Results for α_s from the decoupling strategy

R. Höllwieser

M. Dalla Brida, F. Knechtli, T. Korzec

A. Nada, A. Ramos, S. Sint, R. Sommer

ALPHA Collaboration

ALPHA 1912.06001

Lattice Symposium, ZOOM/GATHER@MIT, 7/30/2021

Decoupling Strategy

- $N_f = 0$ QCD is an eff. theory for $N_f = 3$ QCD with 3 heavy quarks at mass M, pert. dec. rel. at scale M
- matching: $\Lambda^{(0)} = \Lambda^{(3)} P_{3,0}(M/\Lambda^{(3)})$ *[Bruno et al. (2015)]*
- then: low energy quantities agree:
  $\bar{g}^{(3)}(\mu_{\rm dec}, M) = \bar{g}^{(0)}(\mu_{\rm dec}) + \mathcal{O}(\frac{\Lambda^2}{M^2}, \frac{\mu_{\rm dec}^2}{M^2})$
- known to high precision non-perturbatively:
  $\bar{g}^{(0)}(\mu_{\rm dec}) \overset{\beta(\bar{g})}{\longleftrightarrow} \Lambda^{(0)}/\mu_{\rm dec}$ *[M. Dalla Brida, A. Ramos (2019)]*
- challenge: $\mu_{\rm dec} << M << a^{-1} \Rightarrow$ finite volume $\bar{g}$
  $\mu_{\rm dec} = L^{-1} \Rightarrow$ larger M, smaller a *[Luscher et al. (1991)]*

$\Rightarrow$ compute $\bar{g}^{(3)}(\mu_{\rm dec}, M)$ at known $\mu_{\rm dec} << M$ in cont.

$\Rightarrow$ set $\bar{g}^{(0)}(\mu_{\rm dec}) = \bar{g}^{(3)}(\mu_{\rm dec}, M)$

$\Rightarrow$ obtain: $\Lambda^{(3)} = \mu_{\rm dec} \frac{\Lambda^{(0)}}{\mu_{\rm dec}} P_{3,0}^{-1}(M/\Lambda^{(3)})$

Renormalization

- known LCP: $(L/a, \beta, m_0)$ *[ALPHA 2017]* *[HQET 2018]*
  $[\bar{g}^{(3)}(\mu_{\rm dec}, M = 0)]^2 = 3.95 \Leftrightarrow \mu_{\rm dec} = 789(15)$MeV
- need: $(L/a, \beta', m_0')$ for $[\bar{g}^{(3)}(\mu, M)]^2$ so that $\mu = \mu_{\rm dec}$,
  $z = LM =$ const. (here: $z = 2, 4, 6, 8, 12$)
- renormalization is non-trivial with Wilson quarks

  $$M = \frac{M}{\bar{m}(\mu)} Z_m(\mu, a)[m_0 - m_{\rm crit.}] \approx \frac{M}{\bar{m}(\mu)} \frac{Z_A(a)}{Z_P(\mu,a)} m_{\rm PCAC},$$

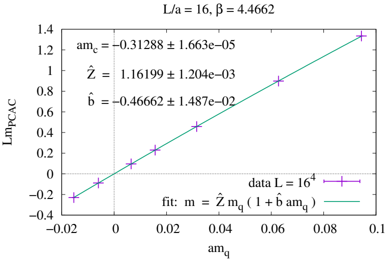
  complicated by $\mathcal{O}(a)$-impr. *[Luscher et al. (1996)]*
  $M/\bar{m}(\mu)$ obtained from *[Campos et al. (2018)]*

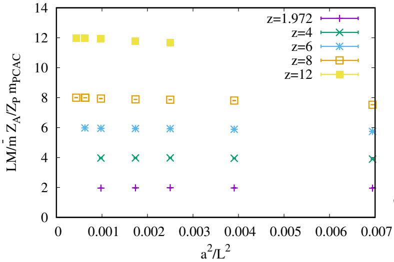
$\Rightarrow$ dedicated renormalization runs to determine

$$(L/a, \beta, m_0)|_{\mu_{\rm dec}, M=0} \overset{Z_m, b_m, b_g}{\underset{m_{\rm crit.}}{\longrightarrow}} (L/a, \beta', m_0')|_{\mu_{\rm dec}, LM={\rm const.}}$$

Massless & Massive Simulations

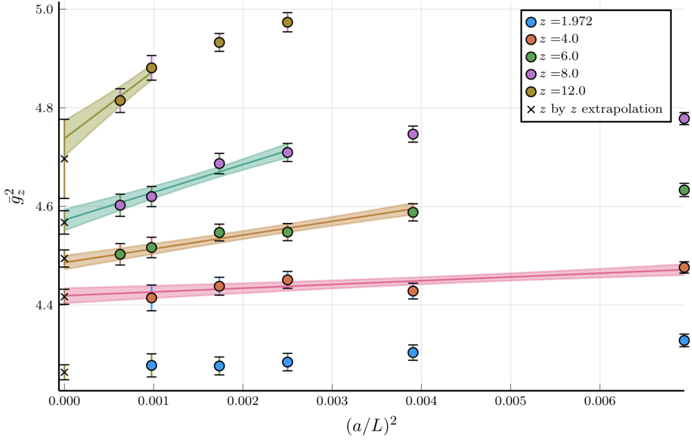
- $N_f = 3$ QCD with $\mathcal{O}(a)$-imp. Wilson quarks and tree-level Symanzik $\mathcal{O}(a^2)$-imp. gauge action
- finite volume $T \times L^3$ with Dirichlet / per. (SF) bcs.
- GF couplings $\bar{g}^2(\mu) = \mathcal{N}^{-1} t^2 \langle E_{\mathrm{mag}}(t) \rangle |_{\mu^{-1} = \sqrt{8t} = cL}$
- massless renormalization runs for $L/a = 12 \ldots 40$
- massive simulations for $M = 1.6 \ldots 9.5$ GeV, $T = 2L$





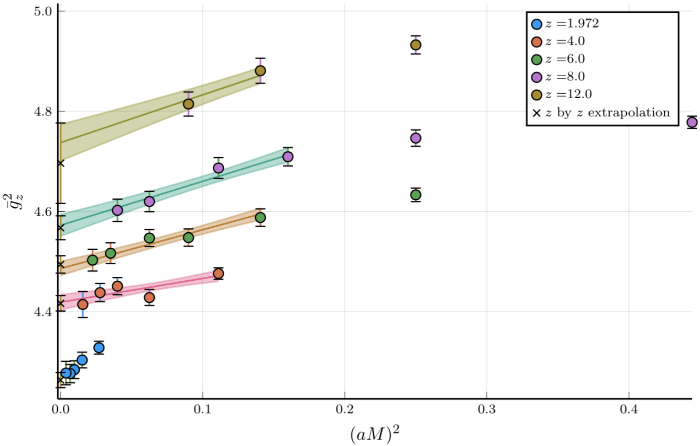
Continuum extrapolations, $z \geq 4, aM \leq 0.4$

- $\bar{g}^2(z_i) = c_i + p_1 \left[\alpha_s(a^{-1})\right]^{\hat{\Gamma}_1} (a/L)^2 + p_2 \left[\alpha_s(a^{-1})\right]^{\hat{\Gamma}_2} (aM)^2$
- Symanzik exp., no (a^2M/L) ➜ *S. Sint, this session*
- $\log(a)$ corr. to a^2 scaling ➜ *N. Husung, this session*



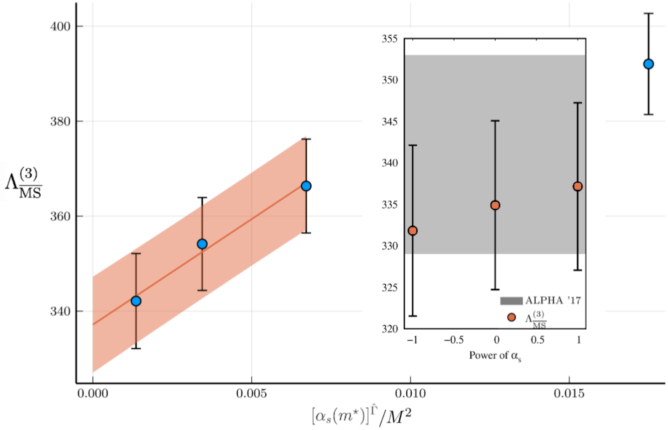
Continuum extrapolations, $z \geq 4, aM \leq 0.4$

- $\bar{g}^2(z_i) = c_i + p_1 \left[\alpha_s(a^{-1})\right]^{\hat{\Gamma}_1} (a/L)^2 + p_2 \left[\alpha_s(a^{-1})\right]^{\hat{\Gamma}_2} (aM)^2$
- $\hat{\Gamma}_1, \hat{\Gamma}_2 \in [-1, \ldots, 1]$ are consistent with $\hat{\Gamma}_1 = \hat{\Gamma}_2 = 0$



$M \to \infty$ extrapolations, $z \geq 6$

- $\Lambda^{(3)}_{\overline{\rm MS}}(z) = A + \frac{B}{z^2}\,[\alpha_s(m^\star)]^{\hat{\Gamma}}$ with $\bar{m}_{\overline{\rm MS}}(m^\star) = m^\star$
- boundary $\alpha(m_\star)/z$ estimated negligible by explicit computation in pure gauge theory + pert. matching



Preliminary Results & Errors

- min. χ^2 gives $\Lambda^{(3)}_{\overline{\rm MS}} = 332(10),\ 331(11)$ for $c = 0.3,\ 0.4$
- $\sim 1\sigma$ smaller but in agreement with the old result $\Lambda^{(3)}_{\overline{\rm MS}} = 341(12)$MeV *[ALPHA 1706.03821]*
- largely independent computation, only the scale $1/\sqrt{8t_0^\star} = \mu^\star_{\rm ref} \ldots \mu_{\rm dec} \Rightarrow \sim 40\%$ error^2 in common
- four-loop prediction for $\Lambda^{(5)}_{\overline{\rm MS}}/\Lambda^{(3)}_{\overline{\rm MS}}$ yields

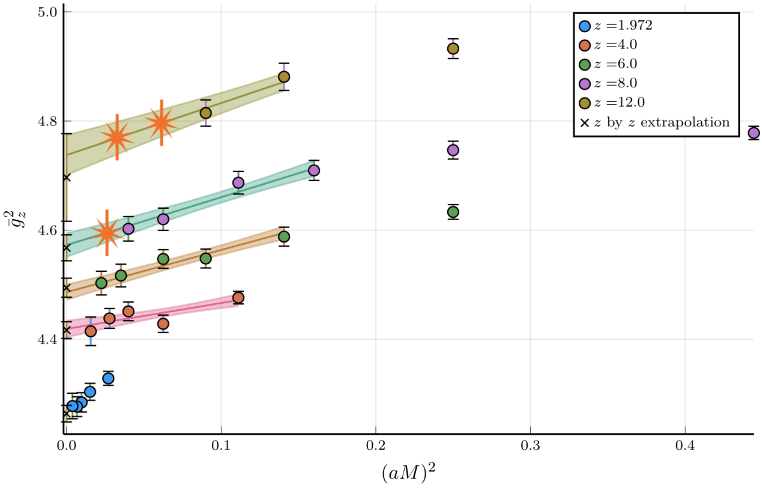
$$\alpha_s(M_Z) = 0.11784(75)(24)$$

- $\Lambda^{(0)}/\mu_{\rm dec}$ error $\approx 1.5\%$ needs to be improved
- $\mu_{\rm dec}$ error $\approx 2\%$ ➜ *B. Strassberger, this session*

$\Rightarrow \Lambda^{(3)}_{\overline{\rm MS}}$ error $1 - 2\% \Rightarrow \alpha_s(M_Z)$ error 0.4% feasible, but requires significant work, *e.g.* $\hat\Gamma$ in $[\alpha(m^\star)]^{\hat\Gamma}$

Ongoing simulations

$L/a = 48, 64$ at $z = 8, 12$ to improve cont. extrapolations



Acknowledgements

THANK YOU

M. Dalla Brida, F. Knechtli, T. Korzec

A. Nada, A. Ramos, S. Sint, R. Sommer

ALPHA Collaboration