,A_2}{\mathrm{Re}\,A_2} - \frac{\mathrm{Im}\,A_0}{\mathrm{Re}\,A_0}\right]\right\} \qquad (\omega = \mathrm{Re}\,A_2/\mathrm{Re}\,A_0)$$

Lellouch-Lüscher finite volume correction

Renormalization matrix

$$A_I = F\frac{G_F}{2}V_{us}^* V_{ud}\sum_{i,j}[z_i(\mu) + \tau y_i(\mu)]Z_{ij}(\mu)\langle(\pi\pi)_I|Q_j^{lat}|K\rangle$$

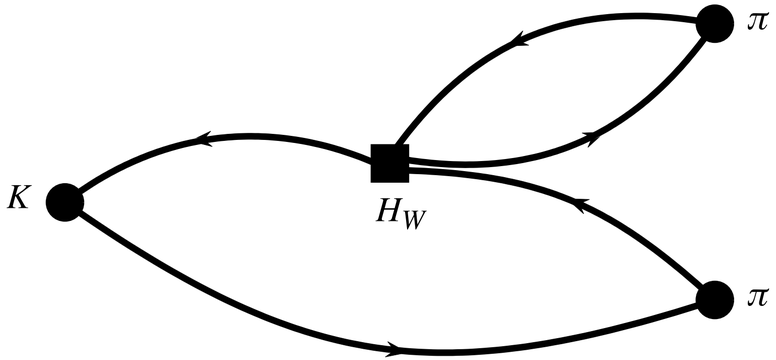
Wilson coefs. pQCD | LQCD (+pQCD) | LQCD

- δ_I, F being determined via π-π scattering work w/ GEVP & Lüscher formalism
- $A_I = <(\pi\pi)_I|H_W|K>$ from 3pt correlation functions
- I = 0 challenging — disconnected diagrams, power divergences

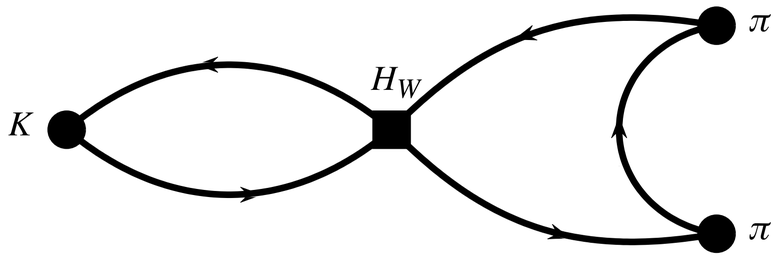
$K \to \pi\pi$ calculation

- 214 configurations (on $24^3 \times 64$), physical pion & kaon masses
- All-to-all quark propagators
  - 2,000 low modes for light quarks (no low mode for strange)
  - high-mode part: spin, color and time dilutions => 768 inversions
- 28 (5 independent) interpolation $\pi\pi$ operators
  - $\pi_{p=(0,0,0)}\pi_{p=(0,0,0)}$, $\pi_{p=(0,0,1)}\pi_{p=(0,0,-1)}$, $\pi_{p=(0,1,1)}\pi_{p=(0,-1,-1)}$, $\pi_{p=(1,1,1)}\pi_{p=(-1,-1,-1)}$ & σ
  - to control effects from various states

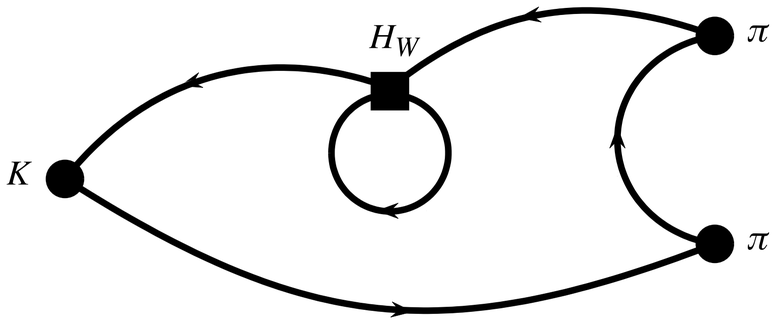
4 types of diagrams



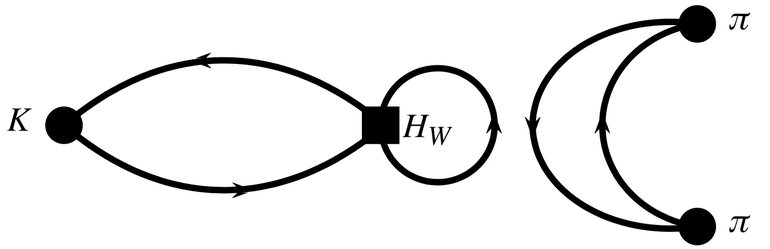
type 1



type 2



type 3



type 4

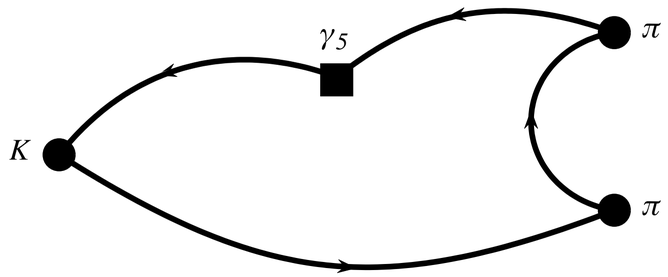
Subtraction of quadratic divergence

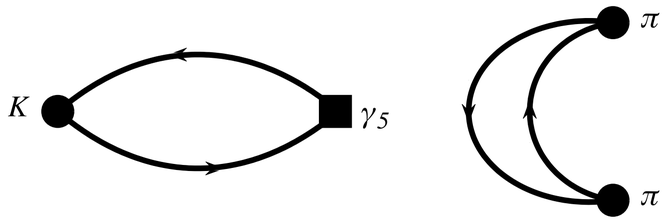
- Loop diagrams (types3,4) have a^{-2} divergence due to mixing with bilinear operators

- Subtraction
  - $Q'_i = Q_i - \alpha_i \bar{s}\gamma_5 d$
  - Condition: $\langle Q'_i(t)K(0)^\dagger \rangle = 0$

$$\alpha_i = \frac{\langle Q_i(t)K(0)^\dagger \rangle}{\langle \bar{s}\gamma_5 d(t)K(0)^\dagger \rangle}$$

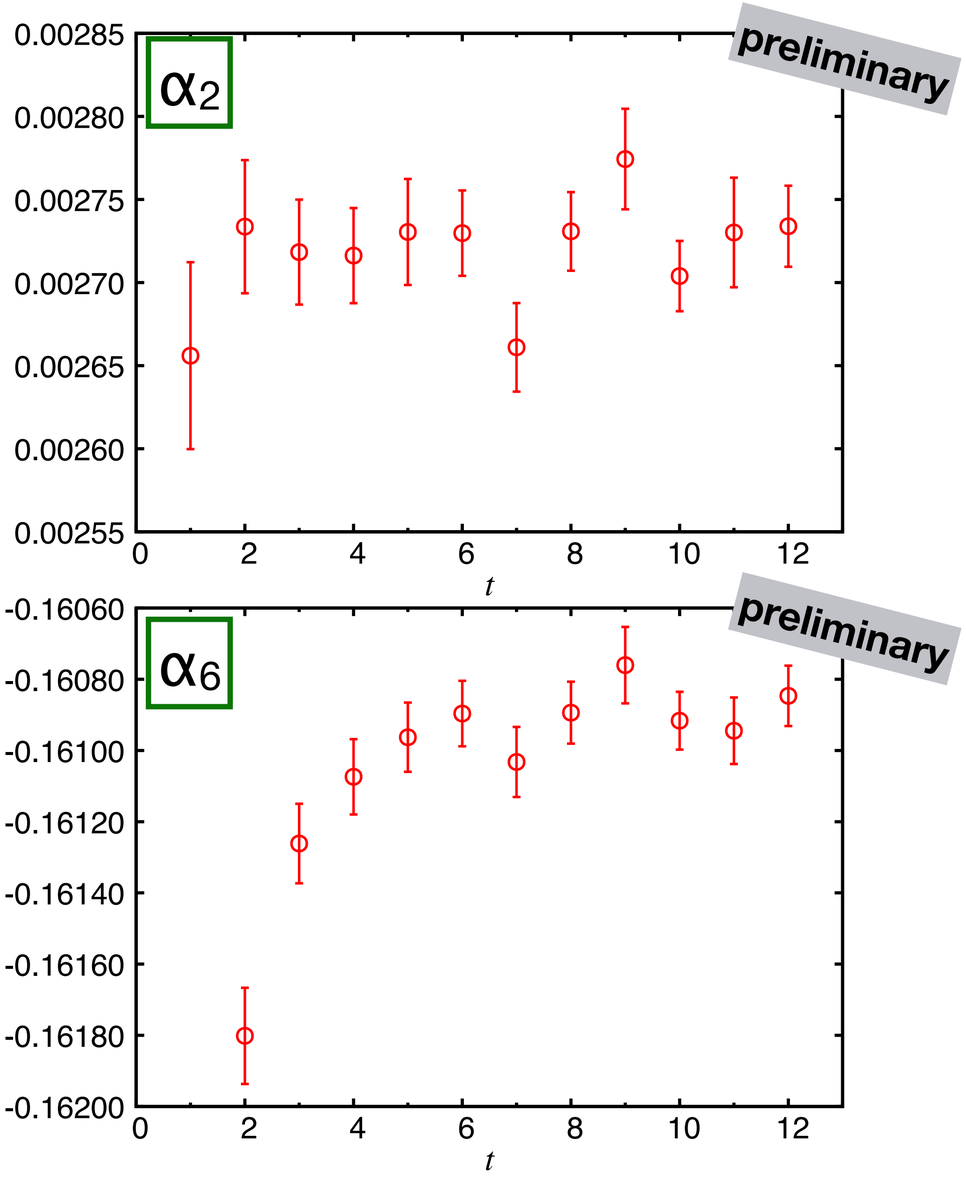
- Additional contractions





α_i

- Ratio α_6 / α_2
  - no a^{-2} divergence
  - less dependent on a
  - $\alpha_6 / \alpha_2 \approx -60$ from this calculation
  - $\alpha_6 / \alpha_2 \approx -50$ from G-parity calculation

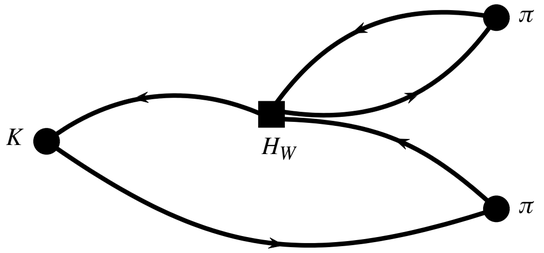


type 4 dominates stats. error

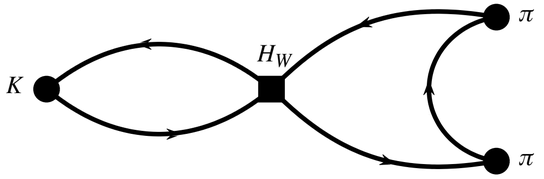
- Previous G-parity calculation
  - types 1,2: averaged over every 8 time translations
  - types 3,4: averaged over every time translation

- types 1,2 still expensive but no need of such precision

  → cost reduction?



type 1



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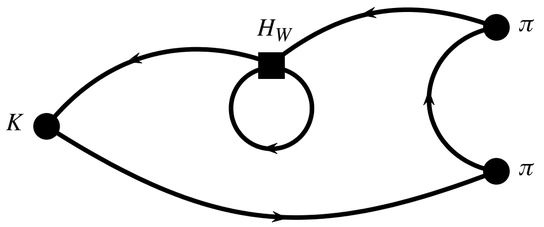

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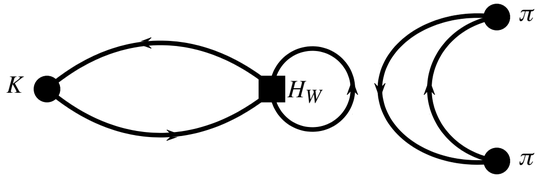
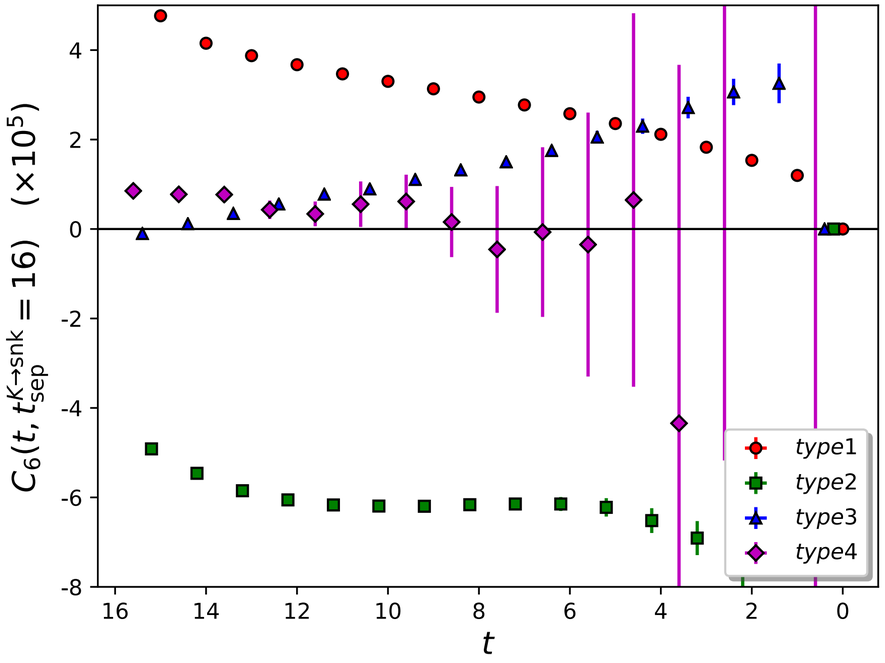


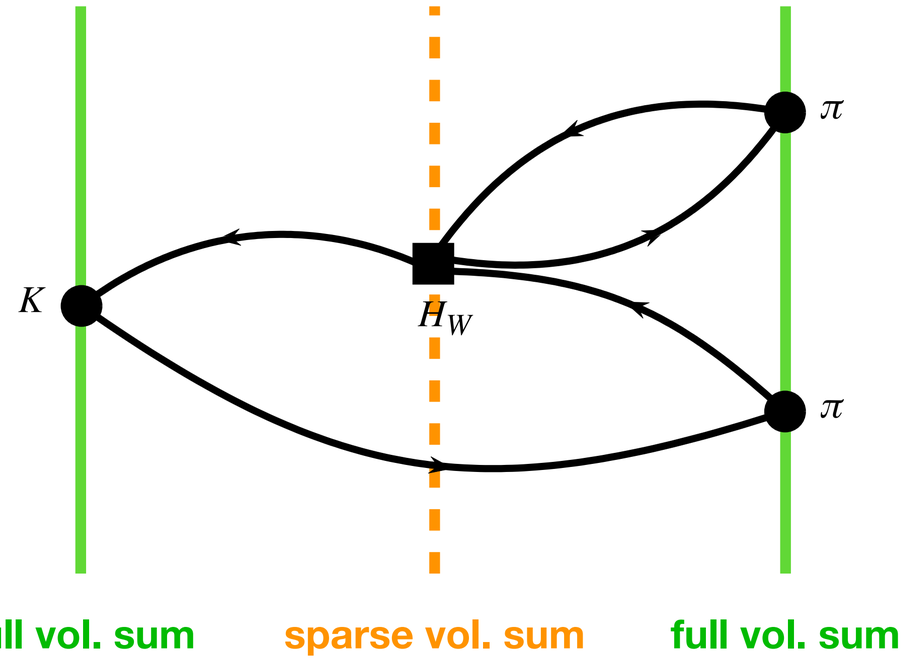
type 4



RBC/UKQCD, PRD 102,054509

Sparsening H_W

- Contraction cost mostly promotional to volume of H_W
- G-parity calculation: summed H_W over whole 3D volume
- This calculation: volume of H_W ($24^3 \rightarrow 8^3$: 27x speed up) for types 1 & 2

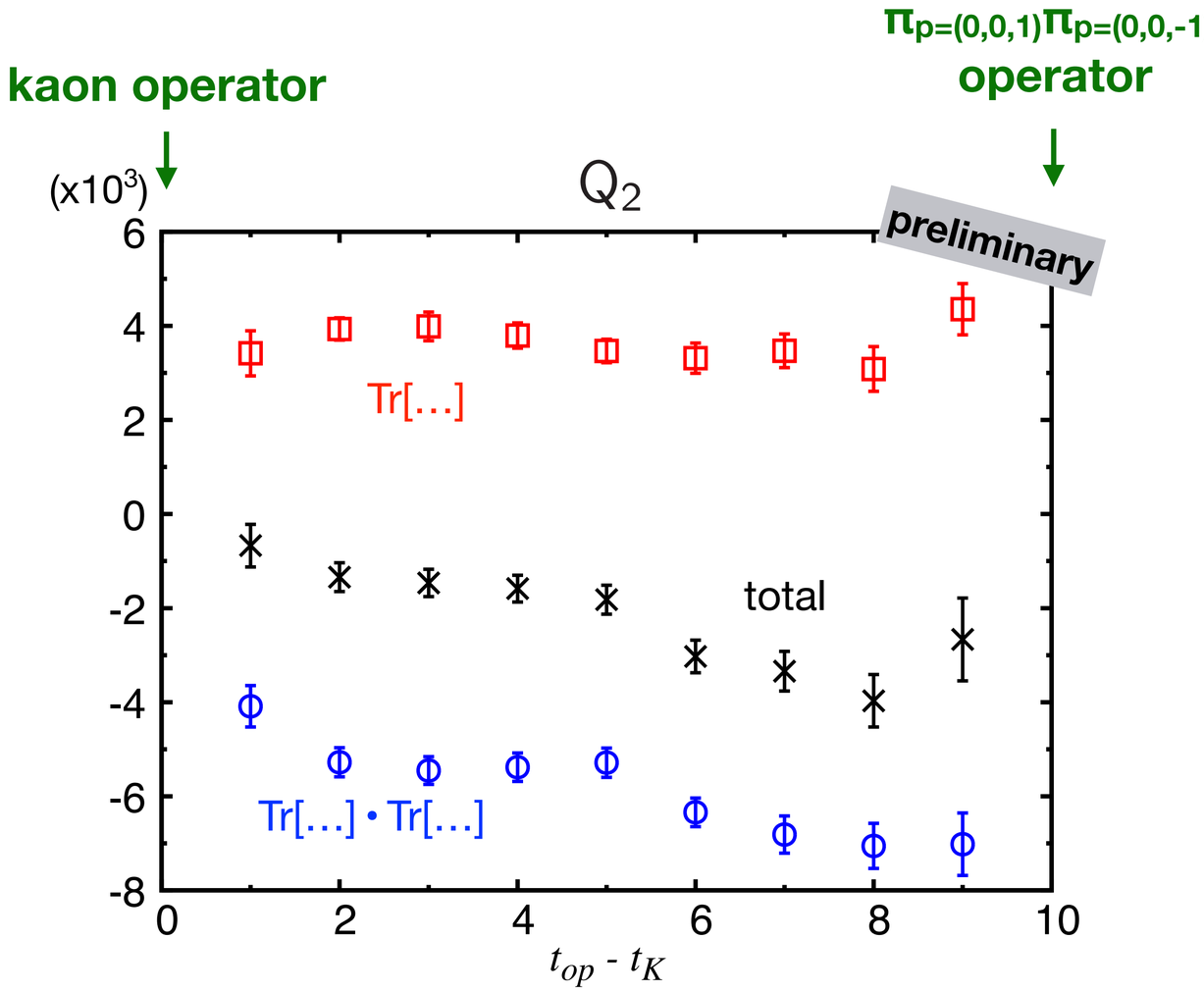


Preliminary result (I = 2)

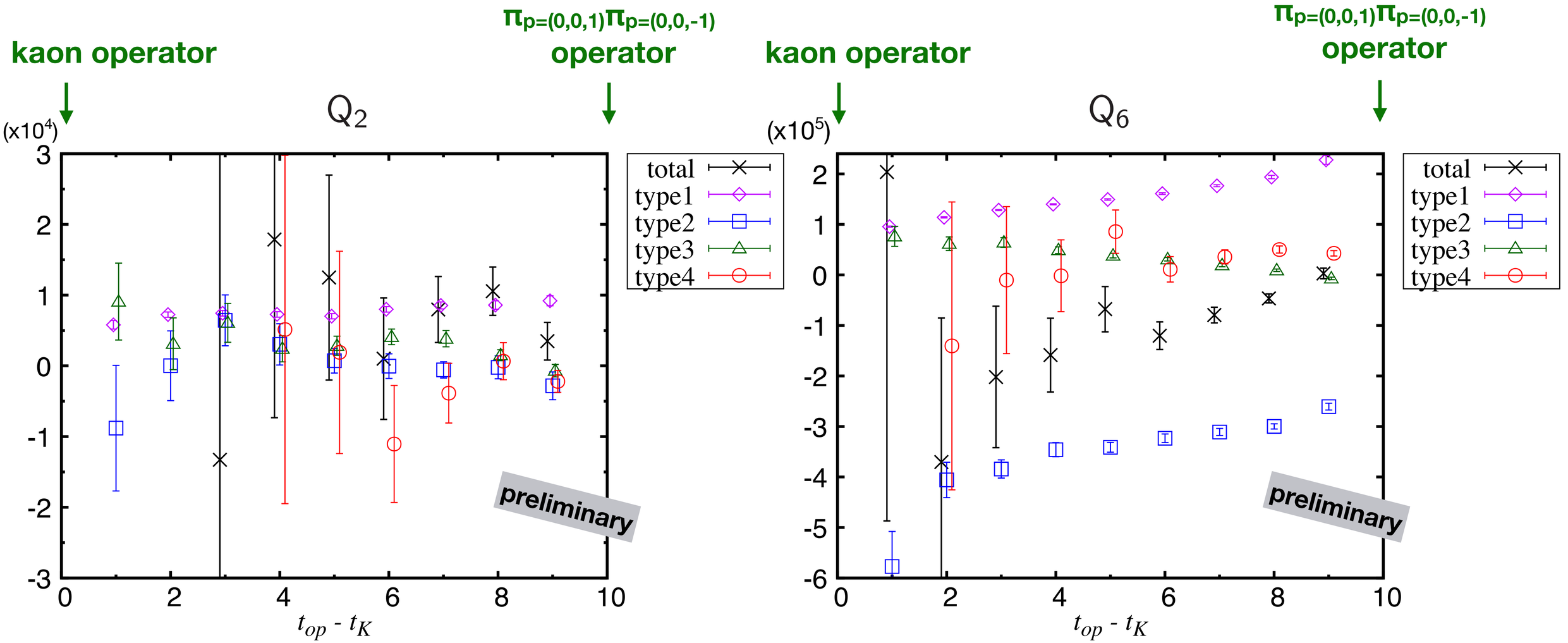
- I = 2 mode contributed by type1 only

- some easy checks
  - Two specific contractions opposite sign (from previous calculation)
  - $Q_1 = Q_2$ (for I = 2)

    ↑ $Q_1 - Q_2 = Q_3 - Q_4$ (Fierz identity) & absence of Q_3 & Q_4 for I = 2



I = 0 correlation functions



- Sparsen H_W for types1,2 – still much more precise than type4

Summary

- Purpose
  - New independent calculation of K → ππ decays
  - Periodic-BC study gives prospect of introducing QED/IB effects

- Continuum limit of K → ππ amplitudes & ε'
  - 24^3 x 64, a^{-1} = 1.0 GeV, 250 confs
    (measurement almost done, data being analyzed)
  - 32^3 x 64, a^{-1} = 1.4 GeV, 200 confs
    (measurement on going)

- Cost reduction by sparsening H_W for types1,2 successful