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Non-Perturbative Bounds for Semileptonic Decays in Lattice QCD

Based on arXiv:2105.02497v1 [hep-lat]

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Outline

We revisited a method introduced by L. Lellouch in an existing work **[L. Lellouch, Nucl. Phys. B 479 (1996)]** bringing novelties and new results.

The talk is organized as follows:

1) How the method works;

2) The main novelties of our paper;

3) Applications and Main Results;

4) Outlook to future goals.

How the method works: the starting point

The imaginary part of the longitudinal and transverse polarization functions are related to their derivatives with respect q^2 by

$$\chi_{0^+}(q^2) = \frac{\partial}{\partial q^2}\left[q^2\Pi_{0^+}(q^2)\right] = \frac{1}{\pi}\int_0^\infty dz \frac{z Im\Pi_{0^+}}{(z-q^2)^2}, \qquad \chi_{1^-}(q^2) = \frac{1}{2}\left(\frac{\partial}{\partial q^2}\right)^2\left[q^2\Pi_{1^-}(q^2)\right] = \frac{1}{\pi}\int_0^\infty dz \frac{z Im\Pi_{1^-}}{(z-q^2)^3},$$

where for a generic current J

$$Im\Pi_{0^+,1^-} = \frac{1}{2}\sum_n \int d\mu(n)(2\pi)^4\delta^{(4)}(q-p_n)\,|\langle 0|J|n\rangle|\,.$$

We can restrict our attention to a subset of hadronic states and thus produce, using analyticity, a strict inequality

$$\boxed{\frac{1}{2\pi i}\int_{|z|=1}\frac{dz}{z}\,|\phi(z,q^2)f(z)|^2 \leq \chi(q^2),}$$

where f is a generic form factor and ϕ an associated kinematical function which may contain subtraction of resonances.

How the method works: inner product formalism and the matrix

We can introduce the inner product

$$\langle g \,|\, h \rangle = \frac{1}{2\pi i} \int_{|z|=1} \frac{dz}{z} \bar{g}(z) h(z) \,.$$

Using this formalism, the inequality can be simply written as

$$0 \leq \langle \phi f \,|\, \phi f \rangle \leq \chi(q^2) \,.$$

Introducing the function $g_t(z) = \dfrac{1}{1 - \bar{z}(t) z}$ and using the definition of the inner product we can define

$$\langle g_t \,|\, \phi f \rangle = \phi(z(t), q^2) f(z(t)), \qquad \langle g_{t_m} \,|\, g_{t_n} \rangle = \frac{1}{1 - z(t_l)\bar{z}(t_m)} \,.$$

$z(t)$ is such that

$$\frac{1+z}{1-z} = \sqrt{\frac{t_+ - t}{t_+ - t_-}}$$

where for $D \to K$

$$t_\pm = (m_D \pm m_K)^2$$

We can use these three quantities to build a matrix!!

$$\mathbf{M} = \begin{pmatrix} \langle \phi f | \phi f \rangle & \langle \phi f | g_t \rangle & \langle \phi f | g_{t_1} \rangle & \cdots & \langle \phi f | g_{t_n} \rangle \\ \langle g_t | \phi f \rangle & \langle g_t | g_t \rangle & \langle g_t | g_{t_1} \rangle & \cdots & \langle g_t | g_{t_n} \rangle \\ \langle g_{t_1} | \phi f \rangle & \langle g_{t_1} | g_t \rangle & \langle g_{t_1} | g_{t_1} \rangle & \cdots & \langle g_{t_1} | g_{t_n} \rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n} | \phi f \rangle & \langle g_{t_n} | g_t \rangle & \langle g_{t_n} | g_{t_1} \rangle & \cdots & \langle g_{t_n} | g_{t_n} \rangle \end{pmatrix} .$$

The values $t_1, t_2, \ldots, t_n$ correspond to the squared 4-momenta at which the FFs have been computed while the first element is the quantity directly related to the susceptibility $\chi(q^2)$. The point t is the unknown point where we want to extract the value of the FF.

How the method works: the bounds

The positivity of the inner product guarantees that

$$\det M \geq 0.$$

This condition leads to a constraints on the form factor f computed in the generic unknown point t

$$f_{lo}(t) \leq f(t) \leq f_{up}(t),$$

where

α and $\Delta_1(t)$ are determinants of minors of M depending only on kinematical factors.

$$f_{lo(up)}(t) = f(t) \mp \frac{1}{\alpha\phi}\sqrt{\Delta_1(t)\Delta_2^f}$$

Δ_2^f depends on the FF and on χ but not on t. It is a crucial quantity because, depending on the susceptibility, it contains information on the unitarity!

The crucial point is that $\Delta_1(t) > 0 \; \forall t \longrightarrow \Delta_2^f$ must be positive!

If $\Delta_2^f > 0$ the unitarity is always satisfied $\forall t$!

$f_{lo(up)}$ are defined if unitarity is satisfied. Then, the bounds that we can obtain imposing $\Delta_2^f > 0$ in the Dispersive Matrix method always satisfy unitarity!!!

The novelties of our work

The DM method allows to reconstruct the interval of the possible values of the form factor in a <u>generic</u> point t in *<u>a total model independent way and without any assumption or truncation</u>* (differently from CLN, BGL…) starting from (also few) known points and the susceptibilities!

→ **No assumptions on the q^2 functional dependence of the FF!!**

With respect to the proposal by L. Lellouch and other previous studies, we introduced two main novelties:

1) We determined **non perturbatively** all the relevant two-point current correlation functions on the lattice which are fundamental to implement the dispersive bounds (i.e. the susceptibilities χ that appear in the matrix). We also proposed to reduce **discretisation errors** of the two-point correlation functions by using a combination of non-perturbative and perturbative subtractions which were found very effective in the past;
2) A quite simpler treatment of the lattice **uncertainties** with respect to the method proposed by Lellouch.

Long story… See the paper arXiv:2105.02497v1 [hep-lat] in Sections IV and VII.

Using the matrix method we can compute the lower/upper bounds of $f_{0(+)}(t)$ once we have chosen our set of $2(n+1)$ input data $\{\chi_{0^+(1^-)}, f_{0(+)}(t_1), \ldots, f_{0(+)}(t_n)\}$.

How to propagate the uncertainties related to these quantities?

The novelties of our work: statistical uncertainties and KC

We build a multivariate Gaussian distribution of N_{boot} bootstrap events both for the form factors extracted from the three-point functions and for the susceptibilities (***properly correlated*** if we have access to the data of the simulations) in our numerical simulation and covariance matrix $\Sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$.

To take into account in our analysis the Kinematical Constraint for each of the N_{boot} events we define

$$f^*_{lo}(0) = min[f_{+,lo}(0), f_{0,lo}(0)],$$

$$f^*_{up}(0) = max[f_{+,up}(0), f_{0,up}(0)] .$$

$$\rightarrow \quad f^*_{lo}(0) \le f(0) \le f^*_{up}(0) .$$

If we consider $f(0)$ to be uniformly distributed in this range, we can generate N_0 values, obtaining a sample of $\bar{N}_{boot} = N_{boot} \times N_0$, that we can add to the input data set as a new point at $t_{n+1} = 0$.

$$\mathbf{M}_C = \begin{pmatrix} \langle \phi f|\phi f\rangle & \langle \phi f|g_t\rangle & \langle \phi f|g_{t_1}\rangle & \cdots & \langle \phi f|g_{t_n}\rangle & \langle \phi f|g_{t_{n+1}}\rangle \\ \langle g_t|\phi f\rangle & \langle g_t|g_t\rangle & \langle g_t|g_{t_1}\rangle & \cdots & \langle g_t|g_{t_n}\rangle & \langle g_t|g_{t_{n+1}}\rangle \\ \langle g_{t_1}|\phi f\rangle & \langle g_{t_1}|g_t\rangle & \langle g_{t_1}|g_{t_1}\rangle & \cdots & \langle g_{t_1}|g_{t_n}\rangle & \langle g_{t_1}|g_{t_{n+1}}\rangle \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \langle g_{t_n}|\phi f\rangle & \langle g_{t_n}|g_t\rangle & \langle g_{t_n}|g_{t_1}\rangle & \cdots & \langle g_{t_n}|g_{t_n}\rangle & \langle g_{t_n}|g_{t_{n+1}}\rangle \\ \langle g_{t_{n+1}}|\phi f\rangle & \langle g_{t_{n+1}}|g_t\rangle & \langle g_{t_{n+1}}|g_{t_1}\rangle & \cdots & \langle g_{t_{n+1}}|g_{t_n}\rangle & \langle g_{t_{n+1}}|g_{t_{n+1}}\rangle \end{pmatrix} .$$

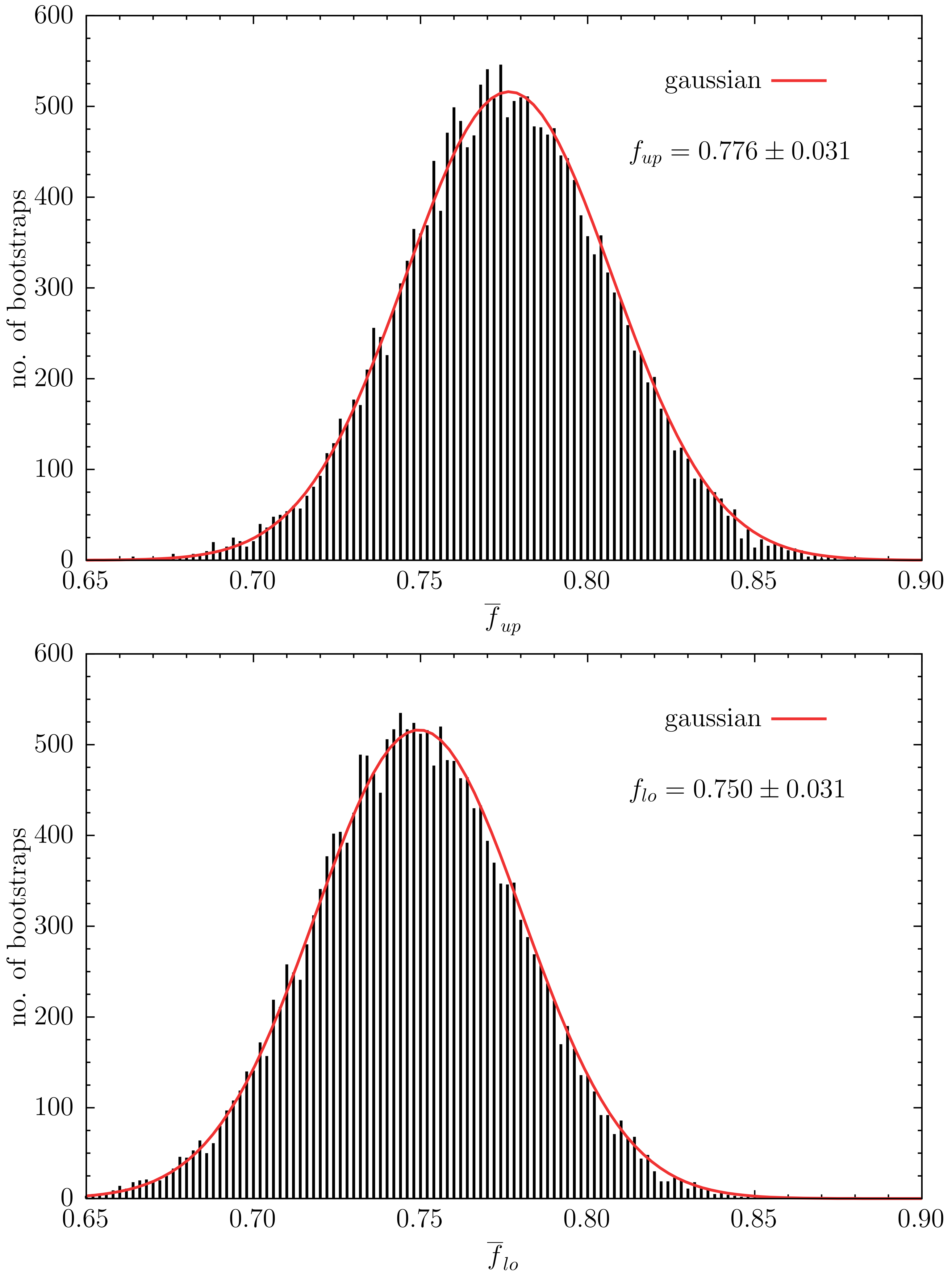
We can do the analysis as before using now a further information that takes into account the KC. This can be done for each of the N_0 events.

At the end of this second analysis we recombine the N_0 events choosing

$$\bar{f}_{lo}(t) = min[f^1_{lo}(t), \ldots, f^{N_0}_{lo}(t)],$$

$$\bar{f}_{up}(t) = max[f^1_{up}(t), \ldots, f^{N_0}_{up}(t)] .$$

The novelties of our work: Recombinations of the Bootstraps



To recombine the N_{boot} events we generate the corresponding histograms and fit them with a Gaussian Ansatz. We can then extract for every value of t average values $f_{lo(up)}(t)$, standard deviations $\sigma_{lo(up)}$ and the corresponding correlation $\rho_{lo,up}(t) = \rho_{up,lo}(t)$.

By combining the flat distribution that we have between f_{lo} and f_{up} with a multivariate Gaussian distribution necessary to mediate over the whole set of bootstrap events, we can obtain the final values for the form factor $f(t)$ and its variance $\sigma_f^2(t)$ using

$$f(t) = \frac{f_{lo}(t) + f_{up}(t)}{2},$$

$$\sigma_f^2(t) = \frac{1}{12}[f_{up}(t) - f_{lo}(t)]^2 + \frac{1}{3}[\sigma_{lo}^2(t) + \sigma_{up}(t)^2 + \rho_{lo,up}(t)\sigma_{lo}(t)\sigma_{up}(t)]\,.$$

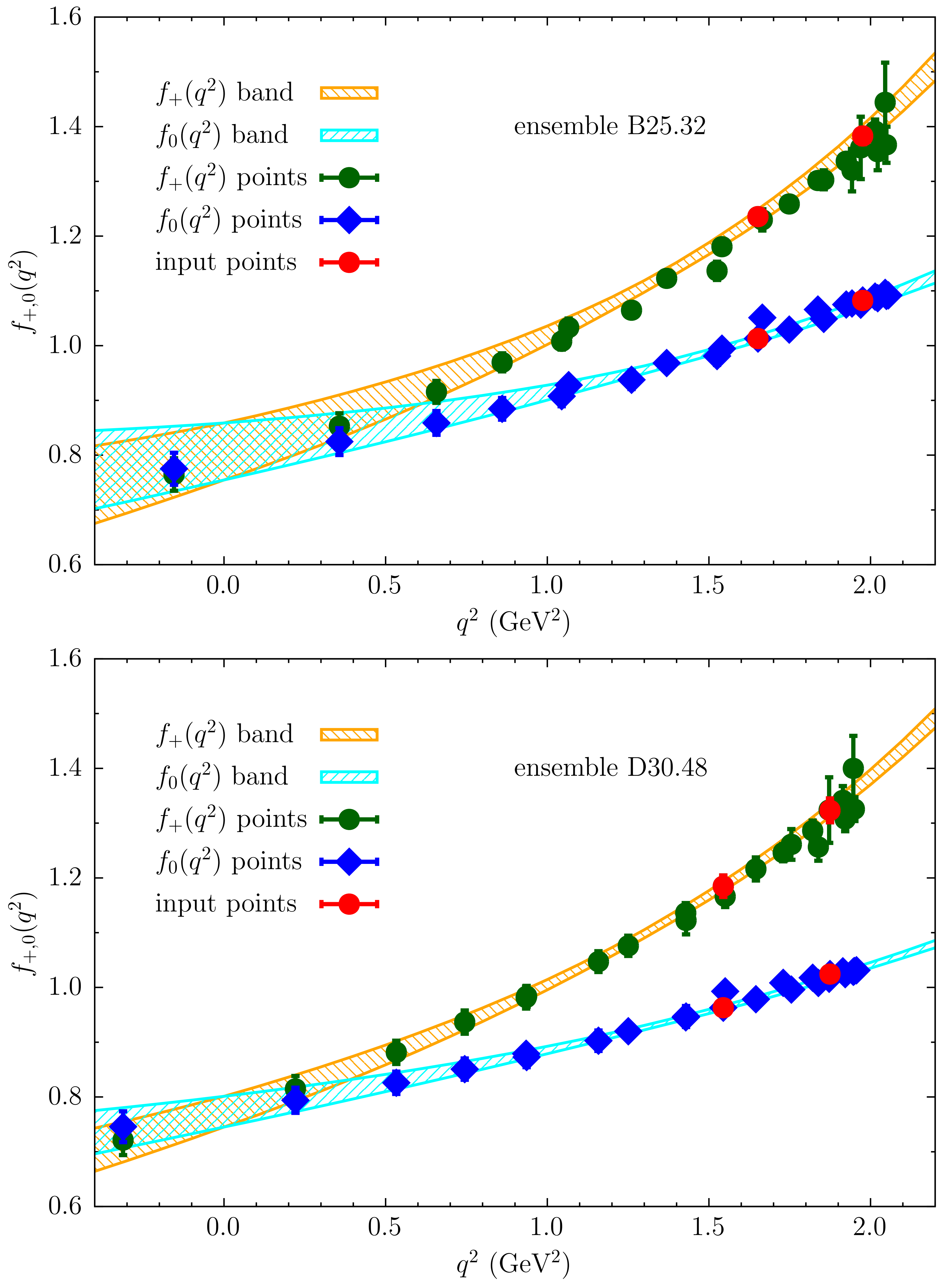
Is the method effective?

We have access to original data of **[V. Lubicz et al. [ETM], Phys. Rev. D 96 (2017)].**

Then, we redid the analysis using the matrix method with the susceptibilities computed **on the same ensembles and bootstrap by bootstrap**.

The idea has been to mimic what happens in lattice calculations of B decays where all the lattice data are concentrated at $q^2 = q^2_{max}$. Then we have chosen only two points at large values of q^2 and we tested the ability of the method to make prediction at small q^2.

The great advantage of studying the $D \to K$ decay is that we can compare our results obtained with the unitarity procedure to the ones obtained from a direct calculation of the form factors that cover indeed all the kinematical region in q^2.



The red points are the only data used as input for the DM method!!

The figures show the bands obtained by using as inputs only the red points and the rest of the lattice points that are not used as input in our analysis in the case of the ETMC ensembles B25.32 and D30.48.

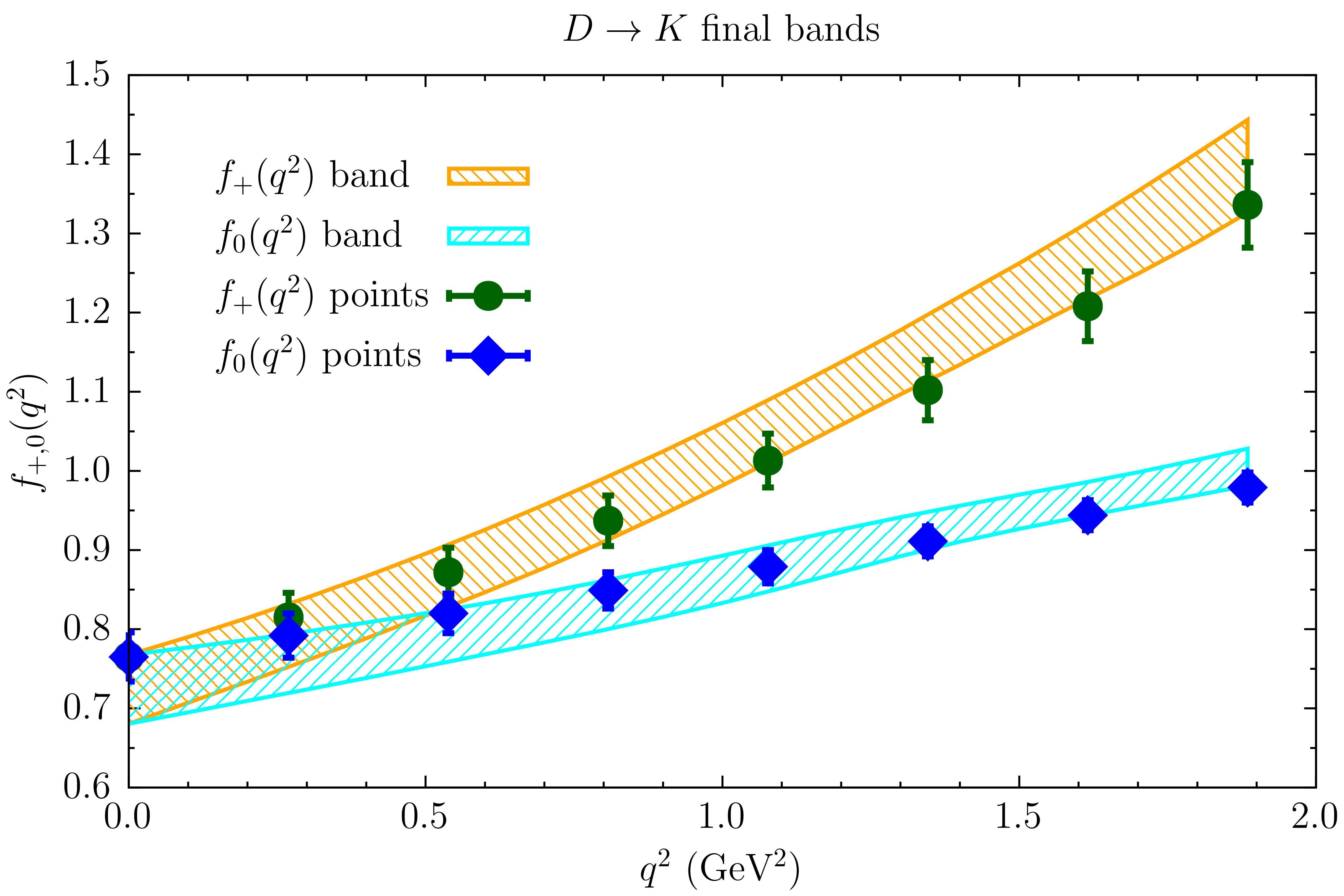
The agreement is excellent!

These results suggest that it will be possible to obtain quite precise determinations of the form factors for B decays by combining form factors at large q^2 with the non perturbative calculation of the susceptibilities.

see arXiv:2105.02497v1 [hep-lat] for details!

Results: Continuum Limit

Also after the extrapolation to the physical pion mass and to the continuum limit, the bands agree with the results of **[V. Lubicz et al. [ETM], Phys. Rev. D 96 (2017)]**, indicated with the blue and green points, and exhibit a good precision.

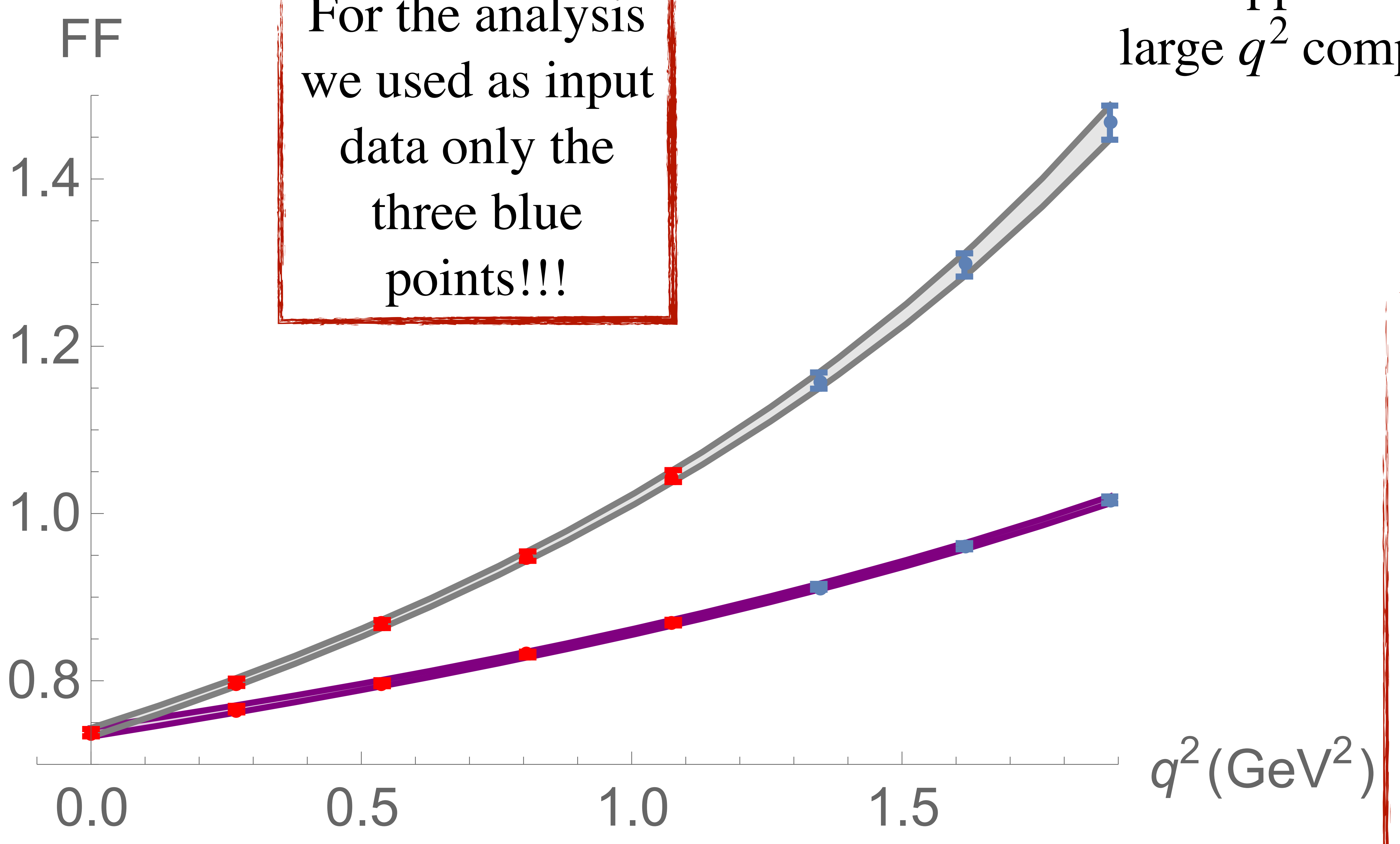


$q^2(GeV^2)$	$f_+(q^2)\|_{LQCD}$	$f_+(q^2)$	$f_0(q^2)\|_{LQCD}$	$f_0(q^2)$
0.0	0.765(31)	0.724(43)	0.765(31)	0.724(43)
0.2692	0.815(31)	0.790(40)	0.792(28)	0.754(37)
0.5385	0.872(31)	0.866(40)	0.820(25)	0.790(33)
0.8077	0.937(32)	0.953(40)	0.849(23)	0.831(31)
1.0769	1.013(34)	1.050(40)	0.879(21)	0.876(29)
1.3461	1.102(38)	1.155(42)	0.911(19)	0.924(24)
1.6154	1.208(44)	1.265(48)	0.944(19)	0.965(21)
1.8846	1.336(54)	1.384(58)	0.979(19)	1.005(23)

This demonstrates that the **Dispersive Matrix (DM) method** allows to make predictions in the whole kinematical range with a quality comparable to the one obtained by the direct calculations, even if only a quite limited number of input lattice data are used!

Recently, a new paper **[Chakraborty et al.: arXiv:2104.09883]** treated the computation of the form factors of $D \to K$ form factors extracting very precise results. For $q^2 = 0$ they found

$$f(0) = 0.7380 \pm 0.0043.$$



For the analysis we used as input data only the three blue points!!!

We applied the dispersive analysis using **only three of the eight** points for each FF at large q^2 computed in the paper. Also with less information we obtain a very close result

$$f(0) = 0.7384 \pm 0.0052.$$

We can make two considerations based on these results:

1) The results of the aforementioned paper surely satisfy unitarity which is always automatically satisfied by the bands builded using the DM method by construction;
2) The fact that we obtain comparable results using much less information suggests that using the DM method it could be unnecessary the computation of lattice points in all the kinematic region. Differently, it could be a better idea to focus on the computation of less but more precise points at high values of q^2.

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Conclusions and Outlook

The Dispersive Matrix method is very effective and precise in its prediction. It contains principally three advantages:

1) The method doesn't rely on any assumption about the functional dependence of the FF on the momentum transferred. Then, in this sense, it is model independent;
2) It's entirely based on first principles. The susceptibilities are non perturbative and we don't have series expansions;
3) It gives very precise and accurate predictions at $q^2 = 0$ even if we insert few data inputs at high values of q^2.

The DM method has been already successfully applied to B ⟶D,D* to obtain new theoretical estimates of Vcb and of the anomalies R(D(*)) (see arXiv:2105.08674 [hep-ph]) and to compute non perturbative constraints for the b->c transition (see arXiv:2105.07851 [hep-lat]).

New applications: work in progress…

Thanks for your attention!