Non-perturbative bounds for the semileptonic $B \to D^{(*)} l\nu_l$ decays and phenomenological applications

Work in collaboration with G. Martinelli and S. Simula
(arXiv:2105.08674 [hep-ph])

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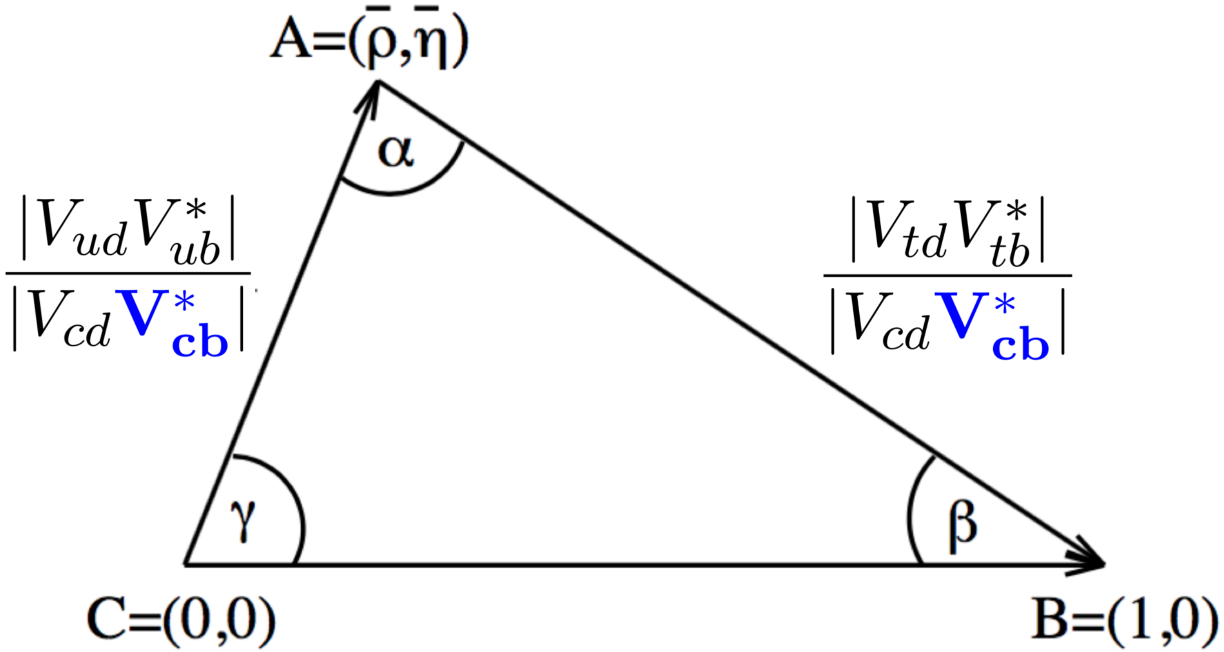


State-of-the-art of the semileptonic $B \to D^{(*)}$ decays

Two critical issues:

- V_{cb} puzzle:

EXCLUSIVE

$$|V_{cb}|_{BGL} \times 10^3 = 39.08(91)$$
$$|V_{cb}|_{CLN} \times 10^3 = 39.41(60)$$

VS

INCLUSIVE

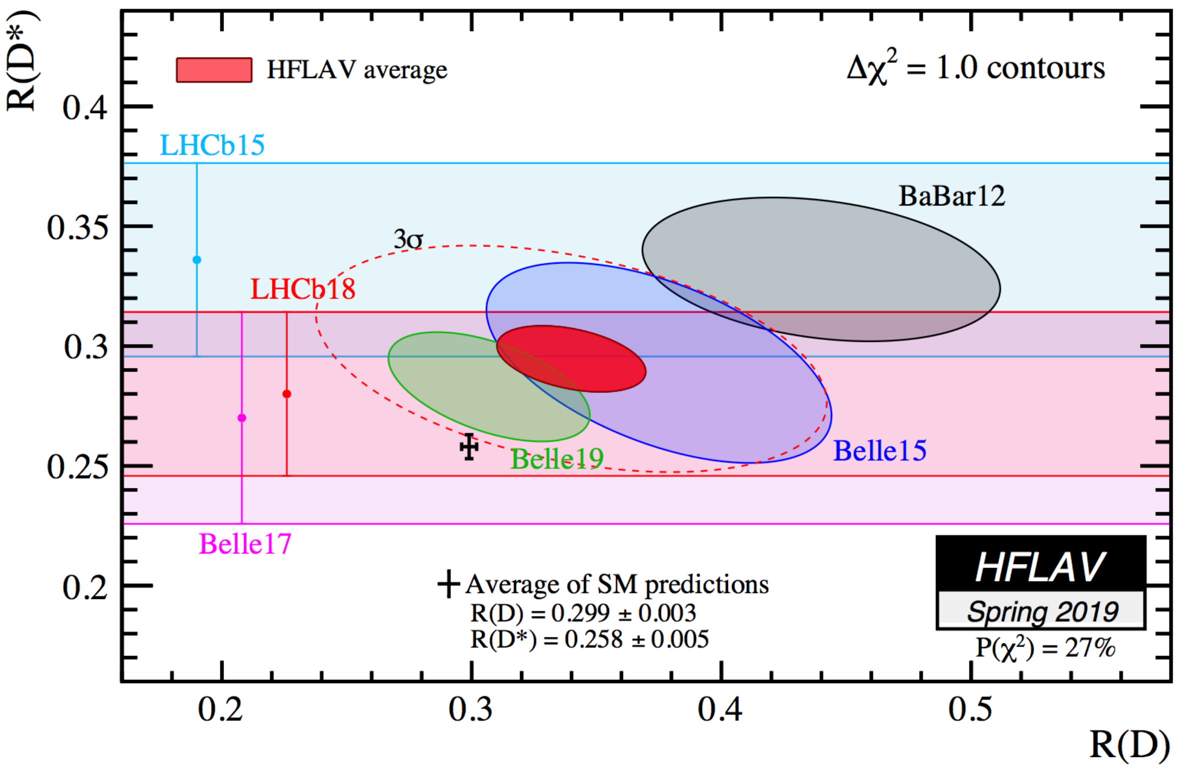
$$|V_{cb}| \times 10^3 = 42.00(65)$$

FLAG Coll., The European Physical Journal C, 80 (2020)

2.6σ discrepancy!!!

- $R_{D^{(*)}}$ anomalies:

$$\mathcal{R}(D) = \frac{\mathcal{B}(B \to D\tau\nu_\tau)}{\mathcal{B}(B \to D\ell\nu_\ell)},$$
$$\mathcal{R}(D^*) = \frac{\mathcal{B}(B \to D^*\tau\nu_\tau)}{\mathcal{B}(B \to D^*\ell\nu_\ell)}$$



3.08σ discrepancy!!!

HFLAV Coll., see https://hflav-eos.web.cern. ch/hflav-eos/semi/spring19/html/RDsDsstar/RDRDs.html

The Form Factors (FFs) entering the semileptonic B → D* decays

Let us focus our attention on the B → D* decays, in which case the $|V_{cb}|$ extraction is particularly challenging!
For massless leptons:

$$\frac{d\Gamma(B \to D^*(\to D\pi)\ell\nu)}{dw\, d\cos\theta_\ell\, d\cos\theta_v\, d\chi} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{4(4\pi)^4} 3 m_B m_{D^*}^2 \sqrt{w^2-1}$$

$$\times B(D^* \to D\pi)\{(1-\cos\theta_\ell)^2 \sin^2\theta_v |H_+|^2$$

$$+ (1+\cos\theta_\ell)^2 \sin^2\theta_v |H_-|^2 + 4\sin^2\theta_\ell \cos^2\theta_v |H_0|^2$$

$$- 2\sin^2\theta_\ell \sin^2\theta_v \cos 2\chi H_+ H_-$$

$$- 4\sin\theta_\ell (1-\cos\theta_\ell) \sin\theta_v \cos\theta_v \cos\chi H_+ H_0$$

$$+ 4\sin\theta_\ell (1+\cos\theta_\ell) \sin\theta_v \cos\theta_v \cos\chi H_- H_0\},$$

$$H_\pm(w) = \boxed{f(w)} \mp m_B m_{D^*} \sqrt{w^2-1}\, \boxed{g(w)}$$

$$H_0(w) = \frac{\boxed{\mathcal{F}_1(w)}}{\sqrt{m_B^2 + m_D^2 - 2 m_B m_D w}}$$

3+1 FFs! The latter is coupled to the lepton mass:

$$H_t = \frac{(m_B + m_{D^*})\sqrt{m_B m_{D^*}(w^2-1)}}{\sqrt{m_B^2 + m_{D^*}^2 - 2 m_B m_{D^*} w}} \boxed{P_1(w)}$$

The dispersive matrix (DM) method for V_{CKM} extraction

Our goal is to describe the FFs using a novel, non-perturbative and model independent approach: starting from the available LQCD computations of the FFs in the high-q^2 regime, we extract the FFs behaviour in the low-q^2 region!

Original idea from L. Lellouch: NPB, 479 (1996)
New developments in Di Carlo et al.: arXiv:2105.02497

Non-perturbative values of the susceptibilities (computing on the lattice the two-point correlation functions of the corresponding quark currents) are needed!!!

See Martinelli et al.: arXiv:2105.07851 for the b → c case

The resulting non-perturbative bands of the FFs will be

- **entirely based on first principles (LQCD, no perturbative evaluation of the susceptibilities, no series expansion)**
- **independent of any assumption on the functional dependence on the momentum transfer**
- **independent of the experimental determinations of the differential decay widths**

See M.Naviglio's talk on July 29th for further details!

→ No HQET, no truncation, no perturbative bounds with respect to the well-known CLN, BCL and BGL parametrizations

LQCD computations and bands of the FFs

$$z = \frac{\sqrt{w+1} - \sqrt{2}}{\sqrt{w+1} + \sqrt{2}}$$

In **arXiv:2105.08674**, two LQCD inputs have been used for our DM method:

- **3 MILC data (squares) for each FF**: preliminary results contained in **arXiv:1912.05886 [hep- lat]**
- **3 JLQCD data (points) for each FF**: preliminary results contained in **arXiv:1912.11770 [hep-lat]**

The non-perturbative susceptibilities for the b → c quark transition have been computed in **arXiv:2105.07851**.

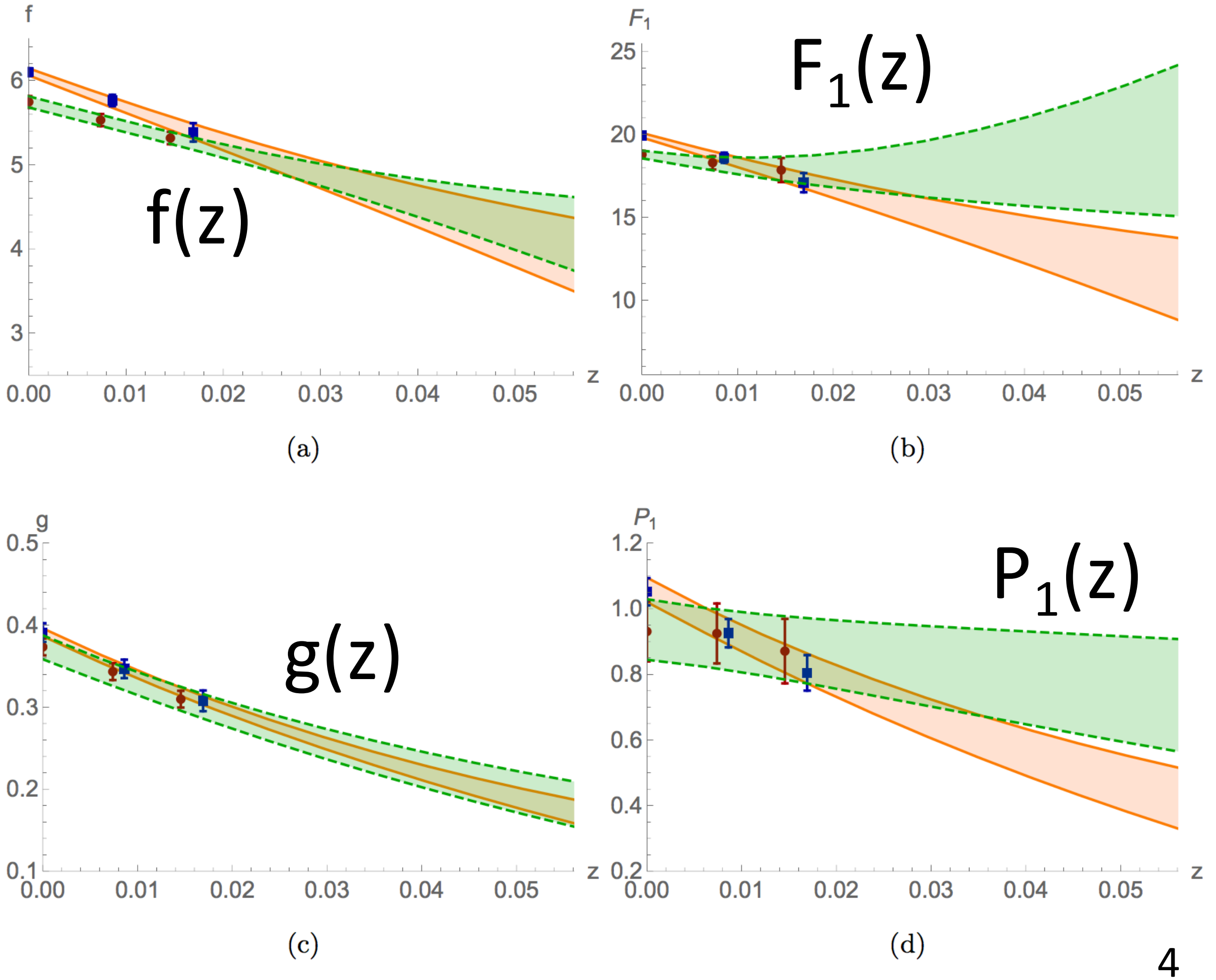
The extrapolated bands of the FFs are:

Orange (solid): unitarity band using MILC

Green (dashed): unitarity band using JLQCD

$$f(z_{max})|_{\rm MILC} = 3.93 \pm 0.44,$$
$$g(z_{max})|_{\rm MILC} = 0.173 \pm 0.014,$$
$$\mathcal{F}_1(z_{max})|_{\rm MILC} = 11.3 \pm 2.5,$$
$$P_1(z_{max})|_{\rm MILC} = 0.42 \pm 0.09,$$

$$f(z_{max})|_{\rm JLQCD} = 4.18 \pm 0.44,$$
$$g(z_{max})|_{\rm JLQCD} = 0.182 \pm 0.027,$$
$$\mathcal{F}_1(z_{max})|_{\rm JLQCD} = 19.6 \pm 4.6,$$
$$P_1(z_{max})|_{\rm JLQCD} = 0.74 \pm 0.17,$$

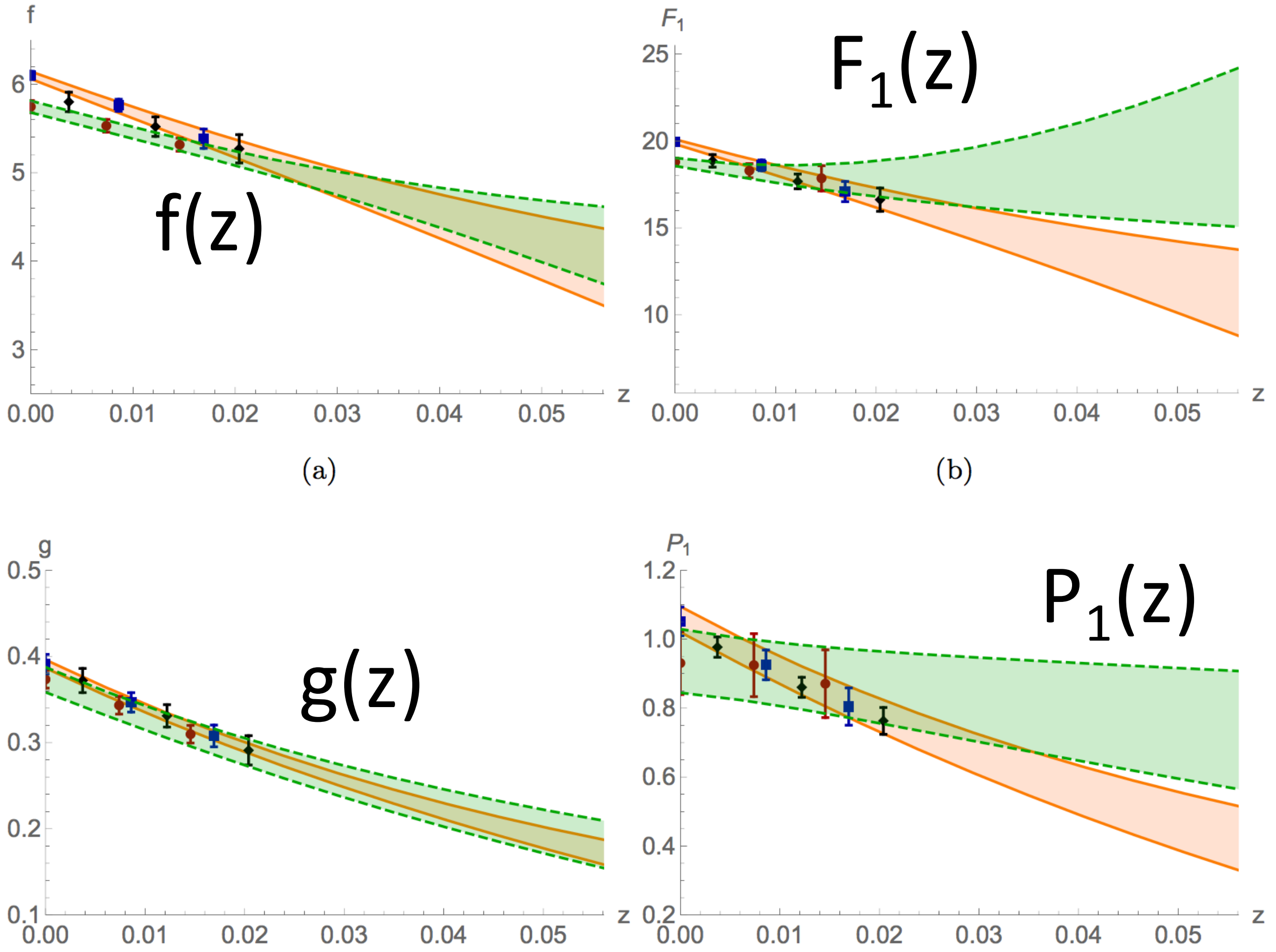


LQCD computations and bands of the FFs

Recently, the final results of the computations of the FFs by the MILC Collaboration have been published in **arXiv:2105.14019**. They are the **black diamonds** in the Figure!!

The whole analysis will be repeated with the final results by the MILC and (once published) the JLQCD Collaboration together with a theoretical improvement of the DM method

Combined final analysis of the B → D and the B → D* decays!!!



(a) (b) (c) (d)

Exclusive Vcb determination through unitarity

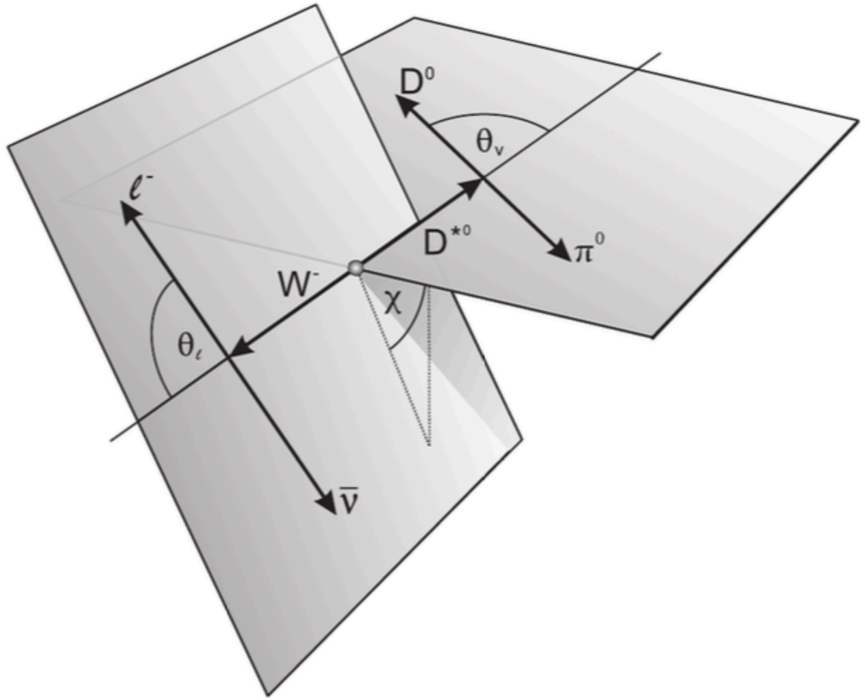
Starting from the FFs bands, we use the experimental data to compute bin-per-bin estimates of Vcb.

NB: the experimental data do NOT enter in the determination of the bands of the FFs!!!

To do it, it is sufficient to compare the two sets of measurements of the differential decay widths

$$d\Gamma/dx, \quad x = w, \cos\theta_l, \cos\theta_v, \chi$$

by the Belle Collaboration **(arXiv:1702.01521, arXiv:1809.03290)** with their *theoretical estimate*, computed through the unitarity bands shown before.

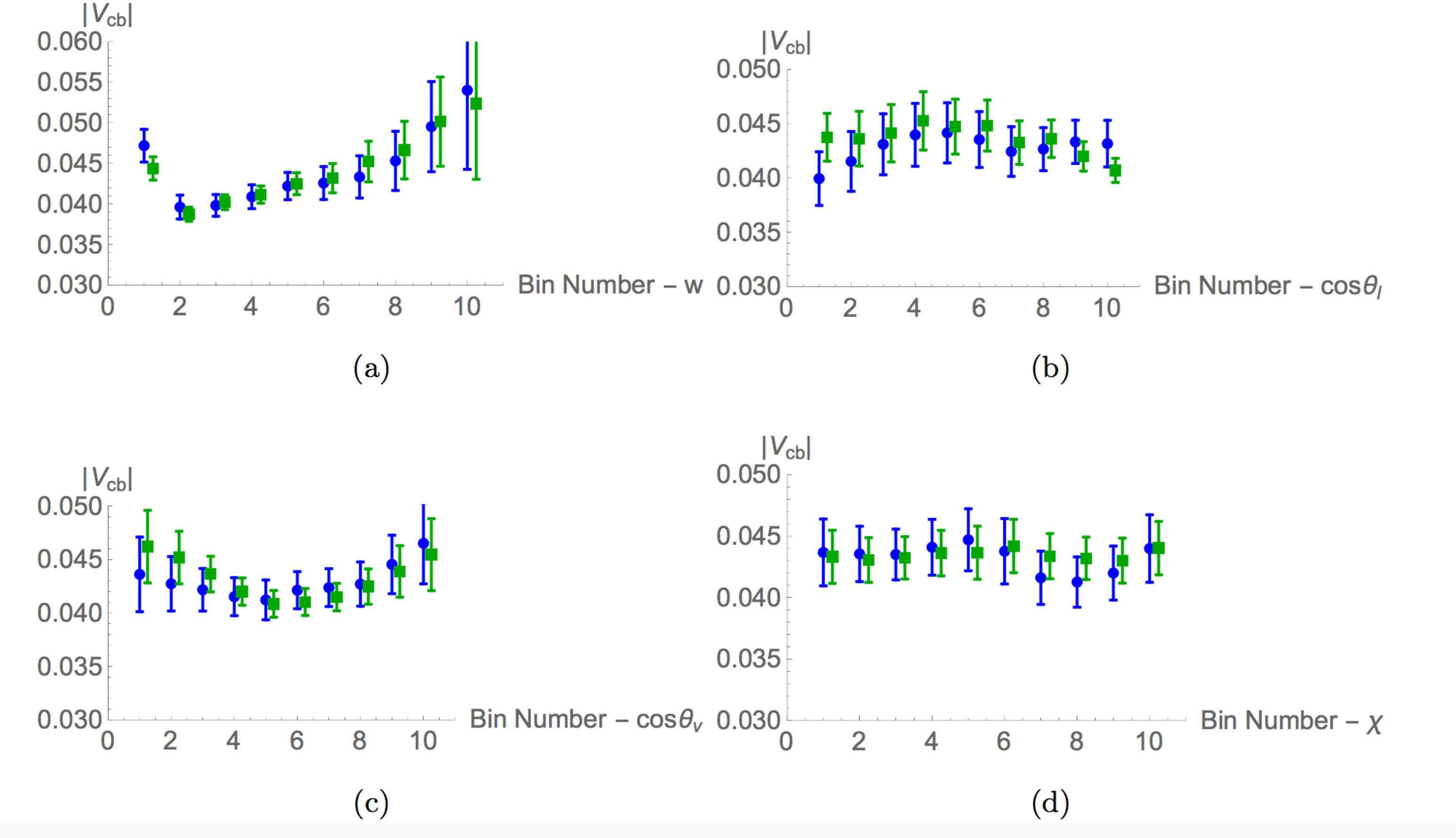


Separate analysis for MILC and JLQCD!!!

$$\frac{d\Gamma(B \to D^*(\to D\pi)\ell\nu)}{dw\, d\cos\theta_\ell\, d\cos\theta_v\, d\chi} = \frac{G_F^2 |V_{cb}|^2 \eta_{EW}^2}{4(4\pi)^4} 3 m_B m_{D^*}^2 \sqrt{w^2-1}$$
$$\times B(D^* \to D\pi)\{(1-\cos\theta_\ell)^2 \sin^2\theta_v |H_+|^2$$
$$+ (1+\cos\theta_\ell)^2 \sin^2\theta_v |H_-|^2 + 4\sin^2\theta_\ell \cos^2\theta_v |H_0|^2$$
$$- 2\sin^2\theta_\ell \sin^2\theta_v \cos 2\chi H_+ H_-$$
$$- 4\sin\theta_\ell(1-\cos\theta_\ell)\sin\theta_v\cos\theta_v\cos\chi H_+ H_0$$
$$+ 4\sin\theta_\ell(1+\cos\theta_\ell)\sin\theta_v\cos\theta_v\cos\chi H_- H_0\},$$

Exclusive Vcb determination through unitarity (MILC case)

The result is



Blue points: arXiv:1702.01521

Green squares: arXiv:1809.03290

To mediate (for each kinematical variable) the various Vcb estimates:

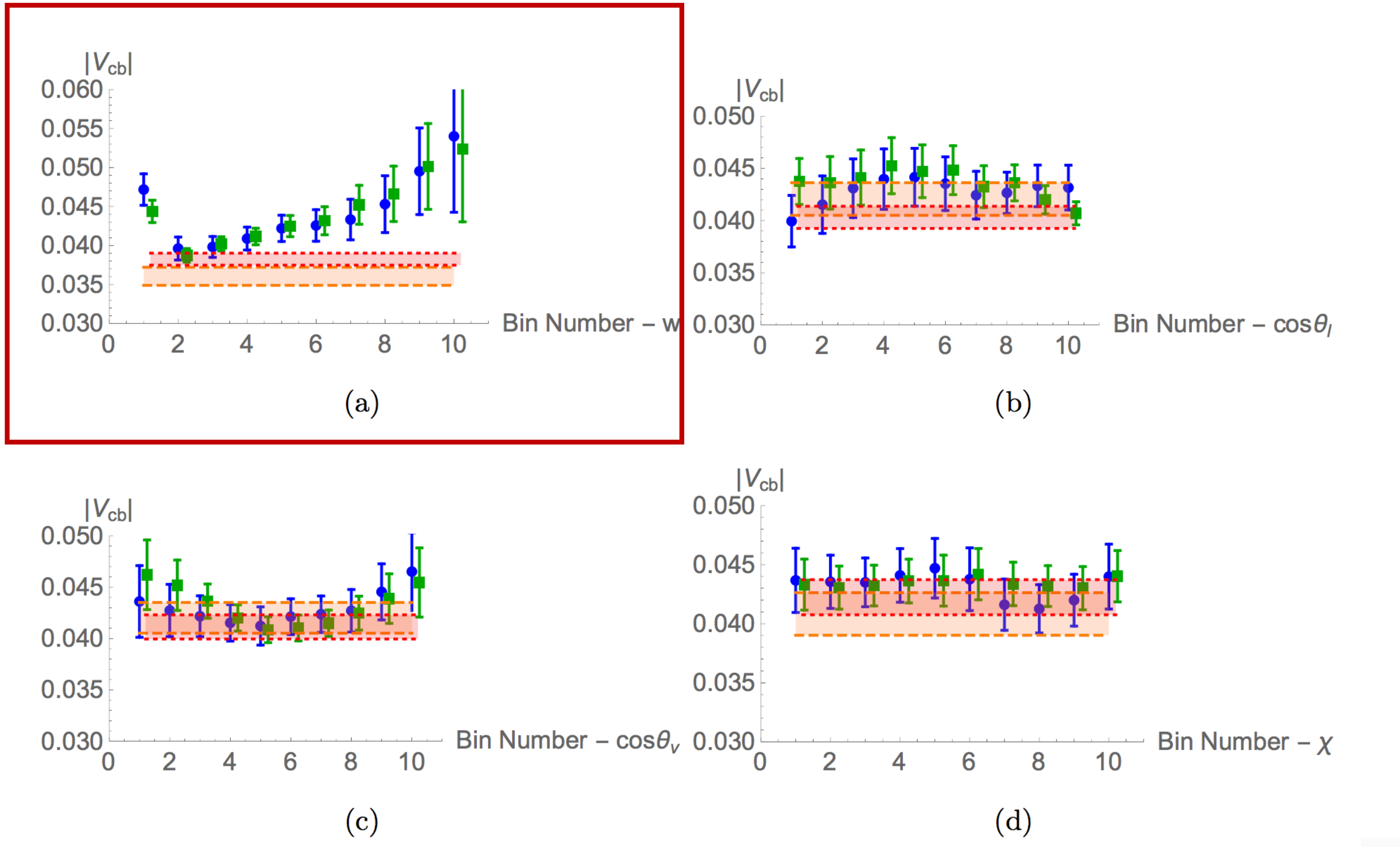
$$|V_{cb}| = \frac{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij} |V_{cb}|_j}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}}, \qquad \sigma^2_{|V_{cb}|} = \frac{1}{\sum_{i,j=1}^{10} (\mathbf{C}^{-1})_{ij}}$$

BUT…

Exclusive Vcb determination through unitarity (MILC case)

… we find

!!!!!



Blue points: arXiv:1702.01521
MEAN: dashed orange band

Green squares: arXiv:1809.03290
MEAN: dotted red band

In the *w* case, there is an evident underestimation of the weighted mean value!
This problem is well-known and has been deeply studied in ***Nucl.Instrum.Meth.A 346 (1994) 306-311***

Alternative strategy?

Exclusive Vcb determination through unitarity (MILC case)

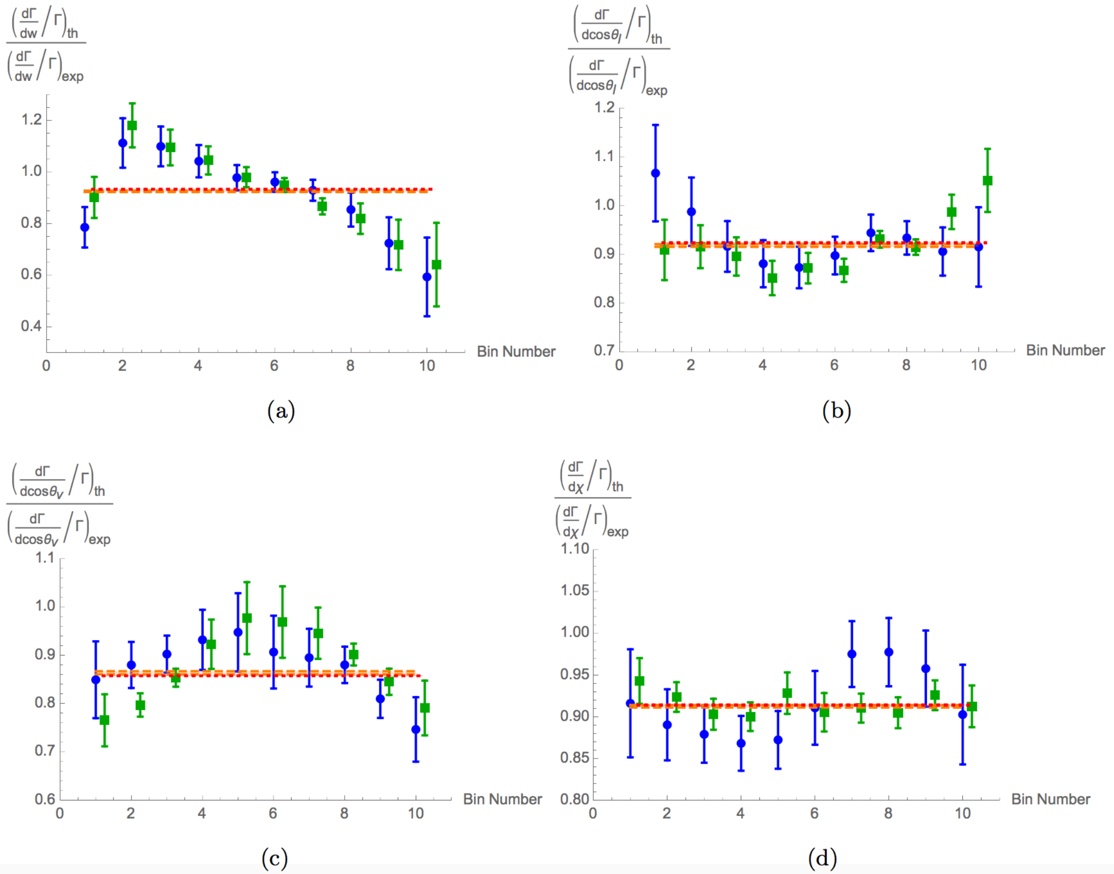
We *suppose* that there is a calibration error in the data. Thus, calling *x* one of the four kinematical variables of interest, we compute the quantity **(dΓ/dx)/Γ** by using the experimental data by Belle.

In this way, we reduce this error since all the points enter in the evaluation of Γ!

By computing the correlations between the various bins of (d**Γ**/dx)/**Γ**, we define a **new experimental covariance matrix** as

Correlation of (d**Γ**/dx)/**Γ** — Experimental errors

$$C_{ij}|_{exp,NEW} = \rho_{ij}|_{ratio} \times \sigma_{i,exp}\sigma_{j,exp}$$



This procedure has two advantages:

1. The new covariance matrix will be free of calibration errors
2. The fact that (for a fixed kinematical variable) the ten bins are ***not* independent** is taken into account

a similar effect has also been discussed in **arXiv:2105.14019**

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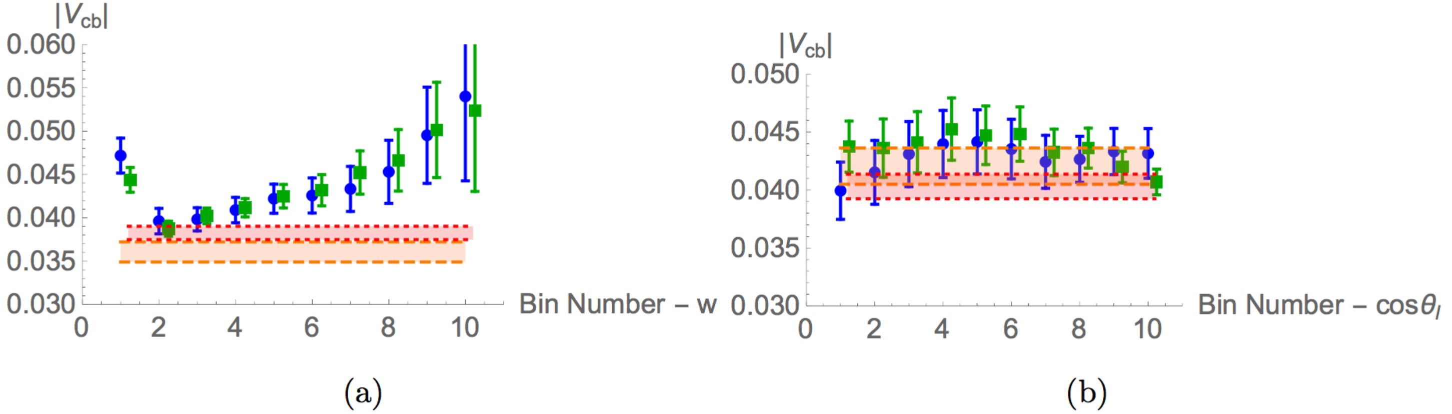
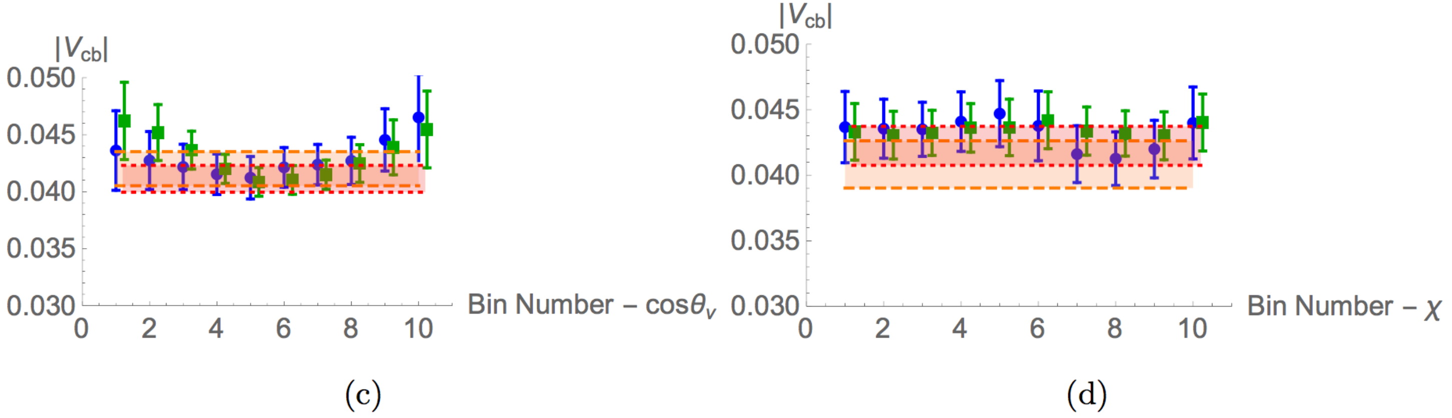
Exclusive Vcb determination through unitarity (MILC case)



BEFORE the modification of the correlations



Blue points:
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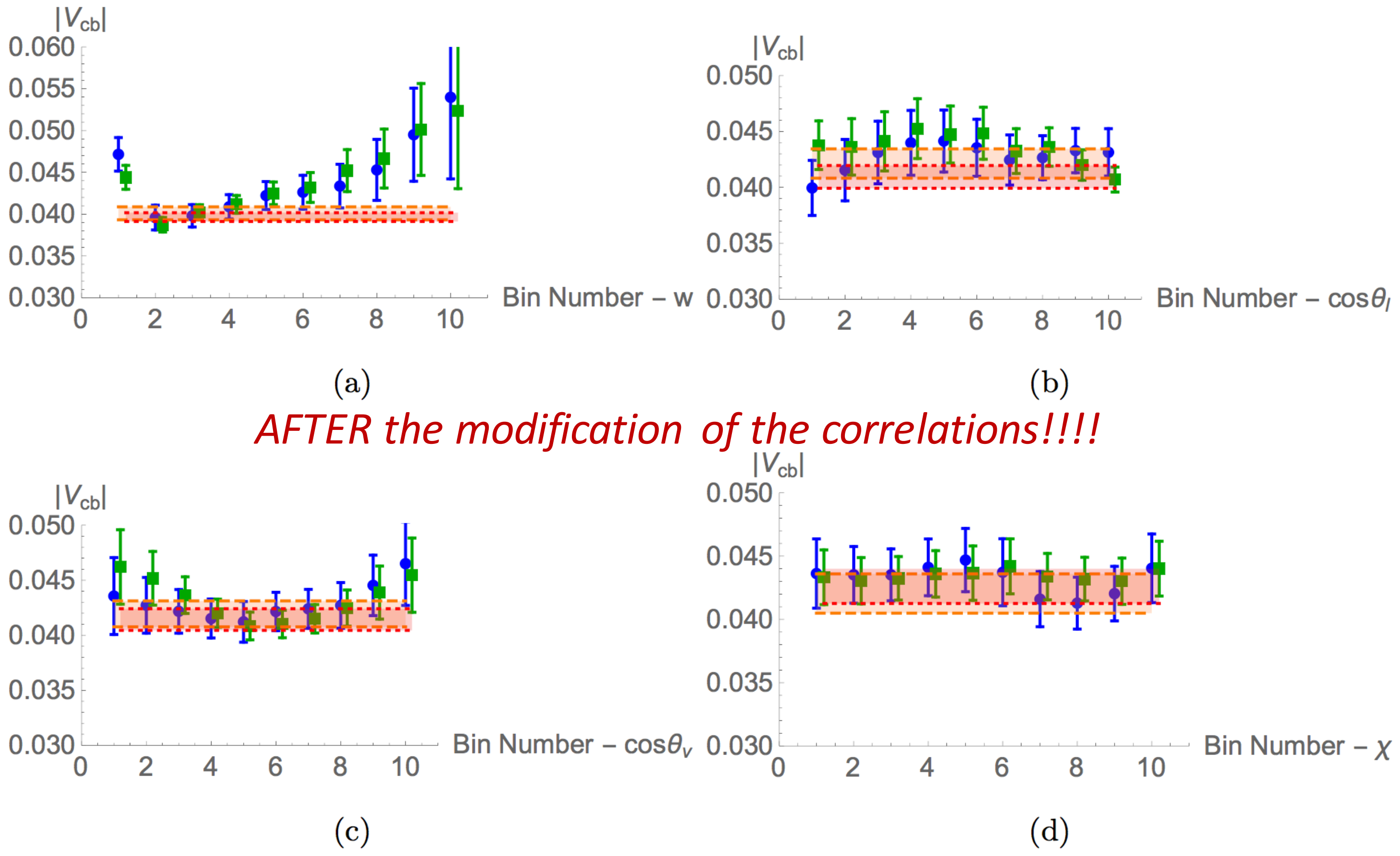
Exclusive Vcb determination through unitarity (MILC case)



Blue points: arXiv:1702.01521

MEAN: dashed orange band

Green squares: arXiv:1809.03290

MEAN: dotted red band

Final MEAN:

$$\mu_x = \frac{1}{N}\sum_{k=1}^{N} x_k,$$

$$\sigma_x^2 = \frac{1}{N}\sum_{k=1}^{N} \sigma_k^2 + \frac{1}{N}\sum_{k=1}^{N}(x_k - \mu_x)^2$$

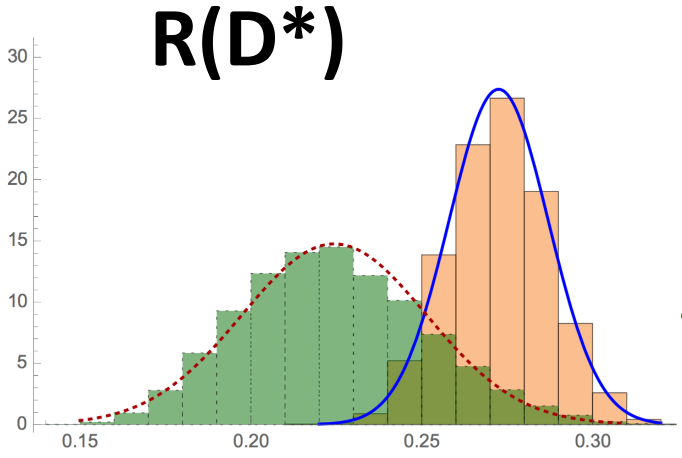
$$|V_{cb}| \times 10^3 = 41.4 \pm 1.5.$$

Compatible with the **inclusive Vcb**:

$$|V_{cb}| \times 10^3 = 42.00(65)$$

R(D*) and the polarization observables

Using the unitarity bands of the FFs, we can compute new *fully-theoretical expectation values* of the anomaly **R(D*)**, the τ-polarization $\mathbf{P_\tau}$ and the D* longitudinal polarization $\mathbf{F_L}$.



MILC: orange (solid) bins
Blue (solid): Gaussian fit

JLQCD: green (dashed) bins
Red (dashed): Gaussian fit

	MILC data [40]	JLQCD data [41]	MILC+JLQCD	Experiments
$R(D^*)$	0.272 ± 0.014	0.224 ± 0.027	0.249 ± 0.021	$0.295 \pm 0.011 \pm 0.008$
$P_\tau(D^*)$	-0.52 ± 0.02	-0.47 ± 0.04	-0.50 ± 0.03	$-0.38 \pm 0.51^{+0.21}_{-0.16}$
$F_L(D^*)$	0.43 ± 0.03	0.50 ± 0.05	0.46 ± 0.04	$0.60 \pm 0.08 \pm 0.04$

Similar to the final value obtained in **arXiv:2105.14019**

Conclusions

We have presented the results of a novel non-perturbative and model-independent analysis of the Form Factors entering the semileptonic $B \to D^*$ decays. By using *only* the information from the lattice in order to contraint the FFs as functions of the momentum transfer, we have extracted new theoretical estimates of Vcb from the experiments:

MILC	**JLQCD**	**combined**
$|V_{cb}| \times 10^3 = 41.4 \pm 1.5.$	$|V_{cb}| \times 10^3 = 40.4 \pm 1.8.$	$|V_{cb}| \times 10^3 = 40.6 \pm 1.6.$

We have also computed new *fully-theoretical expectation values* of **R(D*)**, $\mathbf{P_\tau}$ and $\mathbf{F_L}$.

A similar (and simpler) study has been developed also for the semileptonic $B \to D$ decays, obtaining

$$|V_{cb}| \times 10^3 = 40.7 \pm 1.2.$$

Compatible with the **inclusive Vcb**!

$$|V_{cb}| \times 10^3 = 42.00(65)$$

A new analysis will be published with the new final LQCD computations of the FFs by the MILC and the JLQCD Collaborations, considering the B → D and the B → D* experimental data together!!

BACK-UP SLIDES

Non-perturbative computation of the susceptibilities (*arXiv:2105.07851*)

To compute the susceptibilities on the lattice, we start from the Euclidean correlators:

$$\chi_{0^+}(Q^2) \equiv \frac{\partial}{\partial Q^2}\left[Q^2 \Pi_{0^+}(Q^2)\right] = \int_0^\infty dt\ t^2 j_0(Qt)\ C_{0^+}(t)\ , \quad \xrightarrow{W.\ I.} \quad \frac{1}{4}\int_0^\infty dt'\ t'^4\ \frac{j_1(Qt')}{Qt'}\left[(m_b - m_c)^2 C_S(t') + Q^2 C_{0^+}(t')\right]$$

$$\chi_{1^-}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2 \Pi_{1^-}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\ t^4 \frac{j_1(Qt)}{Qt}\ C_{1^-}(t)$$

$$\chi_{0^-}(Q^2) \equiv \frac{\partial}{\partial Q^2}\left[Q^2 \Pi_{0^-}(Q^2)\right] = \int_0^\infty dt\ t^2 j_0(Qt)\ C_{0^-}(t)\ , \quad \xrightarrow{W.\ I.} \quad \frac{1}{4}\int_0^\infty dt'\ t'^4\ \frac{j_1(Qt')}{Qt'}\left[(m_b + m_c)^2 C_P(t') + Q^2 C_{0^-}(t')\right]$$

$$\chi_{1^+}(Q^2) \equiv -\frac{1}{2}\frac{\partial^2}{\partial^2 Q^2}\left[Q^2 \Pi_{1^+}(Q^2)\right] = \frac{1}{4}\int_0^\infty dt\ t^4 \frac{j_1(Qt)}{Qt}\ C_{1^+}(t)$$

We then perform the *perturbative subtraction* of the contact terms and the lattice artefacts:

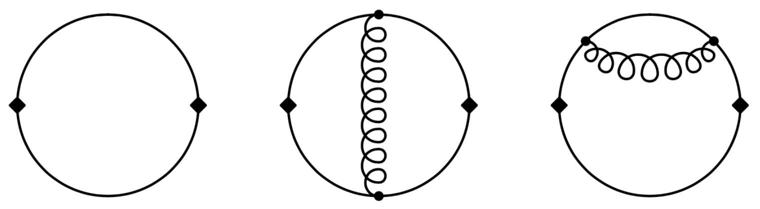
EXAMPLE for the $0^+/1^-$ spin-parity channels!!

$$\Pi_V^{\mu\nu}(Q,a) = \int_{-\pi/a}^{+\pi/a} \frac{d^4k}{(2\pi)^4}\ \mathrm{Tr}\left[\gamma^\mu G_1(k+\frac{Q}{2})\gamma^\nu G_2(k-\frac{Q}{2})\right]$$

$$G_i(p) = \frac{-i\gamma_\mu \mathring{p}_\mu + \mathcal{M}_i(p) - i\mu_{q,i}\gamma_5\tau^3}{\mathring{p}^2 + \mathcal{M}_i^2(p) + \mu_{q,i}^2}$$

Fermion propagator in the Twisted Mass LQCD formalism

Feynman diagrams



Non-perturbative computation of the susceptibilities (*arXiv:2105.07851*)

For the extrapolation to the physical *b*-quark point we have used the ETMC ratio method:

$$R_j(n; a^2, m_{ud}) \equiv \frac{\chi_j[m_h(n); a^2, m_{ud}]}{\chi_j[m_h(n-1); a^2, m_{ud}]} \frac{\rho_j[m_h(n)]}{\rho_j[m_h(n-1)]}$$

to ensure that $\lim_{n\to\infty} R_j(n) = 1$

$$\rho_{0^+}(m_h) = \rho_{0^-}(m_h) = 1 \; ,$$

$$\rho_{1^-}(m_h) = \rho_{1^+}(m_h) = (m_h^{pole})^2$$

All the details are deeply discussed in *arXiv:2105.07851*. In this way, we have obtained **the first lattice QCD determination of susceptibilities of bottom-to-charm-transition current densities:**

	Perturbative	With subtraction	Non-perturbative	With subtraction
$\chi_{V_L}[10^{-3}]$	6.204	$-$	7.52 ± 0.63	7.58 ± 0.59
$\chi_{A_L}[10^{-3}]$	24.1	19.4	25.9 ± 1.8	21.9 ± 1.9
$\chi_{V_T}[10^{-4}\ \mathrm{GeV}^{-2}]$	6.486	5.131	6.76 ± 0.40	5.84 ± 0.44
$\chi_{A_T}[10^{-4}\ \mathrm{GeV}^{-2}]$	3.89	$-$	4.68 ± 0.30	4.69 ± 0.30