

# $V_{cs}$ determination from $D \rightarrow K\ell\nu$

William Parrott  
*2399654p@student.gla.ac.uk*

University of Glasgow



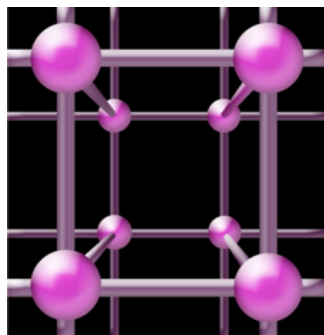
B. Chakraborty, C. Bouchard, C.T.H. Davies, J. Koponen, G.P. Lepage



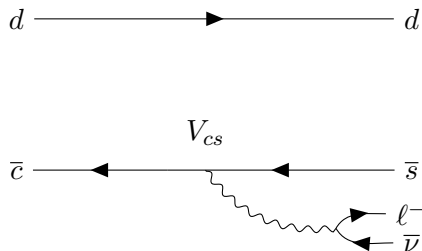
# Overview

I will summarise the results  
in arXiv:2104.09883 (in press)

- ▶ Motivation.
- ▶ Calculation of hadronic form factors.
- ▶  $D \rightarrow K$  form factor results.
- ▶ Sub 1% accurate determination of  $V_{cs}$   
by three semi-independent methods.



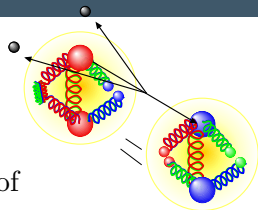
# Motivation



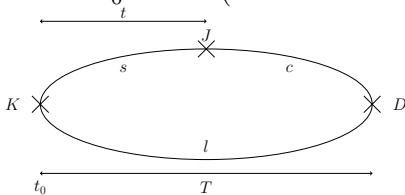
- ▶ Flavour changing weak decays of heavy mesons can test the SM.
- ▶  $D \rightarrow K \ell \nu$  depends on CKM element:  $V_{cs}$ . We calculate form factors for the hadronic part of the decay.
- ▶ CKM unitarity is a good place to look for new physics.
- ▶ We need very precise CKM element determinations to test SM.



# $D \rightarrow K$ form factors



- ▶ Want meson form factors over the full range of  $q^2 = (p_{\text{mother}} - p_{\text{daughter}})^2$  values.
- ▶  $f_0$  and  $f_+$  form factors use matrix elements from 3-point correlation functions with scalar and vector current insertions.
- ▶ Typically use 3 or 4  $T$  values on each ensemble, as well as averaging over 8 or 16  $t_0$  values (4 on finest lattice).



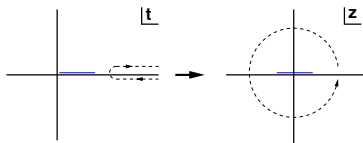
## $D \rightarrow K$ form factors

- ▶ MILC HISQ 2+1+1 ensembles. All valence quarks HISQ.
- ▶ 5 lattice spacings in range 0.15-0.045fm. All with  $m_s/m_l = 5$ , and 3 with physical  $m_l$  too.
- ▶ Charm mass easy to reach on all ensembles and discretisation effects in the HISQ action very small.
- ▶ Cover whole physical  $q^2$  range using twisted b.c.s to give momentum to daughter  $s$  quark.
- ▶ Vector current non-perturbatively renormalised using PCVC.
- ▶ Once we have data on each ensemble, need to extrapolate to the continuum.



## $D \rightarrow K$ form factors

Convert to  $z$  space to perform standard continuum-chiral extrapolation:

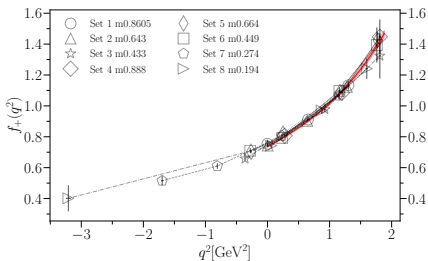
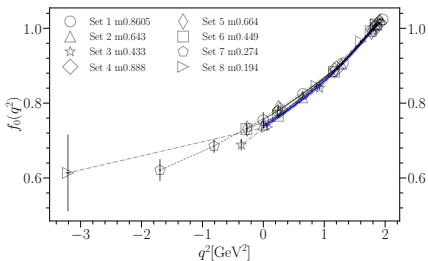


$$f_0(q^2) = \frac{1+L}{1 - \frac{q^2}{M_{D_s^0}^2}} \sum_{n=0}^{N-1} a_n^0 z^n, \quad N=3 \quad (1)$$
$$f_+(q^2) = \frac{1+L}{1 - \frac{q^2}{M_{D_s^*}^2}} \sum_{n=0}^{N-1} a_n^+ \left( z^n - \frac{n}{N} (-1)^{n-N} z^N \right).$$

$$a_n^{0,+} = (1 + \mathcal{N}_n^{0,+}) \sum_{j=0}^2 d_{jn}^{0,+} \left( \frac{am_c^{\text{val}}}{\pi} \right)^{2j}. \quad (2)$$



# $D \rightarrow K$ form factor results



- ▶ We obtain the full  $q^2$  range  $\implies$  can compare bin by bin with exp. partial decay rate data to extract  $V_{cs}$ .



## $D \rightarrow K$ form factor results

Full differential rate includes previously neglected electroweak and EM corrections, as well as terms in  $\epsilon = \frac{m_\ell^2}{q^2}$ ,

$$\frac{d\Gamma^{D \rightarrow K}}{dq^2} = \frac{G_F^2 (\eta_{EW} |V_{cs}|)^2}{24\pi^3} (1 - \epsilon)^2 (1 + \delta_{EM}) \times \left[ |\vec{p}_K|^3 \left(1 + \frac{\epsilon}{2}\right) |f_+(q^2)|^2 + |\vec{p}_K| M_D^2 \left(1 - \frac{M_K^2}{M_D^2}\right)^2 \frac{3\epsilon}{8} |f_0(q^2)|^2 \right], \quad (3)$$

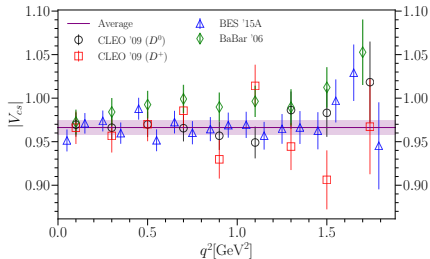
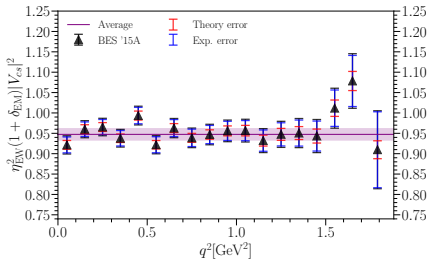
where  $\eta_{EW} = 1.009(2)$ . We take  $\delta_{EM}$  as a 0.5% (1%) error for  $K^0$  ( $K^\pm$ ). Final state interactions dominate  $\delta_{EM}$  - hence  $K^0$  smaller.

We can use this to extract  $V_{cs}$  in three different ways:





Method 1: Using  $q^2$  binned differential decay rates,

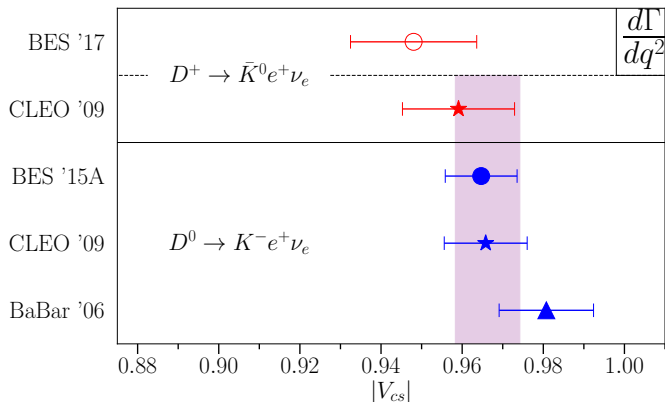


- ▶ Experimental error dominates each bin, theory dominates final result.

$$\Delta_i \Gamma = \int_{q_i^2}^{q_{i+1}^2} \frac{d\Gamma}{dq^2} dq^2$$



# $V_{cs}$ results

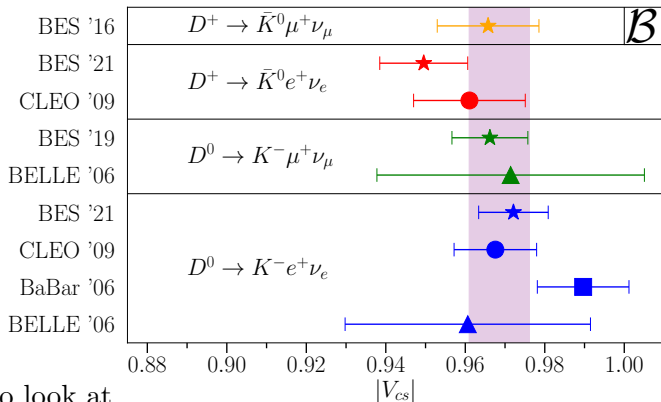


- ▶ Preferred method using whole  $q^2$  range and multiple experiments.
- ▶ Theory still (just) dominates error, but can also be improved with future binned experimental data (with correlations).



# $V_{cs}$ results

Method 2: Using total branching fraction for all 4 decay modes,



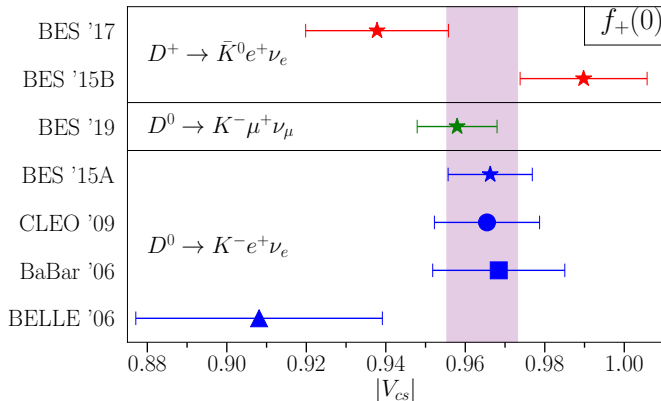
Can also look at

$$R_{\mu/e} = \mathcal{B}_\mu / \mathcal{B}_e$$

$$\mathcal{B} = \tau_D \int_0^{q_{\max}^2} \frac{d\Gamma}{dq^2} dq^2$$



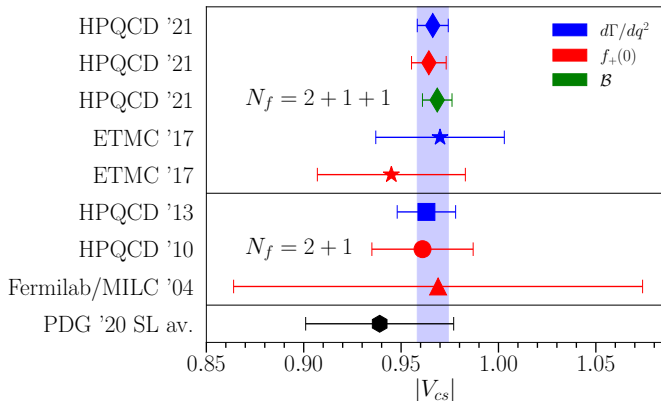
## Method 3: Using $|V_{cs}|f_+(0)$



$$f_+(0)\eta_{EW}\sqrt{(1+\delta_{EM})}|V_{cs}| = 0.7180(33) \quad (\text{HFLAV, arXiv:1612.07233})$$



# $V_{cs}$ results



HPQCD '21 results:

$$|V_{cs}|^\Gamma = 0.9663(53)_{\text{latt}}(39)_{\text{exp}}(19)_{\eta_{\text{EW}}}(40)_{\delta_{\text{EM}}}$$

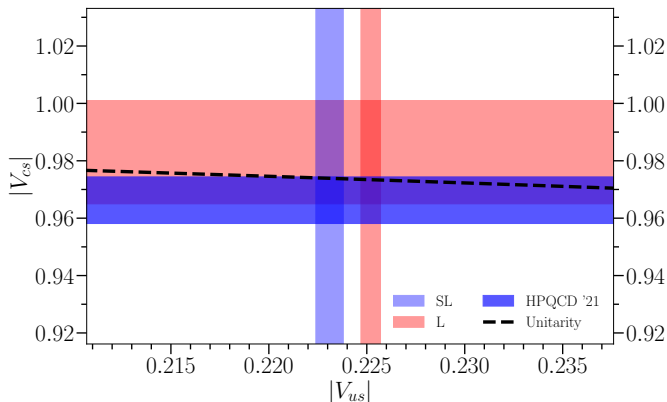
$$|V_{cs}|^{\mathcal{B}} = 0.9686(54)_{\text{latt}}(39)_{\text{exp}}(19)_{\eta_{\text{EW}}}(30)_{\delta_{\text{EM}}}$$

$$|V_{cs}|^{f_+(0)} = 0.9643(57)_{\text{latt}}(44)_{\text{exp}}(19)_{\eta_{\text{EW}}}(48)_{\delta_{\text{EM}}}$$

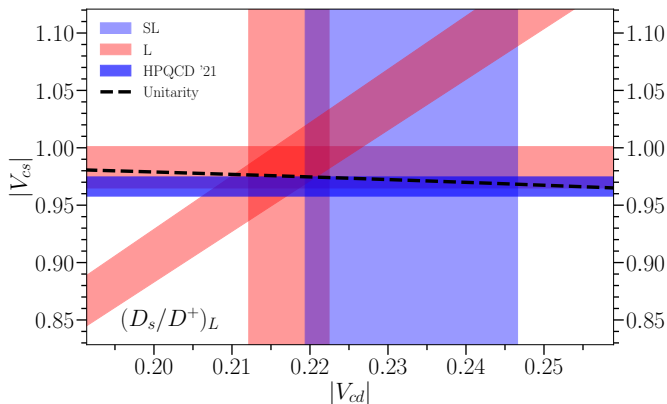


# $V_{cs}$ results

$|V_{cs}| = 0.9663(80)$ , big improvement on current PDG sl value of  $0.939(38)$



# $V_{cs}$ results



Additional constraint from leptonic  $D_s$  and  $D^+$  decays, combined with lattice decay constants.



# Conclusions

$$|V_{cd}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 0.9826(22)_{V_{cd}}(155)_{V_{cs}}(1)_{V_{cb}}$$
$$|V_{us}|^2 + |V_{cs}|^2 + |V_{ts}|^2 = 0.9859(2)_{V_{us}}(155)_{V_{cs}}(1)_{V_{ts}}$$

- ▶  $|V_{cs}| = 0.9663(80)$  determination from  $D \rightarrow K\ell\nu$  using bin by bin comparisons with experimental differential decay rate.
- ▶ First determination showing  $V_{cs}$  to be significantly lower than 1 and first sub 1% uncertainty.
- ▶ Agrees well with determinations from 2 other methods.
- ▶ Theory and experimental error similar. EM error also a large contribution.
- ▶ New generation of lattice calculation, more stats, or EM work needed for theory improvement.
- ▶ Can also look for BSM physics in  $R_{\mu/e}$ .

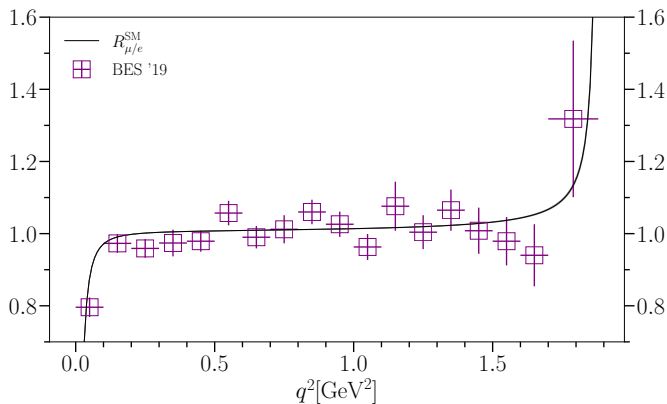
Thanks for listening. Any questions?



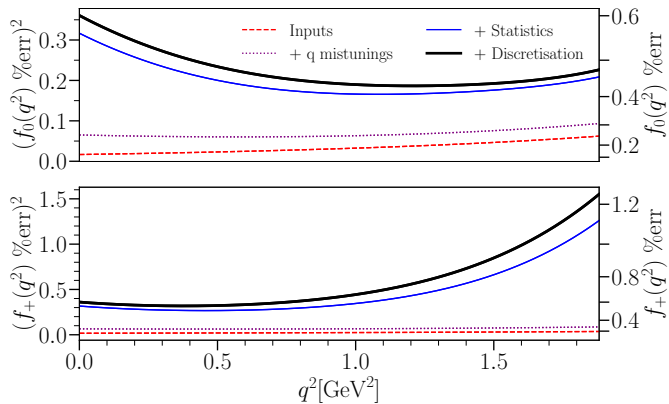


# Extra Slides

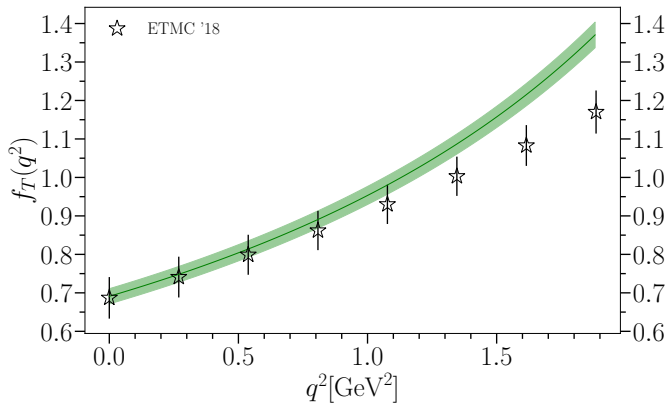
$$R_{\mu/e} = \frac{\mathcal{B}_\mu}{\mathcal{B}_e}.$$



# Extra Slides



# Extra Slides



$$\begin{aligned}
 Z_V \langle K | V^0 | \hat{D} \rangle = & \\
 f_+(q^2) \left( p_D^0 + p_K^0 - \frac{M_D^2 - M_K^2}{q^2} q^0 \right) & \quad (6) \\
 + f_0(q^2) \frac{M_D^2 - M_K^2}{q^2} q^0, &
 \end{aligned}$$

$$\langle K | S | D \rangle = \frac{M_D^2 - M_K^2}{m_c - m_s} f_0(q^2), \quad (7)$$

$$Z_T \langle \hat{K} | T^{i0} | \hat{D} \rangle = \frac{2iM_D p_K^i}{M_D + M_K} f_T(q^2), \quad (8)$$



$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \quad (9)$$

$$\frac{m_l}{m_s} \approx \frac{M_\pi^2}{M_{\eta_s}^2} \quad (10)$$

