Non-perturbative renormalization of the flavour-singlet local vector current with O(a)-improved Wilson fermions

M I C H E L E   P E P E

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IN COLLABORATION WITH

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C E R N (S w i t z e r l a n d)

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Introduction

- Lattice QCD: composite operators need, in general, renormalization

$$O^R = Z_O\, O^{latt} + \sum_k Z_{O_k}\, O_k^{latt} \qquad dim(O_k^{latt}) \leq dim(O^{latt})$$

Various schemes: RI'-MOM, Schrödinger functional, Wilson flow, …

- Shifted boundary conditions: very convenient scheme in several cases

$T_{\mu\nu}$
  - SU(3) Yang-Mills theory (no mixing)
  - QCD (with mixing)

V_μ
  - flavour – singlet vector current
  - flavour – non-singlet vector current

Using Ward Identities:
Bochicchio et al., NPB 262 (1985) 331

G. Martinelli, C. Sachrajda, A. Vladikas
NPB 358 (1991) 212

Recent accurate calculation in the SF framework:
M. Dalla Brida, T. Korzec, S. Sint, P. Vilaseca,
Eur. Phys. J. C79 (1) (2019) 23

- Unique advantage: 1-point functions, higher accuracy and cheaper

QFT with shifted boundary conditions

L. Giusti and H. Meyer,
PRL 2011, JHEP 2011 and 2013

- Kinematic conditions can be changed without affecting renormalization: finite temperature, fermionic phases (chemical potential), fixed boundary conditions, …

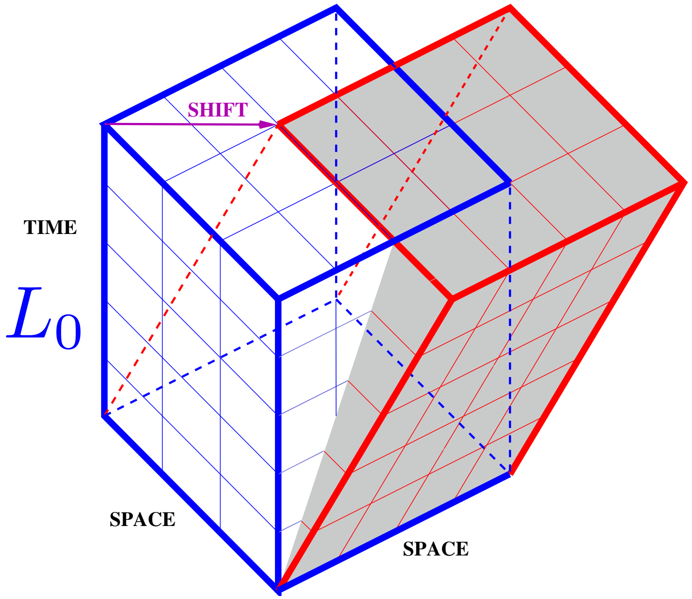
- Shifted boundary conditions: finite temperature + shift ξ in the temporal direction

A thermal quantum field theory
in a moving reference frame

Easy to implement

Monte Carlo simulations

Pert. calculations for improvement



$$U_\mu(x_0 + L_0, \mathbf{x}) = U_\mu(x_0, \mathbf{x} - L_0\xi) \qquad \overline{\psi}(x_0 + L_0, \mathbf{x}) = -\overline{\psi}(x_0, \mathbf{x} - L_0\xi) \qquad \psi(x_0 + L_0, \mathbf{x}) = -\psi(x_0, \mathbf{x} - L_0\xi)$$

$$U_\mu(x_0, \mathbf{x} + L_k\mathbf{k}) = U_\mu(x_0, \mathbf{x}) \qquad \overline{\psi}(x_0, \mathbf{x} + L_k\mathbf{k}) = \overline{\psi}(x_0, \mathbf{x}) \qquad \psi(x_0, \mathbf{x} + L_k\mathbf{k}) = \psi(x_0, \mathbf{x})$$

M. Dalla Brida, L. Giusti, M. Pepe,
JHEP 04 (2020) 043

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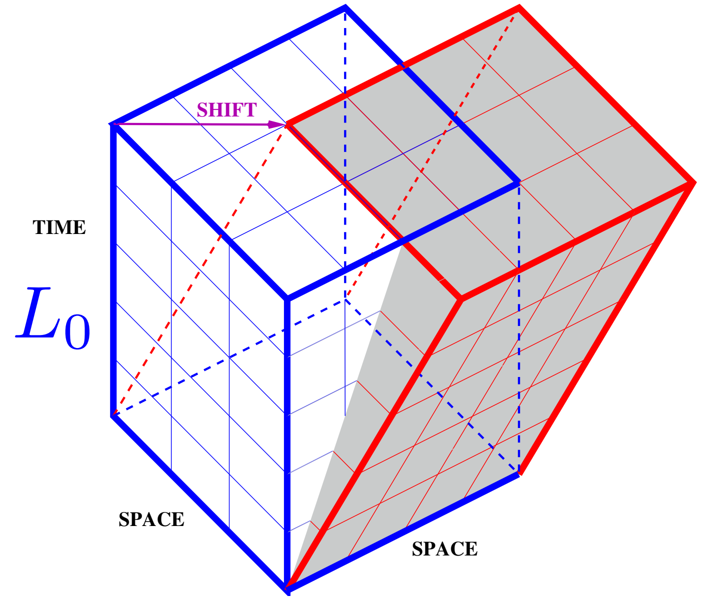

$$U_\mu(x_0 + L_0, \mathbf{x}) = U_\mu(x_0, \mathbf{x} - L_0\xi) \qquad \overline{\psi}(x_0 + L_0, \mathbf{x}) = -\,\mathrm{e}^{-i\theta_0}\,\overline{\psi}(x_0, \mathbf{x} - L_0\xi) \qquad \psi(x_0 + L_0, \mathbf{x}) = -\,\mathrm{e}^{i\theta_0}\,\psi(x_0, \mathbf{x} - L_0\xi)$$

$$U_\mu(x_0, \mathbf{x} + L_k\mathbf{k}) = U_\mu(x_0, \mathbf{x}) \qquad \overline{\psi}(x_0, \mathbf{x} + L_k\mathbf{k}) = \overline{\psi}(x_0, \mathbf{x}) \qquad \psi(x_0, \mathbf{x} + L_k\mathbf{k}) = \psi(x_0, \mathbf{x})$$

M. Dalla Brida, L. Giusti, M. Pepe,
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- We can switch on a phase θ_0 for fermion fields in the temporal direction

Flavour – singlet vector current

- Non-anomalous abelian part of QCD chiral symmetry: conserved on the lattice

$$V^c_\mu(x) = \frac{1}{2}\left[\overline{\psi}(x+\hat{\mu})\,U^\dagger_\mu(x)(\gamma_\mu+1)\,\psi(x) + \overline{\psi}(x)\,U^\dagger_\mu(x)(\gamma_\mu-1)\,\psi(x+\hat{\mu})\right]$$

in the vacuum: $\langle V^c_\mu(x)\rangle = 0$ but when $\theta_0 \neq 0$ then $\langle V^c_0(x)\rangle \neq 0$

One can also write $\langle V^c_0(x)\rangle = \frac{1}{V}\frac{\partial}{\partial\theta_0}\log Z$ where $Z = \int d\overline{\psi}\,d\psi\,dU\,\mathrm{e}^{-S_{QCD}}$

- local non-conserved definition

$V^l_\mu(x) = \overline{\psi}(x)\,\gamma_\mu\,\psi(x)$ but it renormalizes: $V^R_\mu(x) = Z_V V^c_\mu$

computationally simpler and possibly smaller lattice artifacts

- improved definition: $\hat{V}^{c,l}_\mu(x) = V^{c,l}_\mu(x) + a\,c^{c,l}_V\,\partial_\nu t_{\mu\nu} + \mathcal{O}(a^2)$

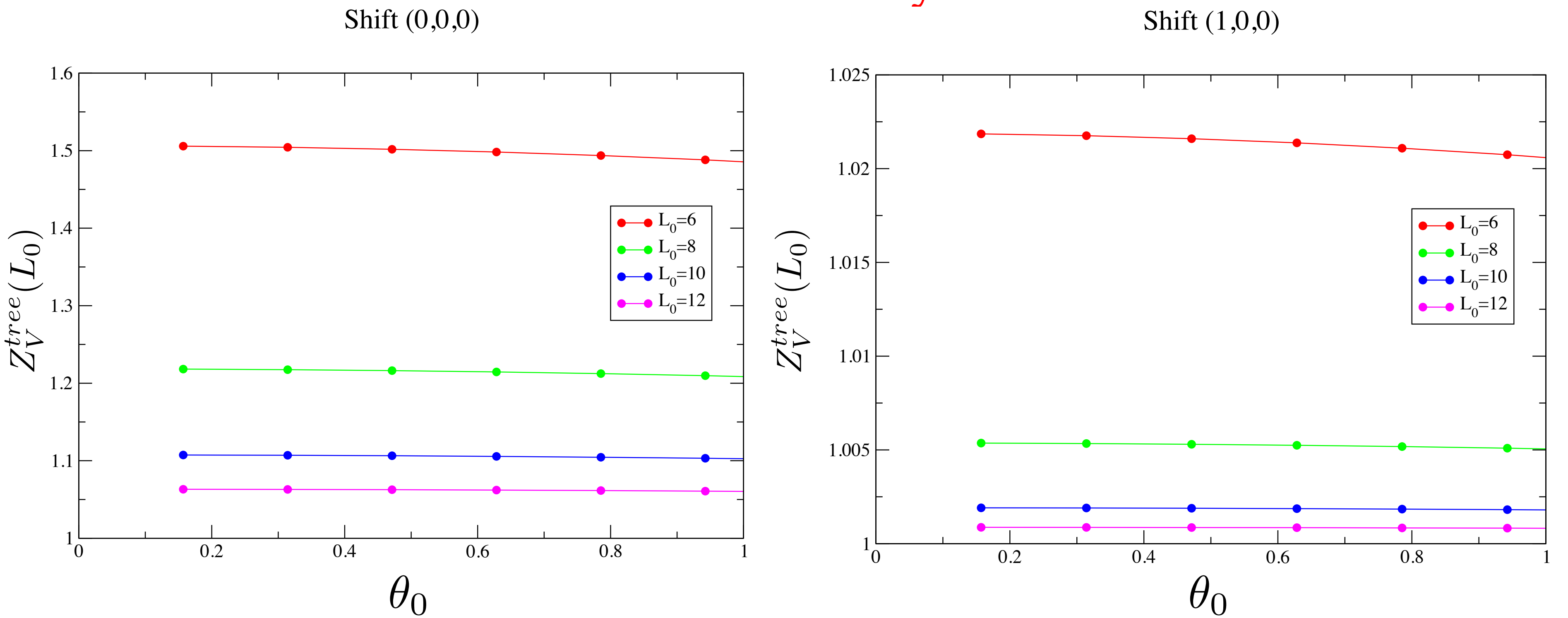
- One can define the renormalization constant as:

$$Z_V(g_0^2) = \lim_{1/L_0\to 0}\frac{\langle V^c_0\rangle}{\langle V^l_0\rangle}$$

it does not contribute

in principle no need for shifted boundary conditions, $\theta_0 \neq 0$ is enough but ….

Perturbation theory



- Largely different relevance of lattice artifacts
- It turns out to be more convenient to consider the shift $\xi = (1, 0, 0)$
- We consider a tree-level improved definition:

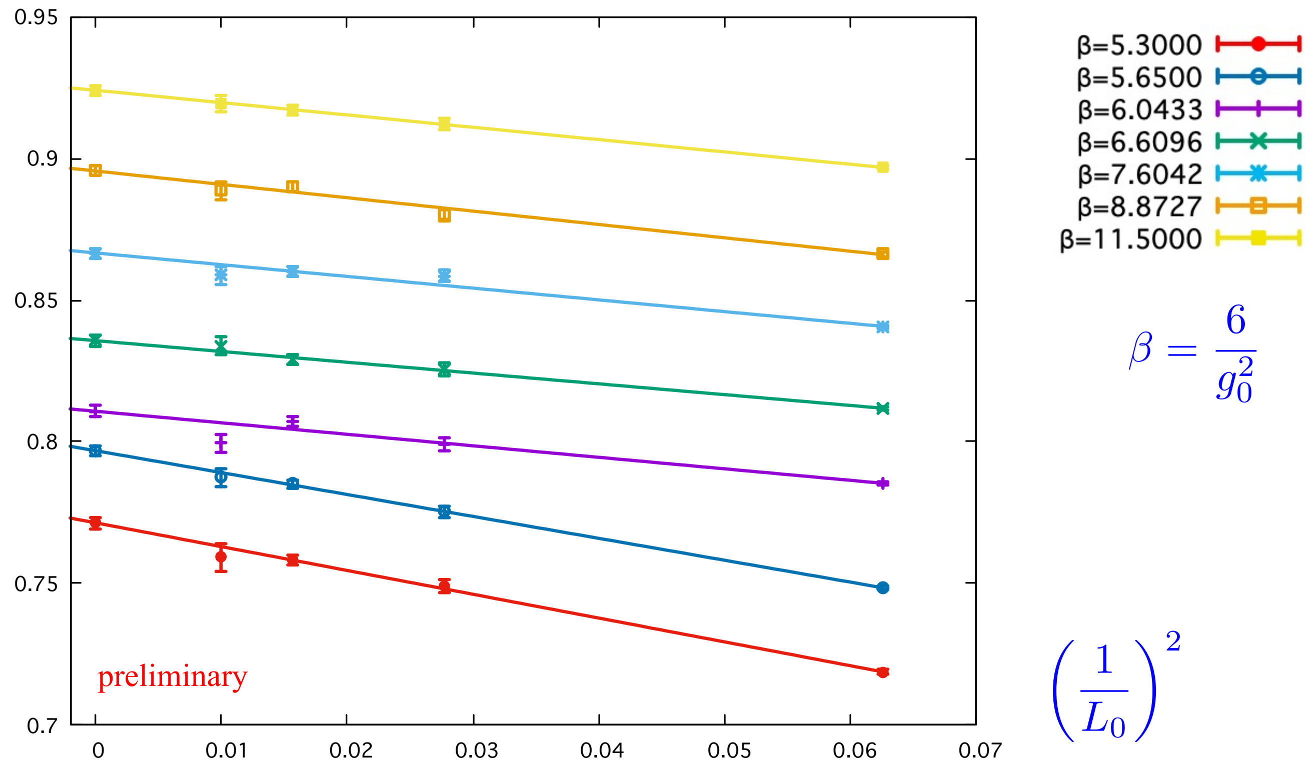
$$Z_V^{0-imp}(g_0^2, L_0) = \frac{\langle V_0^c \rangle}{\langle V_0^l \rangle} \frac{1}{Z_V^{tree}(L_0)} \qquad Z_V(g_0^2) = \lim_{1/L_0 \to 0} Z_V^{0-imp}(g_0^2, L_0)$$

- We have computed Z_V at 1-loop: better improvement in progress

The numerical study

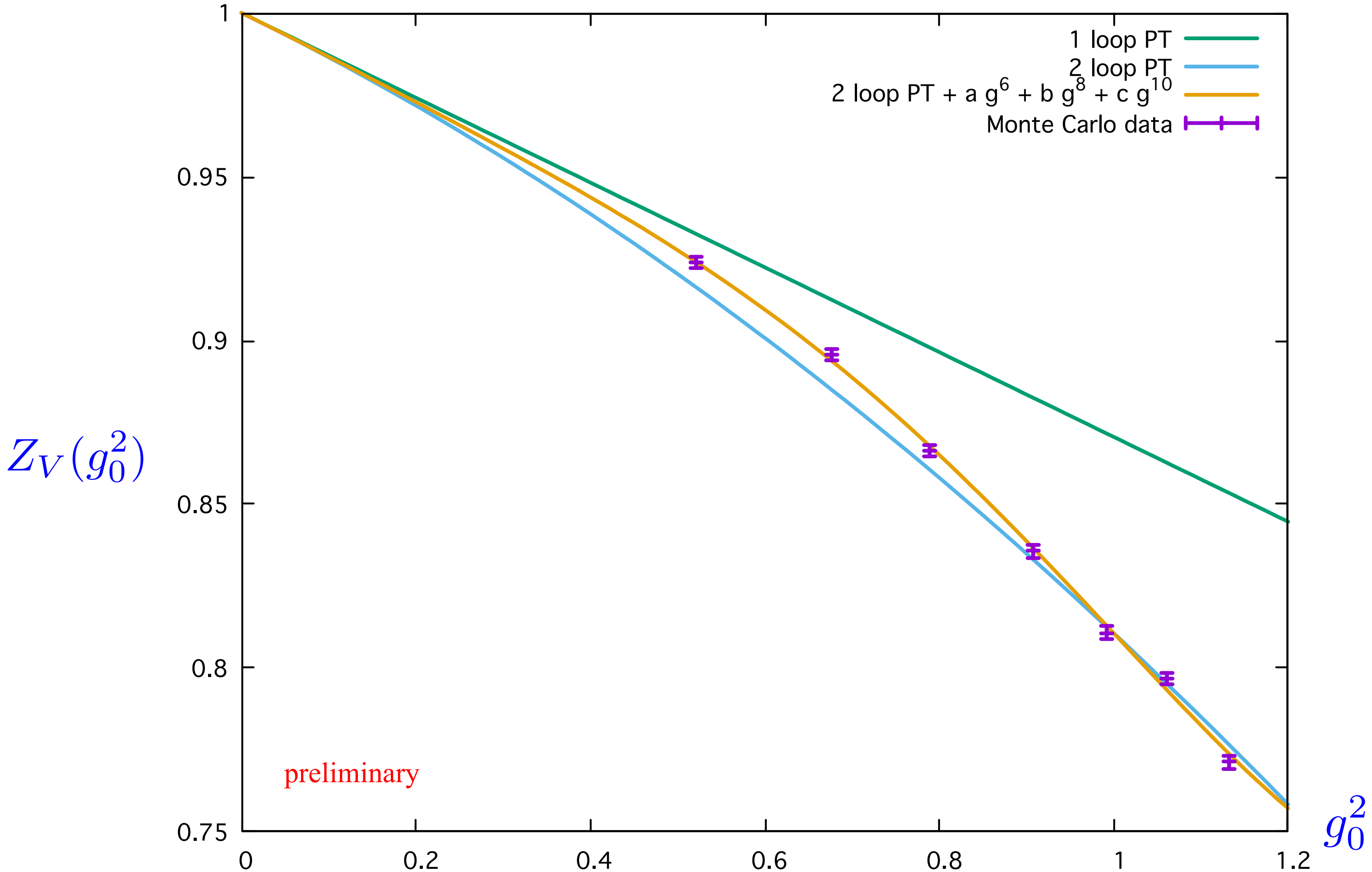
- $N_f = 3$ O(a)-improved massless Wilson fermions with the plaquette Wilson action
- Lattice size: $96^3 \times L_0$ with $L_0 = 4, 6, 8, 10$ at $\theta_0 = \frac{\pi}{6}$ and $\xi = (1, 0, 0)$
- We have considered 7 values of the gauge coupling g_0^2 in the range [0.52, 1.13]
- Extrapolation to $L_0 \to \infty$ turns out to be very good with a ~ 2.5‰ accuracy

$Z_V^{0-imp}(g_0^2, L_0)$



Statistics
L_0= 4, 6 ➔100 traj.
L_0= 8, 10 ➔700 traj.

The final result for Z_V



- The 2-loop perturbative result turns out to work fairly well at the level of 1%.

A. Skouroupathis, H. Panagopoulos
Phys.Rev.D 79 (2009) 094508

Conclusions and perspectives

- Shifted boundary conditions: convenient scheme to compute $Z_V(g_0^2)$ non-perturbatively. First application in QCD.

- Final accuracy at a few ‰ level thanks to 1-point observable and small lattice artifacts.

- Quite good agreement agreement with 2-loop Perturbation Theory at 1% level

- The same method can be used for the flavour non-singlet current

- Work in progress: similar approach to compute the renormalization constants of the energy-momentum tensor in QCD ➔ QCD thermodynamics up to T ~ 100 GeV