Charm physics with a tmQCD mixed action approach

Alessandro Conigli

A. Bussone, J. Frison[1], G. Herdoíza[2], C. Pena, D. Preti, J.A. Romero, J. Ugarrio

IFT UAM-CSIC

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Instituto de Física Teórica UAM-CSIC

Universidad Autónoma de Madrid

ALPHA Collaboration

EuroPLEx

H2020

[1] Talk on: *"Charmed semileptonics with twisted-mass valence quarks"*

[2] Talk on: *"Scale setting from a mixed action with twisted-mass valence quarks"*

A mixed action approach: motivation

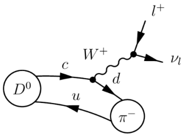
Sea sector: $N_f = 2 + 1$ flavours

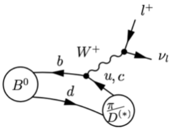
- CLS ensembles with $N_f = 2 + 1$ flavours
- $O(a)$-improved Wilson fermions
- Small lattice spacings without topology freezing

Valence sector: $N_f = 2 + 1 + 1$ flavours

- Wilson twisted mass fermions at full twist
- Automatic $O(a)$-improvement[a] $\implies$ no $O(am_h)$ effects

[a] apart from tiny residual



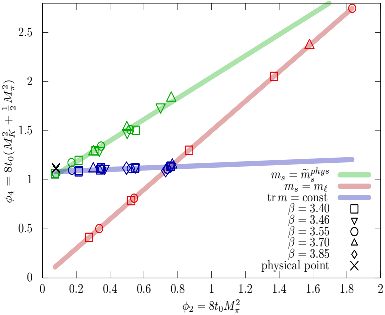


Sea sector - CLS ensembles [Lüscher and Schaefer, JHEP 1107 036 Bruno et al. JHEP 1502 043 - 1712.04884 - 2003.13359]

- Lüscher-Weisz tree-level improved gauge action
- $N_f = 2 + 1$ non-perturbatively $\mathcal{O}(a)$-improved Wilson fermions
- Open boundary conditions in time
  - Topological charge flows smoothly
    [Lüscher and Schaefer, 1206.2809]
  - Avoid topological freezing
  - Reliable estimates for fine values of the lattice spacing
- Mass corrections to ensure $\Phi_4 = \Phi_4^{\text{phys}}$ LCP
  [M. Bruno, T. Korzec, S. Schaefer, 1608.08900]

$$\langle \mathcal{O} \rangle \to \langle \mathcal{O} \rangle + \sum_{i=1}^{N_f} \Delta m_{q,i} \frac{\mathrm{d}\langle \mathcal{O} \rangle}{\mathrm{d}m_{q,i}}$$

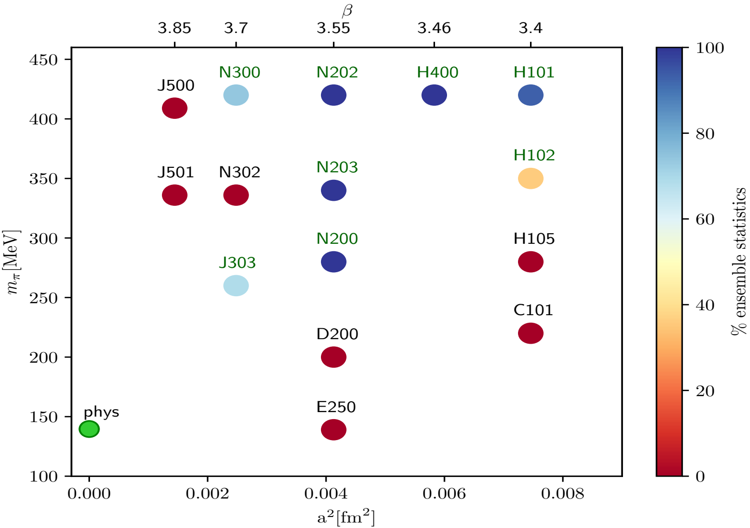
[Plot by J. Simeth]



Lattice spacings :

$a = 0.087,\ 0.077,\ 0.065,\ 0.050,\ 0.039$ fm

CLS ensembles



Lattice spacings : $a = 0.087, 0.077, 0.065, 0.050, 0.039$ fm

Valence sector [Frezzotti, Grassi, Sint, Weisz, hep-lat/0101001 Frezzotti, Rossi, hep-lat/0306014 - Pena, Sint, Vladikas hep-lat/0405028]

Wilson twisted mass Dirac operator

$$D_{tm} = \frac{1}{2}\sum_{\mu=0}^{3}\left[\gamma_\mu(\nabla_\mu^\star + \nabla_\mu) - a\nabla_\mu^\star\nabla_\mu\right] + \frac{i}{4}ac_{SW}\sum_{\mu,\nu=0}^{3}\sigma_{\mu\nu}\hat{F}_{\mu\nu} + \mathbf{m_0} + i\gamma_5\boldsymbol{\mu_0}$$

At full twist $\boldsymbol{m_0} = \mathbb{1}\boldsymbol{m}_{\rm cr}$ $\boldsymbol{\mu_0} = \mathrm{diag}(\mu_l, -\mu_l, -\mu_s, +\mu_c)$

- Twisted axial symmetry is only softly broken

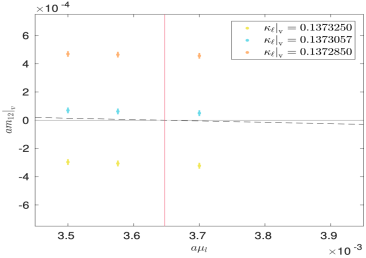
$$\partial_\mu \tilde{V}_\mu^a = -2\mu_q \epsilon^{3ab} P^b \quad \Longrightarrow \quad Z_P = \frac{1}{Z_\mu}$$

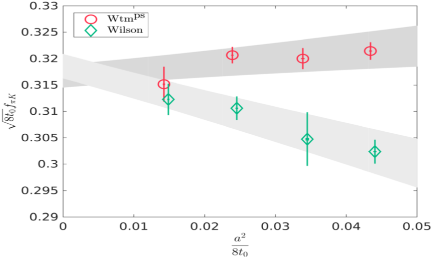
- Automatic $O(a)$ improvement for physical observables
    - no parameter tuning
    - residual lattice artefacts of $O(a g_0^4 \,\mathrm{tr}\, M_q^{\rm sea})$
    - absence of lattice artefacts proportional to $\boldsymbol{\mu_0}$

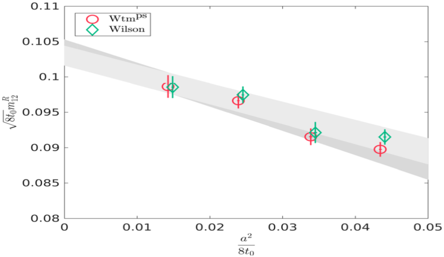
Matching conditions

[1711.06017, 1812.05458, 1903.00286] talk by G. Herdoíza (Friday)

- Achieve full twist imposing standard light quark mass to vanish
- Match the sea and valence quark masses to recover unitarity
  - Consider a tuning grid in the $(k_l|_v, \mu_{0,l}, \mu_{0,s})$ plane
  - Impose $\Phi_2|_v = \Phi_2|_s, \;\; \Phi_4|_v = \Phi_4|_s$

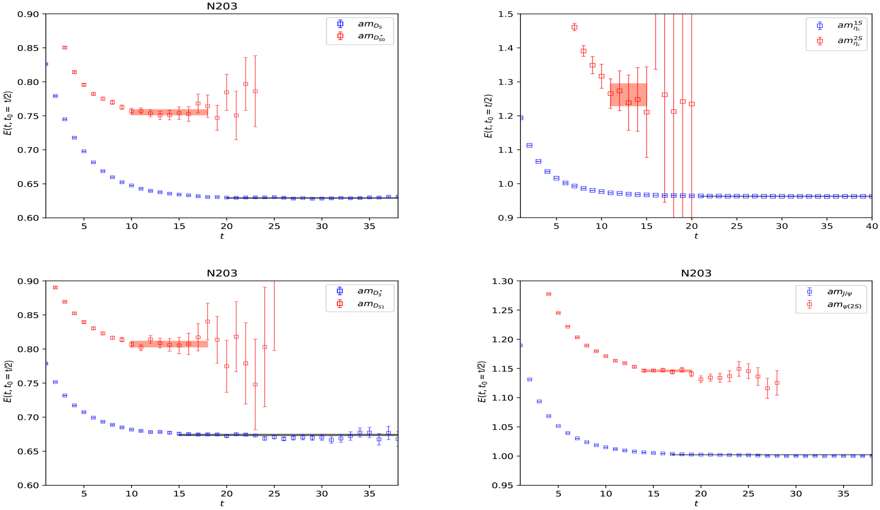






Extraction of meson masses [1010.0202 - 2002.12347]

- Observables extracted from a GEVP (only connected components for charmonia)
  [P. de Forcrand et al. hep/lat0404016 - 1208.2855 - 2005.01845]



Chiral-continuum extrapolations

[ALPHA, 2101.02694]

- General ansatz for the functional form:
  Renormalization and running of quark mass [ALPHA, 1802.05243]

$$\sqrt{8t_0}\mathcal{O}^{\text{cont}}(\phi_2, \phi_H, 0) = p_0 + p_1\phi_2 + p_3\phi_H, \qquad \phi_2 = 8t_0 m_\pi^2, \quad \phi_H = \sqrt{8t_0}m_H$$

$$c_{\mathcal{O}}(\phi_2, \phi_H, a) = \frac{a^2}{8t_0}\left(c_1 \; + c_2\phi_2 + c_3\phi_H^2\right) + \frac{a^4}{(8t_0)^2}\left(c_4 + c_5\phi_H^4\right)$$

- Combined models:

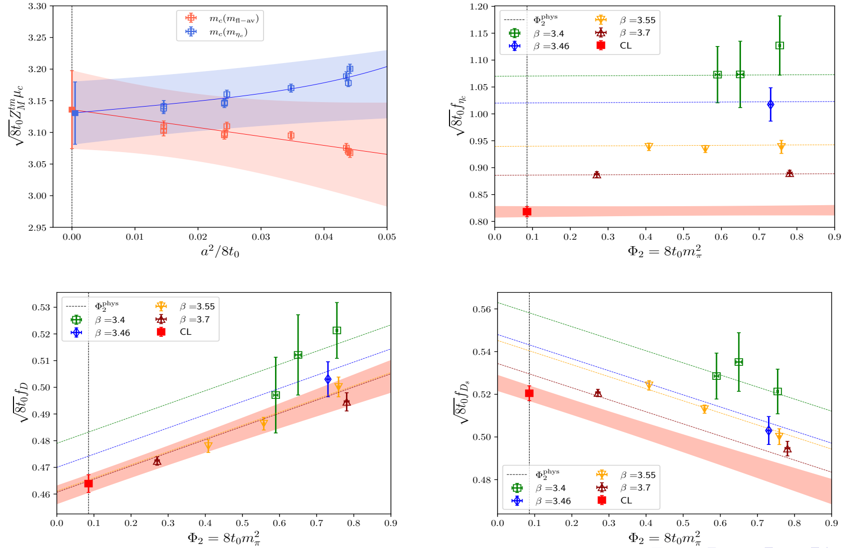
$$\sqrt{8t_0}\mathcal{O}^{\text{linear}}(\phi_2, \phi_H, a) = \sqrt{8t_0}\mathcal{O}^{\text{cont}}(\phi_2, \phi_H, 0) + c_{\mathcal{O}}(\phi_2, \phi_H, a)$$

$$\sqrt{8t_0}\mathcal{O}^{\text{non-lin}}(\phi_2, \phi_H, a) = \sqrt{8t_0}\mathcal{O}^{\text{cont}}(\phi_2, \phi_H, 0) \times (1 + c_{\mathcal{O}}(\phi_2, \phi_H, a))$$

- Address systematic effects by classifying data in different categories
  - Fixing the charm: flavour-averaged D-meson mass, charmonium[3] η_c
  - Consider all ensembles or exclude the ones with $\beta = 3.40$

[3] Neglecting disconnected contributions

Chiral-continuum extrapolation PRELIMINARY



Model average [ALPHA, 2101.02694]

$$\langle M_c \rangle = \sum_{n=1}^{N} w_n \langle M_c \rangle_n \qquad \sigma^2_{M_c} = \sum_{n=1}^{N} w_n \langle M_c \rangle_n^2 - \left(\sum_{n=1}^{N} w_n \langle M_c \rangle_n \right)^2$$

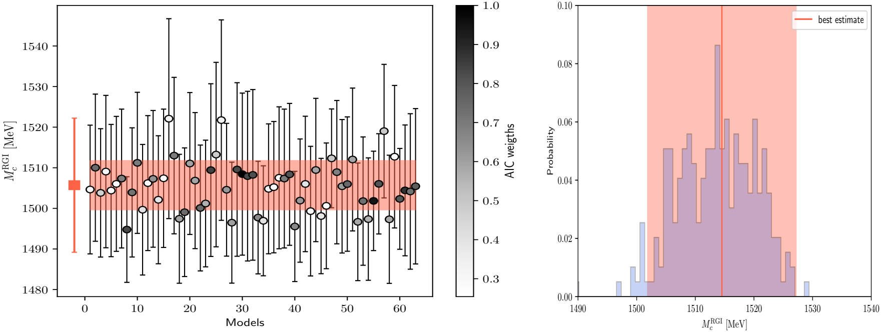
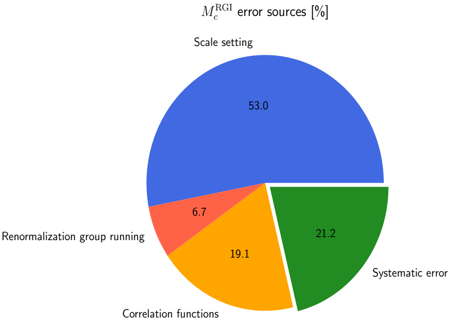


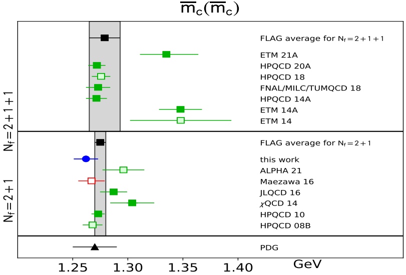
Figure: Left: model average procedure for a representative category. Right: histogram with central values for all categories.

Analysis software and error contributions

- ADerrors.jl by Alberto Ramos https://gitlab.ift.uam-csic.es/alberto/aderrors.jl
  MC data analysis with:
  - Γ-method [Wolff, hep-lat/03060174; Bruno, Sommer, in preparation]
  - Automatic Differentiation [A. Ramos, 1809.01289]
  - $\chi^2_{\text{exp.}}$ fitting routine [Bruno, Sommer, in preparation]
- Juobs.jl by J. Ugarrio and AC https://gitlab.ift.uam-csic.es/jugarrio/juobs



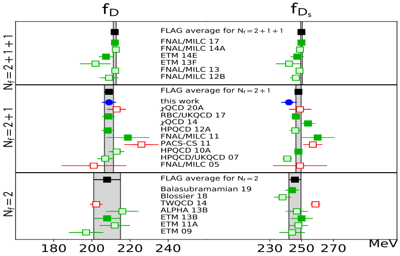
Results comparison[4] PRELIMINARY

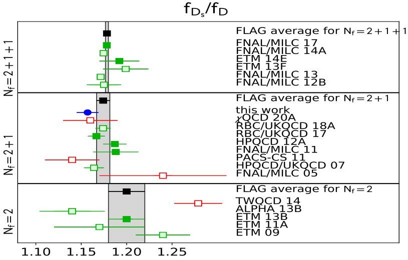


- Charmonium decay constants

$$f_{\eta_c} = 392(8)(9)\ \text{MeV}$$

$$f_{J/\Psi} = 395(10)(4)\ \text{MeV}$$





[4] See talks by S. Bouma (Wednesday), F. Joswig (Thursday)

Conclusions and outlook

Summary

- Charm results from a mixed action setup with $N_f = 2 + 1$ CLS ensembles at four fine lattice spacings and a tmQCD valence action
- Fully non-perturbative $O(a)$-improvement of the observables
- Systematic effects in chiral-continuum extrapolation under control

Future

- We plan to extend this analysis across more ensembles
- Push towards heavier masses to tackle the B sector

Thank You!

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J. Frison: *"Charmed semileptonics with twisted-mass valence quarks"*
G. Herdoíza: *"Scale setting from a mixed action with twisted-mass valence quarks"*

The Generalized Eigenvalue Problem

[M. Lüscher, U. Wolff, Nucl. Phys. B339 (1990) 222-252]

- Consider a set of operators $\{\hat{O}_i\}$, then combine different interpolators to get a matrix of Euclidean space correlation functions

$$C_{ij}(t) := \langle O_i(t) O_j^\dagger(0) \rangle = \sum_{n=1}^{\infty} e^{-E_n t} \psi_{ni} \psi_{nj}^*, \quad i,j = 1, \ldots, N$$

$$\psi_{ni} \equiv (\psi_n)_i = \langle 0 | \hat{O}_i | n \rangle \quad E_n < E_{n+1}$$

- The GEVP is defined as

$$C(t) v_k(t,t_0) = \lambda_k(t,t_0) C(t_0) v_k(t,t_0), \quad k = 1, \ldots, N, \quad t > t_0$$

Assuming that only N states contribute:

$$\lambda_k^{(0)}(t,t_0) = e^{-E_k(t-t_0)}, \qquad v_k(t,t_0) \rightsquigarrow \psi_m$$

GEVP set-up

- We consider the following interpolating fields of different Dirac structures in the twisted basis

$$P = \bar{\psi}\gamma_5\psi \quad A_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi, \quad \mu = 0, \ldots, 3$$

$$C_{PP}(t) = \begin{bmatrix} \langle P(t)P(0)\rangle & \langle P(t+\tau)P(0)\rangle \\ \langle P(t)P(-\tau)\rangle & \langle P(t+\tau)P(-\tau)\rangle \end{bmatrix} \quad C_{VV}(t) = \begin{bmatrix} \langle A_k(t)A_k(0)\rangle & \langle A_k(t+\tau)A_k(0)\rangle \\ \langle A_k(t)A_k(-\tau)\rangle & \langle A_k(t+\tau)A_k(-\tau)\rangle \end{bmatrix}$$

- Then we solve the associated GEVP for the pseudoscalar-pseudoscalar and vector-vector matrix of correlators

$$C_{PP}(t)v_l^P(t,t_0) = \lambda_l^P(t,t_0)C_{PP}(t_0)v_l^P(t,t_0)$$

$$C_{VV}(t)v_l^V(t,t_0) = \lambda_l^V(t,t_0)C_{VV}(t_0)v_l^V(t,t_0)$$

Observables

- Observables extracted by solving the GEVP

$$C(t)v_k(t,t_0) = \lambda_k(t,t_0)C(t_0)v_k(t,t_0), \quad k = 1,\dots,N, \quad t > t_0$$

where

$$C_{ij}(t) := \langle O_i(t)O_j^\dagger(0)\rangle = \sum_{n=1}^{\infty} e^{-E_n t}\psi_{ni}\psi_{nj}^*, \quad i,j = 1,\dots,N$$

$$P = \bar{\psi}\gamma_5\psi \quad A_\mu = \bar{\psi}\gamma_\mu\gamma_5\psi, \quad \mu = 0,\dots,3$$

Effective masses extracted from eigenvalues $\lambda_k(t,t_0)$

$$aE_n^{\mathrm{eff}}(t,t_0) = \log\frac{\lambda_n(t,t_0)}{\lambda_n(t+a,t_0)} + O(e^{-(E_m-E_n)t})$$

Matrix elements extracted from eigenvectors $v_k(t,t_0)$

$$f_{\mathrm{ps}} = \sqrt{\frac{2}{m_p^3L^3}}(\mu_q+\mu_c)|\langle 0|\,P^{c,q}\,|ps\rangle|, \qquad f_{\mathrm{v}}\epsilon_k = Z_A\sqrt{\frac{2}{m_vL^3}}|\langle 0|\,A_k\,|v\rangle|$$

More on matrix elements

[Alpha Collaboration, hep/lat0902.1265v2]

Matrix elements are extracted from the effective operator

$$\hat{\mathcal{A}}_n^{\text{eff}}(t,t_0) = e^{-\hat{H}t}\hat{\mathcal{Q}}_n^{\text{eff}}(t,t_0), \quad |n\rangle = \hat{\mathcal{A}}^\dagger |0\rangle, \quad \hat{H}|n\rangle = E_n |n\rangle$$

$$\hat{\mathcal{Q}}_n^{\text{eff}}(t,t_0) = R_n\left(\hat{O}, v_n(t,t_0)\right)$$

$$R_n = (v_n(t,t_0), C(t)v_n(t,t_0))^{-1/2}\frac{\lambda_n(t_0+t/2,t_0)}{\lambda_n(t_0+t,t_0)}$$

Corrections to the large time asymptotic behaviour are parametrized by

$$e^{-\hat{H}t}\hat{\mathcal{Q}}_n^{\text{eff}}(t,t_0)^\dagger |0\rangle = |n\rangle + \sum_{n'=1}^{\infty} \pi_{nn'}(t,t_0)\,|n'\rangle$$

More on matrix elements

[Alpha Collaboration, hep/lat0902.1265v2]

They can show that

$$\pi_{nn'} = O(e^{-\Delta E_{N+1,n} t_0}) \qquad \text{at} \ \ t - t_0 = \text{const}$$

Thus, matrix elements of a local operator $\hat{X}$ can be computed via

$$\langle 0 | \, \hat{Q}_n^{\text{eff}} e^{-\hat{H}t} \hat{X} e^{-\hat{H}t} (\hat{Q}_n^{\text{eff}})^\dagger \, | 0 \rangle = \langle n | \, \hat{X} \, | n' \rangle + O(e^{-\Delta E_{N+1,n} t_0})$$

And the amplitude between a vacuum and the state $|n\rangle$ is then given by

$$p_n^{\text{eff}} = \langle 0 | \, \hat{Q}_n^{eff} e^{-\hat{H}t} \hat{X} \, | 0 \rangle = R_n(v_n(t, t_0), C_X), \quad (C_X)_j = \langle O_j(0) X(t) \rangle$$

If $\hat{X}$ denotes the time component of an axial current, the decay constant of the associated ground state meson is

$$p_1^{\text{eff}}(t, t_0) = \langle 0 | \, \hat{X} \, | 1 \rangle$$

Mass corrections and line of constant physics

[M. Bruno, T. Korzec, S. Schaefer, 1608.08900]

- Chiral trajectory defined in terms of $\Phi_4 \equiv 8t_0^2(m_K^2 + \frac{1}{2}m_\pi^2)$
- Achieve $\Phi_4 = \Phi_4^{\text{phys}}$ in each ensemble by employing quark mass shift via a Taylor expansion

$$\frac{\mathrm{d}\langle\mathcal{O}\rangle}{\mathrm{d}m_{q,i}} = \left\langle \frac{\partial\mathcal{O}}{\partial m_{q,i}} \right\rangle - \left\langle \mathcal{O}\frac{\partial\mathcal{S}}{\partial m_{q,i}} \right\rangle + \langle\mathcal{O}\rangle\left\langle \frac{\partial\mathcal{S}}{\partial m_{q,i}} \right\rangle$$

- The shift is performed at the level of the observables by

$$\langle\mathcal{O}\rangle \to \langle\mathcal{O}\rangle + \sum_{i=1}^{N_f} \Delta m_{q,i} \frac{\mathrm{d}\langle\mathcal{O}\rangle}{\mathrm{d}m_{q,i}}$$

Assessing the systematics

- Estimate the optimal fit parameters

$$\chi^2_{\text{corrected}} = \frac{\chi^2}{\chi^2_{\text{exp}}} \leq 1$$

- Akaike information criteria as measure for the best fit

$$\text{AIC} = \chi^2_{\text{corrected}} + 2k$$

- Model average

$$\langle M_c \rangle = \sum_{n=1}^{N} w_n \langle M_c \rangle_n \qquad w_n^{\text{AIC}} = N \exp\left(-\frac{1}{2}\text{AIC} \right)$$

- Estimate the systematics

$$\sigma^2_{M_c} = \sum_{n=1}^{N} w_n \langle M_c \rangle^2_n - \Big(\sum_{n=1}^{N} w_n \langle M_c \rangle_n \Big)^2$$

Heavy mass dependence of M_c

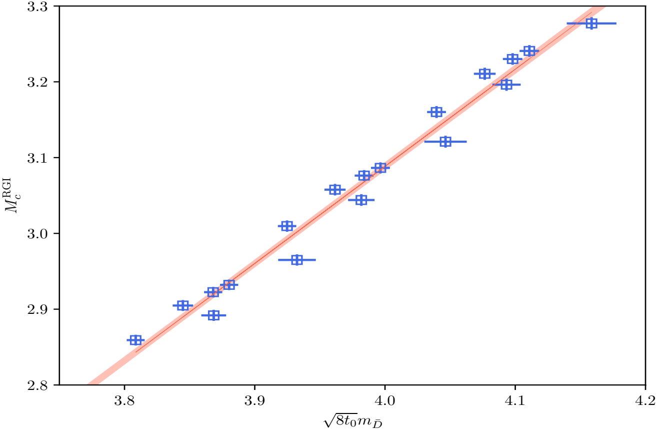

Figure: Charm mass dependence on the dimensionless heavy meson mass $\phi_H = \sqrt{8t_0}m_H$.