Precision bottomonium properties and b quark mass from lattice QCD+QED

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Abstract

Lattice QCD calculations of weak decay matrix elements for hadrons containing b quarks are critical to the flavour physics programme. It is therefore important to have stringent tests of lattice QCD results for b physics in other settings to make sure systematic errors are under control. Here we provide such tests in the bottomonium system with the ground-state hyperfine splitting and the Υ leptonic width, both accurately known from experiment. We give the most accurate lattice QCD results to date for these quantities and also test the impact on them of the b quark's electric charge [1].

Accurate masses for heavy quarks are important for high-precision searches for new physics in Higgs decay. We give here a new determination of the ratio of the masses for b and c quarks that is completely nonperturbative in lattice QCD and includes the calculation of QED effects for the first time [2].

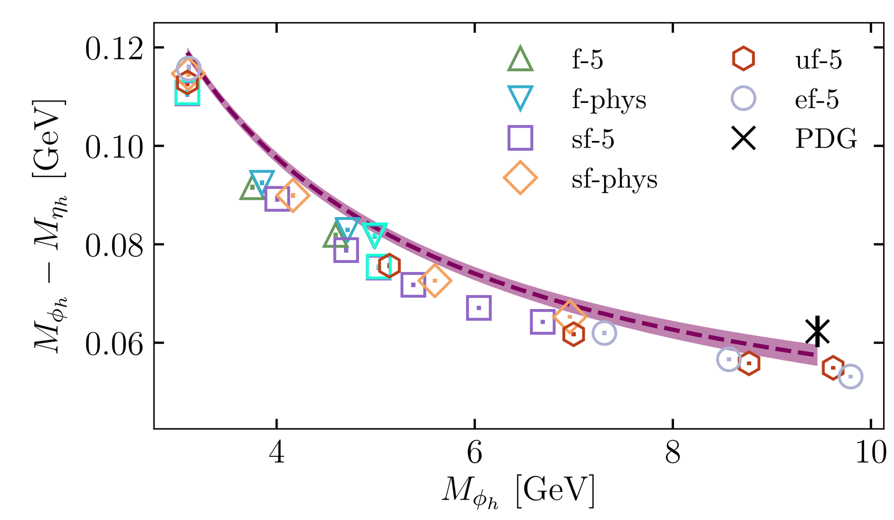
Lattice QCD calculation

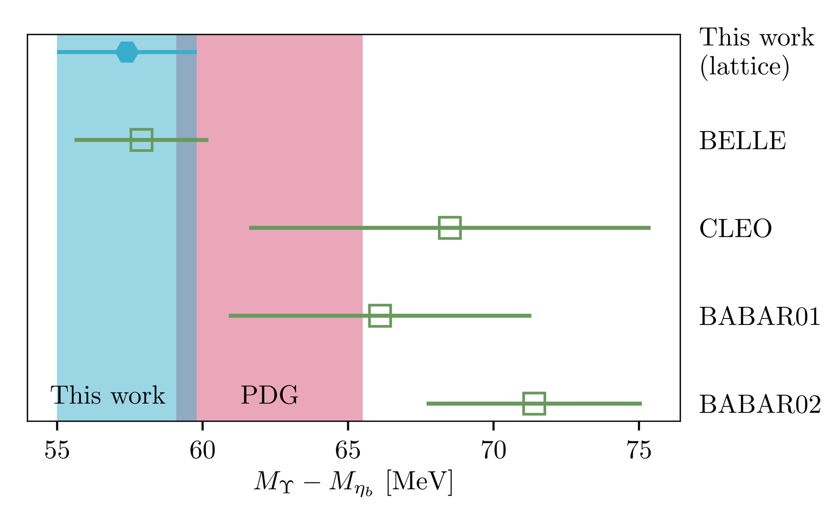
We use the Highly Improved Staggered Quark (HISQ) action on ensembles of gluon field configurations that include 2+1+1 flavours of HISQ quarks in the sea, generated by the MILC collaboration. The ensembles have a range of lattice spacing values from 0.09 fm down to 0.03fm and with u/d sea masses varying from 1/5 that of strange down to their physical value. On each ensemble we calculate pseudoscalar and vector 2-point correlators for HISQ valence quarks with masses m_h from that of charm up to am_h=0.8 or 0.9. On the finest lattices we can reach the b quark mass with am_h=0.65. We calculate correlators including quenched QED on a subset of ensembles.

We determine the masses and amplitudes of the ground state mesons (denoted η_h and ϕ_h) on each ensemble using standard correlator fits. The hyperfine splitting is the difference of masses, M_ϕ-M_η. The vector and pseudoscalar decay constants are determined from the amplitudes in the standard way; both are normalised accurately using lattice Ward identities [3]. We then perform a model-independent fit to these results as a function of lattice spacing and ϕ_h mass, using cubic splines. This allows us to determine results in the continuum limit with physical sea quark masses for the case where the heavy quark is the b quark, i.e. where the ϕ_h has the experimental mass of the Υ. We use this criterion for tuning the b quark mass both in pure QCD and in QCD+QED.

The ratio of quark masses is scheme and scale independent in pure QCD, but not in QCD+QED if the quarks have different electric charges. We calculate the ratio of b and c masses at 3 GeV in the MSbar scheme using a 3 step procedure. The first step is to fit results for m_h/m_c as a function of the ϕ_h mass and lattice spacing in pure QCD. m_c here is the tuned c quark mass, obtained on each ensemble using the experimental J/ψ meson mass [4]. Evaluating the fit function in the physical-continuum limit at the Υ mass then gives the pure QCD m_b/m_c value. The second step is to calculate how much this ratio changes if BOTH the b and c quarks have charge Q=e/3 so that the ratio is still scale-invariant. The third step corrects the final ratio in the MSbar scheme at 3 GeV to take the c quark charge to 2e/3. This is the largest QED effect and is calculated from the results of [4].

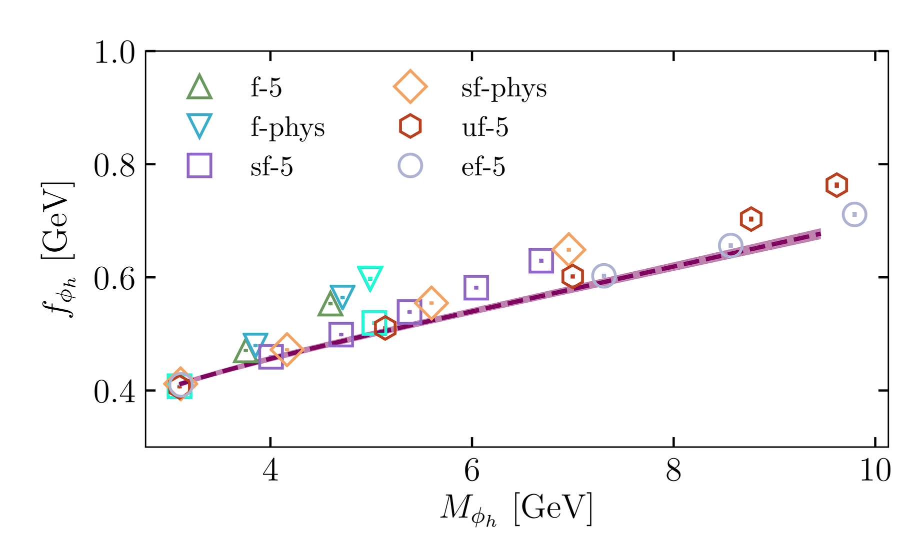
Results - hyperfine splitting

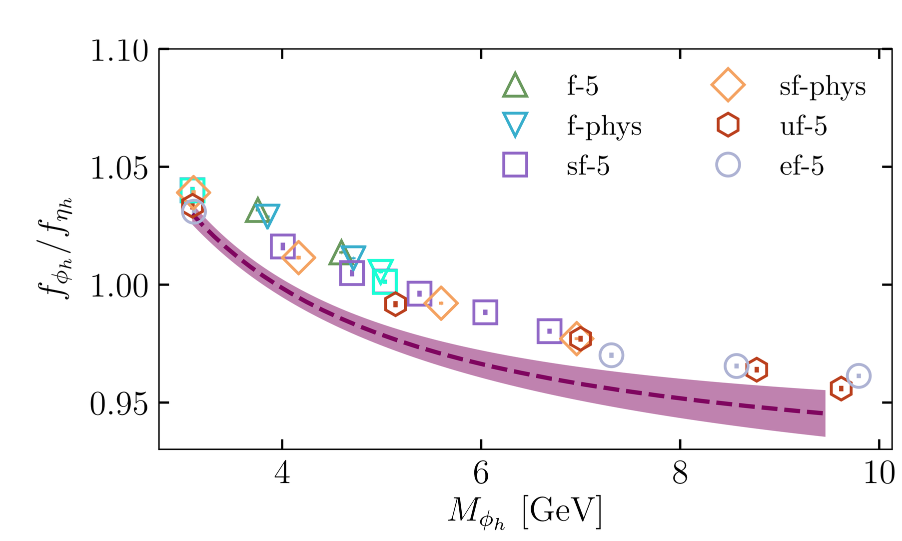


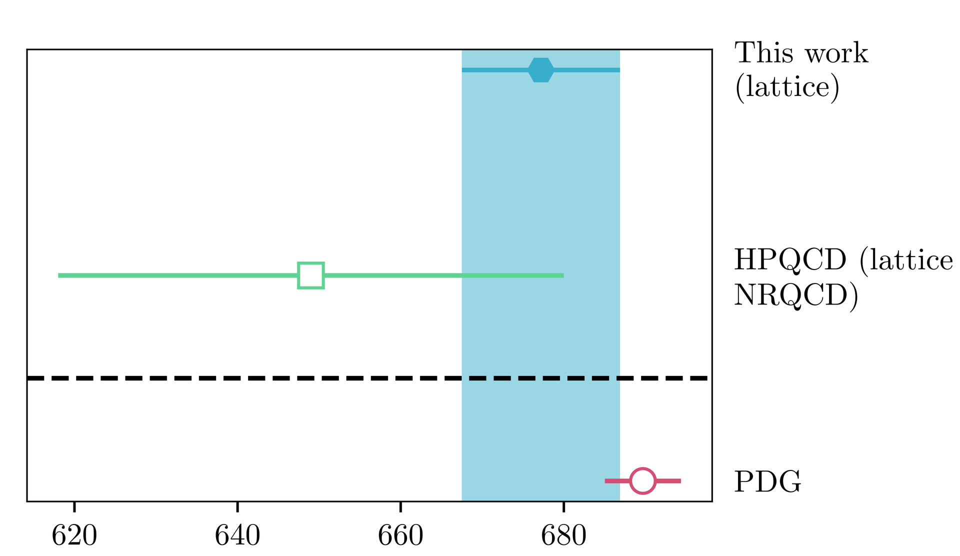


The figures above show our results [1] for the hyperfine splitting as a function of ϕ_h mass along with our fit curve. Our result at the b, i.e. the mass difference between Υ and η_b, is 57.5(2.3) MeV and is compared to experimental results in the lower figure. We see good agreement, particularly with the most recent result from Belle. QED effects here (and below) are tiny.

Results - decay constants

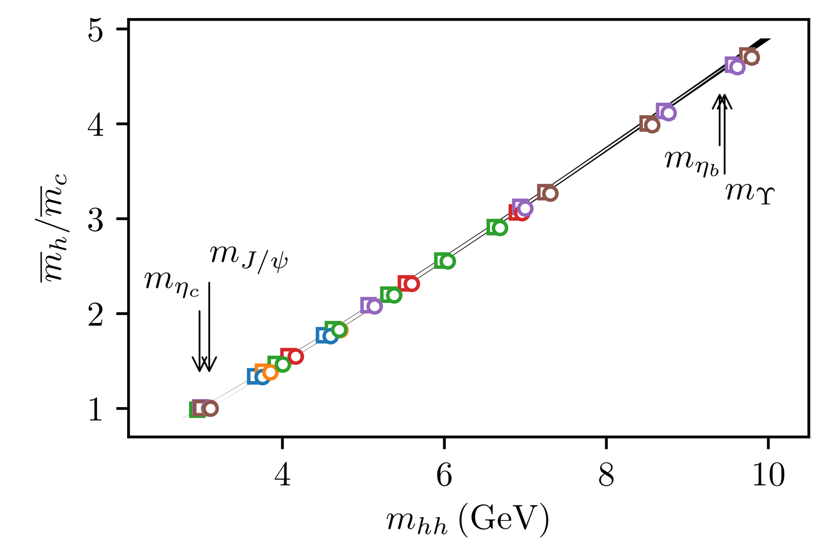


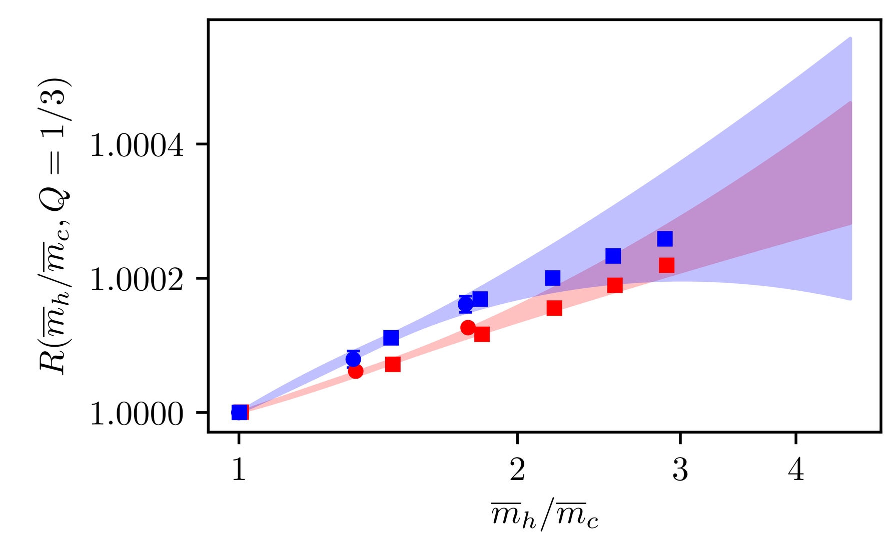


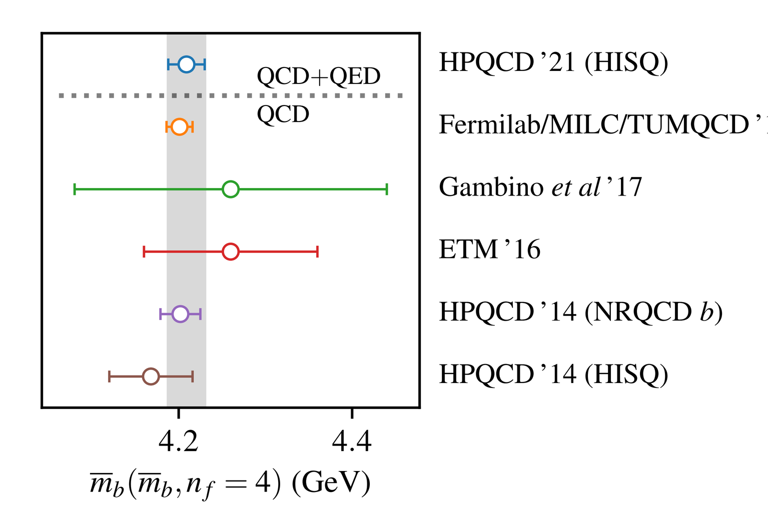


The figures above show the vector meson decay constant and the ratio of vector to pseudoscalar decay constants as a function of the ϕ_h mass [1]. Note that the ratio is larger than 1 at c and falls to less than 1 at b. The lower plot shows the good agreement between our result for the Υ decay constant (677(10) MeV) and that inferred from the experimental Υ leptonic width.

Results - m_b







The top plot [2] shows the ratio of HISQ masses m_h/m_c as a a function of heavyonium meson mass (either ϕ_h or η_h) in pure QCD. Our fit curves (using cubic splines) are also shown. The middle plot shows the relative change in this curve if both quarks have electric charge e/3. This is a tiny (< 0.04%) effect. A larger, 0.13%, QED effect comes from changing the c quark electric charge to 2e/3. We do this at scale 3 GeV using [4]. We obtain, for the ratio of MSbar masses at 3 GeV

$$\left.\frac{\overline{m}_b(3\,\mathrm{GeV})}{\overline{m}_c(3\,\mathrm{GeV})}\right|_{\substack{\mathrm{QCD}\\\mathrm{QED}}} = 4.586(12)$$

We can convert this into a value for m_b, using our m_c result in QCD+QED of 0.9841(51) GeV [4], giving

$$\overline{m}_b(\overline{m}_b)|_{\substack{\mathrm{QCD}\\\mathrm{QED}}} = \begin{cases} 4.209(21)\,\mathrm{GeV} & n_f = 4 \\ 4.202(21)\,\mathrm{GeV} & n_f = 5 \end{cases}$$

from calculations that include QED for the first time. The lower plot above compares this to earlier pure QCD results (that estimated QED effects).

Conclusion: Following our earlier high precision QCD+QED charmonium calculations using HISQ [4], we give here accurate results for Υ and η_b properties. Good agreement with experiment is seen, with few % uncertainties [1]. A new 0.6%-accurate QCD+QED determination of m_b is also given [2].

[1] D. Hatton et al, HPQCD, 2101.08103.
[2] D. Hatton et al, HPQCD, 2102.09609.
[3] D. Hatton et al, HPQCD, 1909.00756.
[4] D. Hatton et al, HPQCD, 2005.01845.

Our calculations used Darwin@Cambridge, part of the UK STFC's DiRAC facility.