The $B \to D^* \ell \nu$ semileptonic decay at non-zero recoil and its implications for $|V_{cb}|$ and $R(D^*)$

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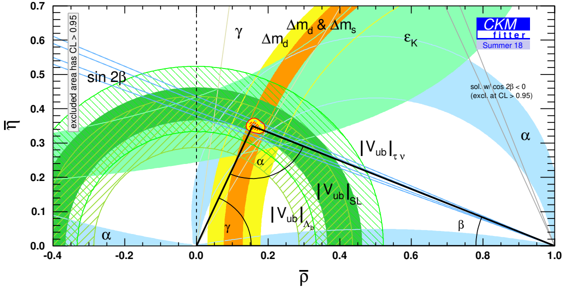
The V_{cb} matrix element: Tensions

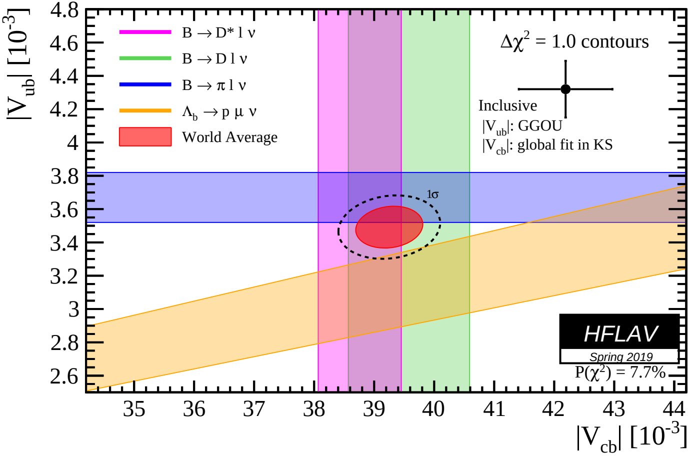
$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$\lvert V_{cb}\rvert\,(\cdot 10^{-3})$	PDG 2016	PDG 2018	PDG 2020
Exclusive	39.2 ± 0.7	41.9 ± 2.0	39.5 ± 0.9
Inclusive	42.2 ± 0.8	42.2 ± 0.8	42.2 ± 0.8

- Matrix must be unitary (preserve the norm)
- **Current tensions (2021) stand at** $\approx 3\sigma$



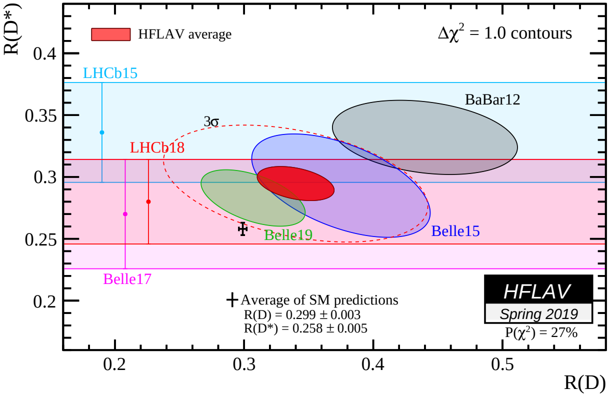
Break: Reminder of $|V_{ub}|$ vs $|V_{cb}|$



Current status of $|V_{ub}|$ vs $|V_{cb}|$ (HFLAV 2019)

The V_{cb} matrix element: Tensions in lepton universality

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current $\approx 3\sigma$ tension with the SM

The V_{cb} matrix element: Measurement from exclusive processes

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \left[\underbrace{K_1(w,m_\ell)}_{\text{Known factors}}\underbrace{|\mathcal{F}(w)|^2}_{\text{Theory}} + \underbrace{K_2(w,m_\ell)}_{\text{Known factors}}\underbrace{|\mathcal{F}_2(w)|^2}_{\text{Theory}}\right] \times |V_{cb}|^2$$

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Can use effective theories (HQET) to say something about $\mathcal{F}(1)$
  - $K_i(w,m_\ell) \propto (w^2-1)^{\frac{1}{2}}$ factor requires extrapolation of experimental data
- $R(D^*)$ requires an extra term that only contributes with the τ

$$R(D^*) = \frac{\int_1^{w_{\text{Max},\tau}} dw\, \left[K_1(w,m_\tau)\,|\mathcal{F}(w)|^2 + K_2(w,m_\tau)\,|\mathcal{F}_2(w)|^2\right] \times \cancel{|V_{cb}|^2}}{\int_1^{w_{\text{Max}}} dw\, \left[K_1(w,0)\,|\mathcal{F}(w)|^2\right] \times \cancel{|V_{cb}|^2}}$$

- It is possible to extract $R(D^*)$ without experimental data!

The V_{cb} matrix element: The parametrization issue

All the parametrizations perform an expansion in the z parameter

$$z = \frac{\sqrt{w+1} - \sqrt{2N}}{\sqrt{w+1} + \sqrt{2N}}$$

- Boyd-Grinstein-Lebed (BGL)

Phys. Rev. Lett. 74 (1995) 4603-4606

Phys.Rev. D56 (1997) 6895-6911

Nucl.Phys. B461 (1996) 493-511

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

  - B_{f_X} Blaschke factors, includes contributions from the poles
  - ϕ_{f_X} is called *outer function* and must be computed for each form factor
  - Weak unitarity constraints $\sum_n |a_n|^2 \leq 1$

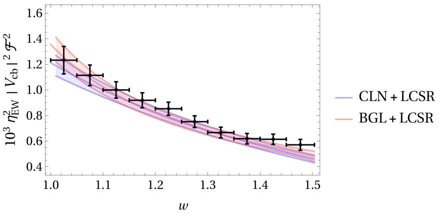
- Caprini-Lellouch-Neubert (CLN)

Nucl. Phys. B530 (1998) 153-181

$$\mathcal{F}(w) \propto 1 - \rho^2 z + cz^2 - dz^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $\mathcal{F}(w)$: four independent parameters, one relevant at $w = 1$

The V_{cb} matrix element: The parametrization issue



From *Phys. Lett. B769 (2017) 441-445* using Belle data from arXiv:1702.01521 and the Fermilab/MILC'14 value at zero recoil

- CLN seems to underestimate the slope at low recoil
- The BGL value of $|V_{cb}|$ is compatible with the inclusive one

$$|V_{cb}| = 41.7 \pm 2.0 (\times 10^{-3})$$

- Latest Belle dataset and Babar analysis seem to contradict this picture
  - From Babar's paper PRL **123**, 091801 (2019) **BGL is compatible with CLN and far from the inclusive value**
  - Belle's paper PRD **100**, 052007 (2019) finds **similar results in its last revision**
- The discrepancy inclusive-exclusive is not well understood
- Data at $w \gtrsim 1$ is **urgently needed** to settle the issue
- Experimental measurements perform badly at low recoil

We would benefit enormously from a high precision lattice calculation at $w \gtrsim 1$

Calculating V_{cb} on the lattice: Formalism

- Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\ \ \rho\sigma}v_B^\rho v_{D^*}^\sigma \boldsymbol{h_V}(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

$$\frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}\left(1+w\right)\boldsymbol{h_{A_1}}(w) - v_B^\nu\left(v_B^\mu \boldsymbol{h_{A_2}}(w) + v_{D^*}^\mu \boldsymbol{h_{A_3}}(w)\right)\right]$$

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The h_X enter in the definition of $\mathcal{F}$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice

Calculating V_{cb} on the lattice: Formalism

- Helicity amplitudes

$$H_\pm = \sqrt{m_B\, m_{D^*}}(w+1)\left(\boldsymbol{h_{A_1}}(w) \mp \sqrt{\frac{w-1}{w+1}}\boldsymbol{h_V}(w)\right)$$

$$H_0 = \sqrt{m_B\, m_{D^*}}(w{+}1)m_B\left[(w-r)\boldsymbol{h_{A_1}}(w) - (w-1)\left(r\,\boldsymbol{h_{A_2}}(w) + \boldsymbol{h_{A_3}}(w)\right)\right]/\sqrt{q^2}$$

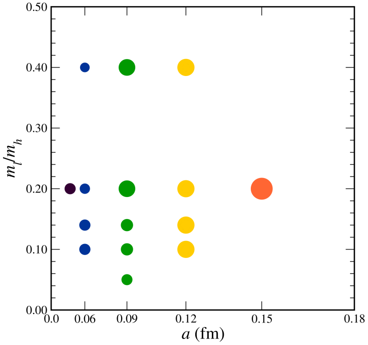
$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)\boldsymbol{h_{A_1}}(w) + (wr-1)\boldsymbol{h_{A_2}}(w) + (r-w)\boldsymbol{h_{A_3}}(w)\right]$$

- Form factor in terms of the helicity amplitudes

$$\chi(w)\,|\mathcal{F}|^2 = \frac{1-2wr+r^2}{12 m_B\, m_{D^*}\,(1-r)^2}\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right)$$

Introduction: Available data and simulations

- Using 15 $N_f = 2 + 1$ MILC ensembles of sea asqtad quarks
- The heavy quarks are treated using the Fermilab action



Analysis: Chiral-continuum fits

- Our data represents the form factors at nonzero a and unphysical m_π
- Extrapolation to the physical pion mass described by EFTs
  - The EFT describe the a and the m_π dependence
- Functional form explicitly known

$$h_{A_1}(w) = \underbrace{\Bigg[1 + \frac{X_{A_1}(\Lambda_\chi)}{m_c^2} + \frac{g^2_{D^*D\pi}}{48\pi^2 f_\pi^2 r_1^2}\mathrm{logs}_{\mathrm{SU3}}(a, m_l, m_s, \Lambda_{QCD})}_{\mathrm{NLO}\ \chi\mathrm{PT} + \mathrm{HQET}}$$

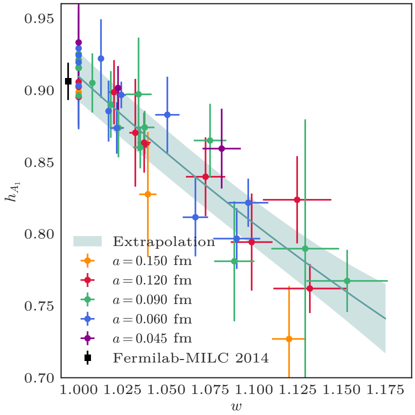
$$\underbrace{+c_1 x_l + c_{a1} x_{a^2}}_{\mathrm{NLO}\ \chi\mathrm{PT}} \underbrace{-\rho^2_{A_1}(w-1) + k_{A_1}(w-1)^2}_{w\ \mathrm{dependence}} \underbrace{+ c_2 x_l^2 + c_{a2} x_{a^2}^2 + c_{a,m} x_l x_{a^2}\Bigg]}_{\mathrm{NNLO}\ \chi\mathrm{PT}} \times$$

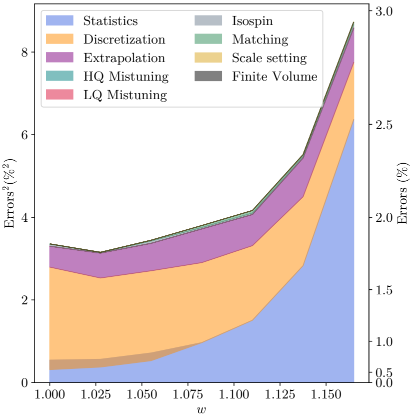
$$\underbrace{\left(1 + \beta_{11}^{A_1}\alpha_s a \Lambda_{\mathrm{QCD}} + \cancel{\beta_{02}^{A_1} a^2 \Lambda^2_{\mathrm{QCD}}} + \beta_{03}^{A_1} a^3 \Lambda^3_{\mathrm{QCD}}\right)}_{\mathrm{HQ\ discretization\ errors}}$$

with

$$x_l = B_0 \frac{m_l}{(2\pi f_\pi)^2}, \qquad x_{a^2} = \left(\frac{a}{4\pi f_\pi r_1^2}\right)^2$$

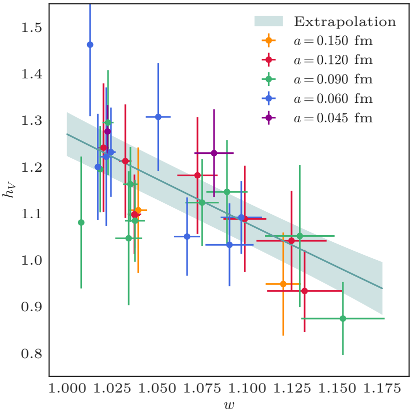
Analysis: Chiral-continuum fits

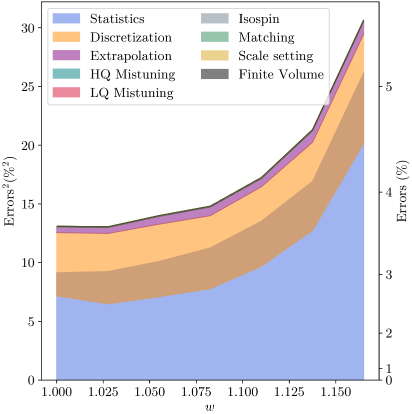




- Combined fit $p - \text{value} = 0.96$
- $h_{A_1}(1) = 0.909(17)$

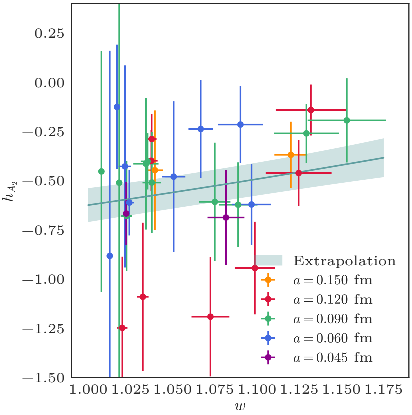
Analysis: Chiral-continuum fits

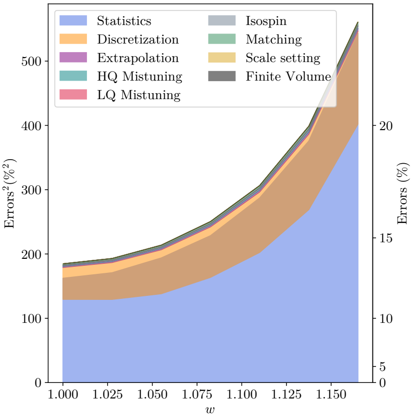




- Combined fit $p - \text{value} = 0.96$
- $h_V(1) = 1.270(46)$

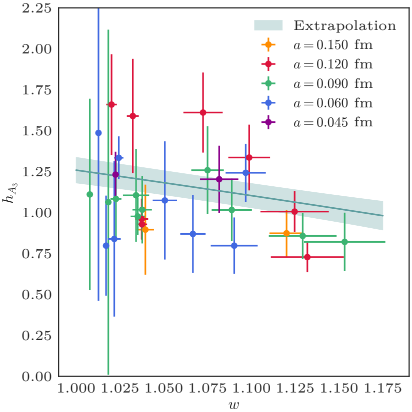
Analysis: Chiral-continuum fits

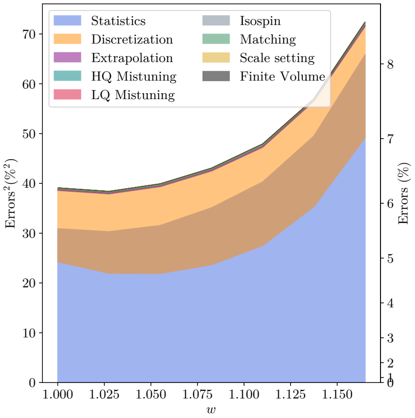




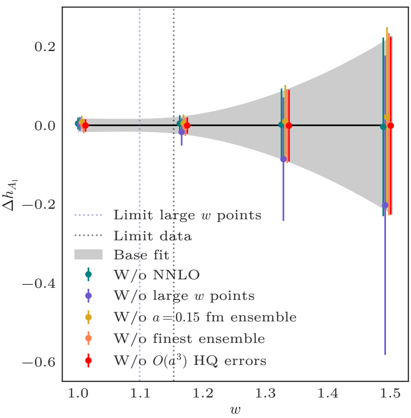
- Combined fit $p - \text{value} = 0.96$
- $h_{A_2}(1) = -0.624(85)$

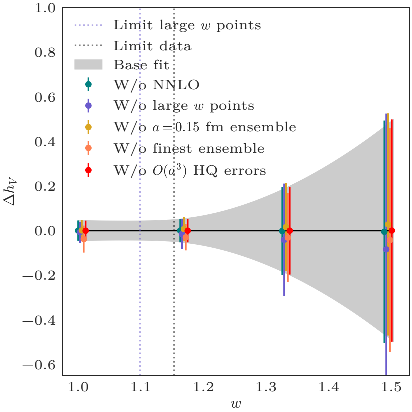
Analysis: Chiral-continuum fits





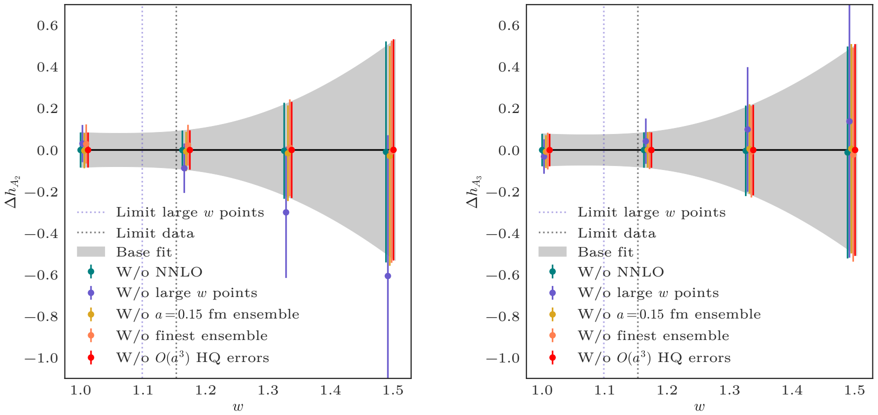
- Combined fit $p - \text{value} = 0.96$
- $h_{A_3}(1) = 1.259(79)$





	Base	W/o NNLO	W/o large w	W/o $a = 0.15$ fm
χ^2/dof	**85.5/110**	86.1/111	71.5/93	79.7/101
		W/o $a = 0.045$ fm	W/o HQ $O(a^3)$	
χ^2/dof		81.9/101	85.6/111	

Results: Stability of chiral-continuum fits



	Base	W/o NNLO	W/o large w	W/o $a = 0.15$ fm
χ^2/dof	**85.5/110**	86.1/111	71.5/93	79.7/101
		W/o $a = 0.045$ fm	W/o HQ $O(a^3)$	
χ^2/dof		81.9/101	85.6/111	

Analysis: z-Expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B769**, 441 (2017), Phys.Lett. **B771**, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w) h_{A_1}(w) = \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j$$

$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j$$

$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0)$
- Constraint $(1+w) m_B^2 (1-r) \mathcal{F}_1(z = z_{\text{Max}}) = (1+r) \mathcal{F}_2(z = z_{\text{Max}})$
- BGL (weak) unitarity constraints

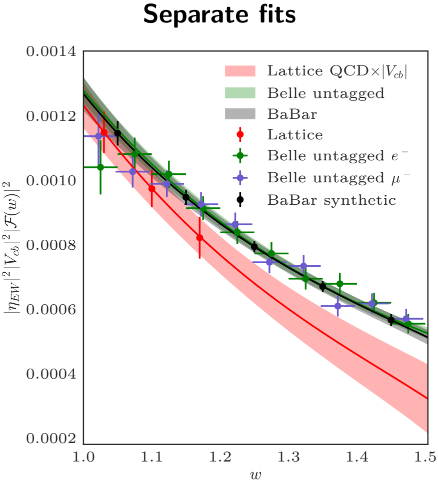
$$\sum_j a_j^2 \le 1, \qquad \sum_j b_j^2 + c_j^2 \le 1, \qquad \sum_j d_j^2 \le 1$$

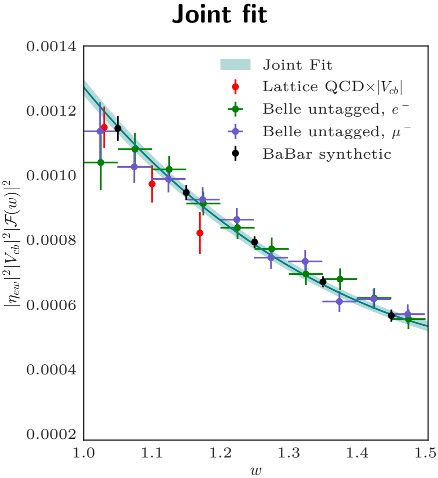
Analysis: z expansion fit procedure

- Several different datasets
  - Our lattice data
  - BaBar BGL fit — arXiv:1903.10002; Phys.Rev.Lett. **123**, 091801 (2019)
    - Generate synthetic data and include the data points to our joint fit
    - Limited by the order of BaBar BGL fit (222) $\rightarrow$ Truncation errors?
    - Fit dominated by Belle data anyway
  - Belle untagged dataset — arXiv:1809.03290; Phys.Rev. D**100**, 052007 (2019)
    - Data binned in four variables: $w, \cos\theta_v, \cos\theta_l$ and χ
    - Same normalization per binning $\sum \text{Bins}(\alpha) = N, \quad \alpha = w, \cos\theta_v, \cos\theta_l, \chi$
    - Correlation matrices should reflect the normalization constraints $\rightarrow$ they don't
    - We use the data as it is published anyway (in Phys.Rev. D, the arXiv correlation matrices are wrong, even on v3!!)
    - Further details arXiv:2104.02094

All the experimental and theoretical **correlations are included** in all fits

Results: Separate fits and joint fit

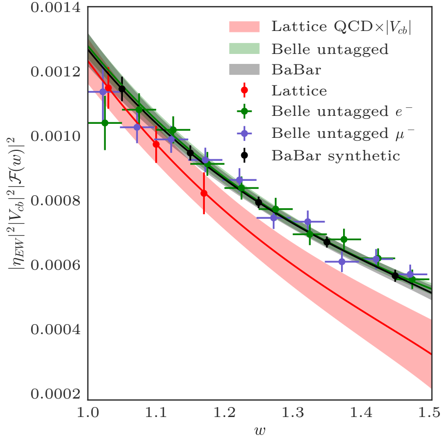




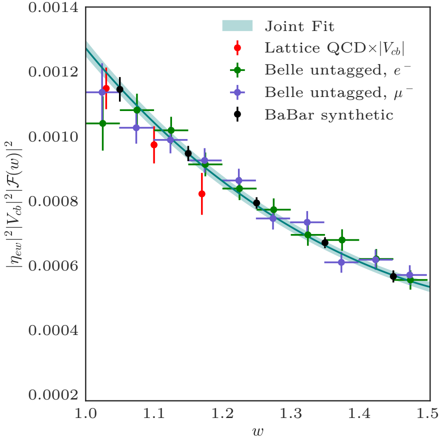
Fit	Lattice	Exp	Lat + Belle	Lat + BaBar	Lat + Exp
p-Value	0.88	0.037	0.015	0.088	0.002

Results: Separate fits and joint fit

Separate fits

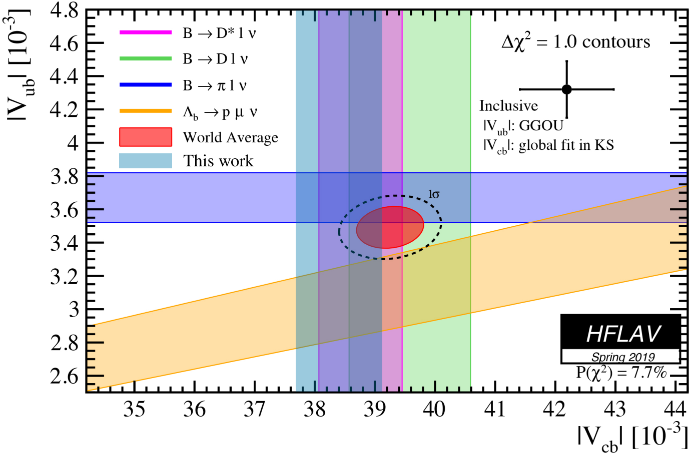


Joint fit



Unblinded, final result $|V_{cb}| = 38.40(74) \times 10^{-3}$

Results: Update of $|V_{ub}|$ vs $|V_{cb}|$

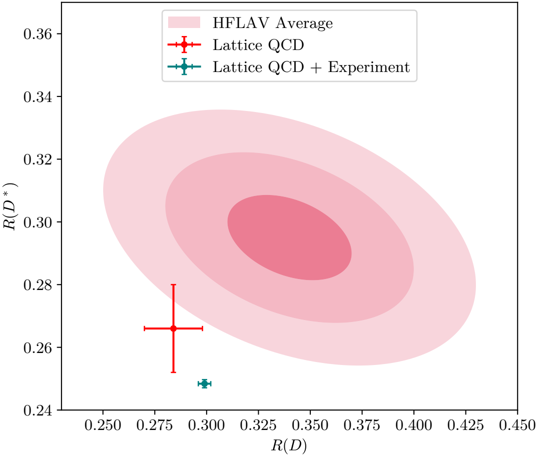


The $|V_{cb}|$ puzzle remains

Results: $R(D^*)$ in context

$$R(D^*)_{\mathbf{Lat}} = 0.265(13) \quad R(D^*)_{\mathbf{Lat+Exp}} = 0.2483(13)$$

Phys.Rev.D**92** (2015), 034506; Phys.Rev.D**100** (2019), 052007; Phys.Rev.D**103** (2021), 079901; Phys.Rev.Lett. **123** (2019), 091801



Conclusions

- This is the **first**, unquenched, completed $B \to D^* \ell\nu$ calculation at nonzero recoil on the lattice
- The **main new information of this analysis** comes from the behavior at small recoil of the form factors
- Our $|V_{cb}|$ agrees with previous determinations and the inclusive-exclusive tension remains unsolved
- Results show $R(D^*)$ very close to the **theoretical prediction** and a reduced tension with experiment
- Main sources of errors of our form factors are
  - Statistics
  - Light- and heavy-quark discretization errors
- Further lattice analysis and refinements of our analysis can potentially settle the $R(D^*)$ issue
  - Pending JLQCD calculation on $B \to D^{(*)} \ell\nu$ form factor on the lattice
  - Next FNAL/MILC calculation of $B_{(s)} \to D^{(*)}_{(s)} \ell\nu$ is in the queue

THANK YOU