Semileptonic $D \to \pi \ell \nu$, $D \to K \ell \nu$ and $D_s \to K \ell \nu$ decays with 2+1f domain wall fermions

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Related RBC/UKQCD talks

- Felix Erben: BSM $B - \bar{B}$ mixing [Wed 06:15 EST]
- Ryan Hill: Semileptonic form factors for $B \to \pi \ell \nu$ decays [Thu 13:45 EST]
- Jonathan Flynn: Form factors for semileptonic $B_s \to K$ and $B_s \to D_s$ decays [Thu 14:00 EST]

This work used the DiRAC Extreme Scaling HPC Service (https://www.dirac.ac.uk)

Data produced using Grid [1] and Hadrons [2]

Outline

1. Introduction
   - Heavy-light semileptonic decays
   - Lattice set up
2. Results
   - Point-wall diagonalisation study
   - First three-point results – R_1 and R_2 ratios
3. Outlook
   - Next steps
   - Summary and outlook

Heavy-light semileptonic decays

We are studying exclusive semileptonic decays of $D_{(s)}$ mesons to $K/\pi\ell\nu$ final states.

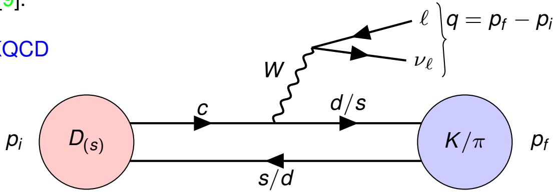
We are computing the form factors parameterising these decays, over the entire physical q^2 range.

When combined with data from experiments such as CLEO-c and BESIII, this will allow us to extract the CKM matrix elements $|V_{cs}|$ and $|V_{cd}|$.

Precise determinations of these quantities are interesting because ultimately they help constrain BSM physics.

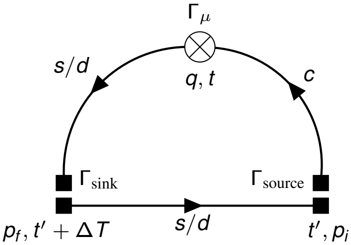
Other collaborations are working towards the same aim, e.g. ETM [3] [4], FermiLab/MILC [5], HPQCD [6] [7] [8] and JLQCD [9].

I present a complementary approach from RBC/UKQCD using domain wall fermions.



Lattice setup

- All domain-wall fermions
- RBC-UKQCD 2+1 flavour [10]
- Stout-smeared Möebius charm
- Shamir strange and down



Exploratory, two-point study

- $\mathbb{Z}_2$ [11] stochastic average **point**-like sources
- Coulomb gauge-fixed **wall**-sources
- "Point-wall" diagonalisation
- I.e. optimal linear combination to remove excited-states
- C1: $a^{-1} = 1.785(5)$ GeV; $(L/a)^3 \cdot T/a = 24^3 \cdot 64$; $m_\pi \approx 340\mathrm{MeV}$
  - 35 configurations $\times$ 16 timeslices

First three-point results

- $D \to \pi\ell\nu$, $D \to K\ell\nu$ and $D_s \to K\ell\nu$
- M1: $a^{-1} = 2.383(9)$ GeV; $(L/a)^3 \cdot T/a = 32^3 \cdot 64$; $m_\pi \approx 304\mathrm{MeV}$
  - 128 configurations $\times$ 1 timeslice

Point-wall diagonalisation – good results from two-point studies

Two-point correlation functions for point/wall interpolating operators with overlap coefficients $A_{f,n} = \langle \Omega | \hat{O}_f | n \rangle$

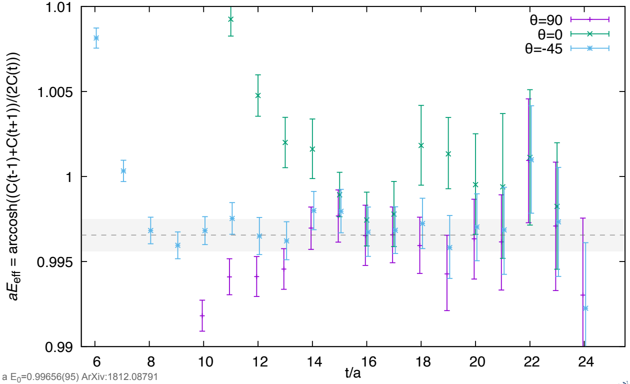
$$C^{(2)}_{fi}(t) = \sum_{n=0}^{\infty} \frac{A_{f,n} A^*_{i,n}}{2E_n} \left(e^{-E_n t} \pm e^{-E_n (T-t)} \right)$$

Linear combinations can be formed to reduce excited-state contamination

$$C^{(2)}_{\text{mixed}}(t) = \frac{C_{PP}}{A_{P,1}} - \frac{C_{WP}}{A_{W,1}}$$

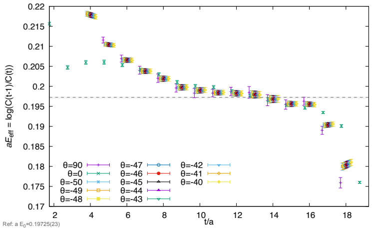
Exploratory study found mixed operators

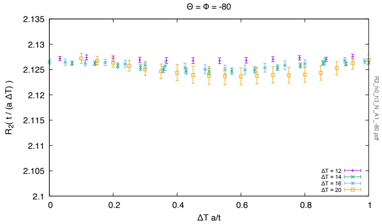
- Plateau earlier than their components
- Compatible with published results
- Showed far smaller error



a E_0=0.99656(95) ArXiv:1812.08791

Point-wall diagonalisation – no clear improvement over wall-sources





Heavy-light three-point functions with current V_μ have the form

$$C^{(3)}_{i\to f}(\Delta T, t) = \sum_{m,n=0}^{\infty} \frac{A_{f,m}A_{i,n}}{4E_{f,m}E_{i,n}} \langle P_f, m|V_\mu|P_i, n\rangle \, e^{-(E_{i,n}-E_{f,m})t}\, e^{-E_{f,m}\Delta T}$$

Linear combinations can again be formed

$$C^{3}_{\rm mixed} \approx \frac{(\gamma P + \delta W)(\alpha P + \beta W)}{4E_f E_i} \langle P_f|V_\mu|P_i\rangle \, e^{-(E_i-E_f)t}\, e^{-E_f\Delta T}$$

We introduced tunable mixing angles ϕ at sink and θ and source

$$\alpha = \frac{\cos\theta}{P_{i_1}} \qquad \beta = \frac{\sin\theta}{W_{i_1}} \qquad \gamma = \frac{\cos\phi}{P_{f_1}} \qquad \delta = \frac{\sin\theta}{W_{f_1}}$$

but found that the optimal mixing angle involved using wall only

Ratios used to examine three-point data

Correlators are constructed for vector current decays
($q = p_f - p_i$ is the momentum transfer to the lepton pair, and $\vec{p} = \frac{2\pi}{L}\vec{n}$)

$$\langle P_f(p_f)|\mathcal{V}^\mu\left(q^2\right)|P_i(p_i)\rangle = f_+\left(q^2\right)(p_i + p_f)^\mu + f_-\left(q^2\right)(p_i - p_f)^\mu$$

We construct (symmetric) double-ratios to extract the matrix element [12]

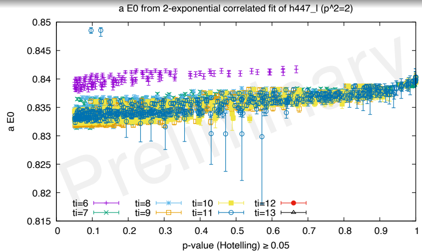
$$R^\mu_\alpha(p_i, p_i) = 2\sqrt{\frac{E_i E_f}{D_\alpha}}\sqrt{C^\mu_{i\to f}(p_i, p_f)\ C^\mu_{f\to i}(p_f, p_i)}$$
$$\approx Z_V \langle P_f(p_f)|V^\mu\left(q^2\right)|P_i(p_i)\rangle$$

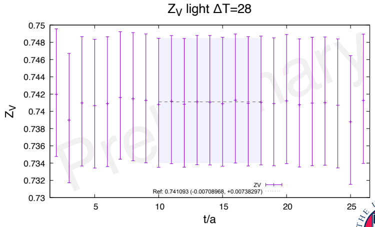
Where we have a choice of denominator using two-point functions $C(p)$

$$R_1 : D_1 = C_i(p_i)\ C_f(p_f)/\left(Z_{V,\mathrm{heavy}} Z_{V,\mathrm{light}}\right)$$

or using three-point functions $C^\mu_{i\to f}$

$$R_2 : D_2 = C^0_{i\to i}(p_i, p_i)\ C^0_{f\to f}(p_f, p_f)$$

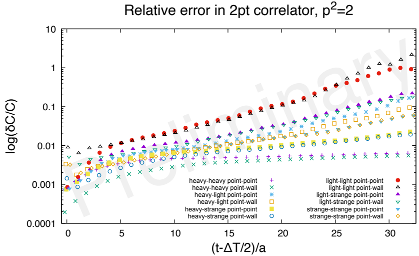


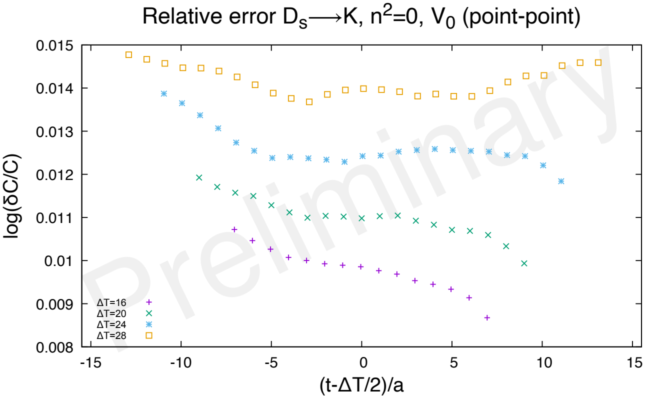


Energies and Z_V extracted

Quality of the raw data

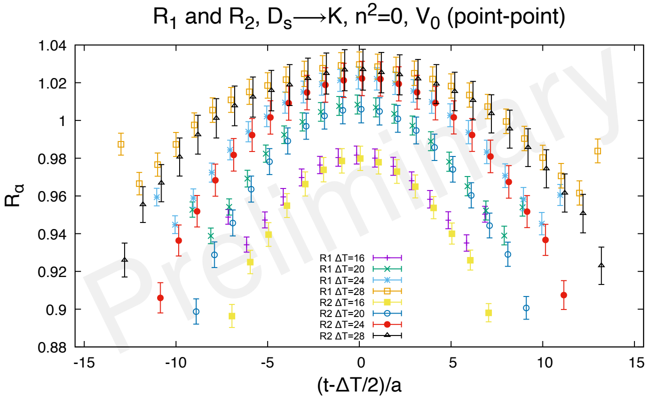
- Error on 3-pt data grows with ΔT
- 2-pt data from timeslice ΔT enters R_1
- δZ_V is small and can be ignored here
- Wall-separation determines ratio error





Error growth $\Longrightarrow$ source-sink separation ΔT must be minimised to obtain %-scale errors

R_1 and R_2 ratios, $D_s \to K$



Observations

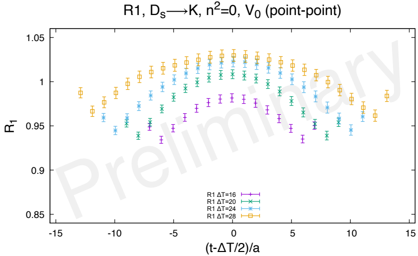
- The R_1 and R_2 ratios are compatible
- Statistical errors of the same order
- Excited-state contamination low ΔT
- Saturates at high ΔT
- We observe error growth at higher ΔT

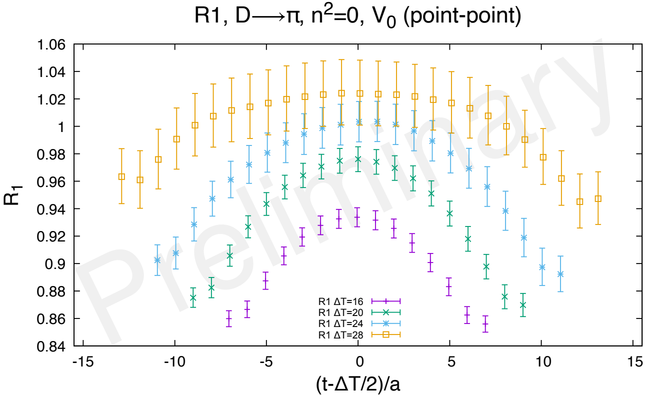
R_1 is $\sim 2\times$ cheaper to produce, so we use R_1 exclusively

Extending to other decays, $D \to \pi$

Observations

- Noise grows as $m_{P_f} \downarrow$
- Excited-states behaviour more pronounced
- Wall data may be better – not yet analysed

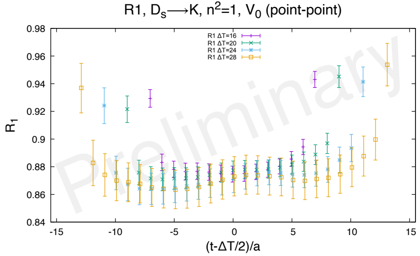


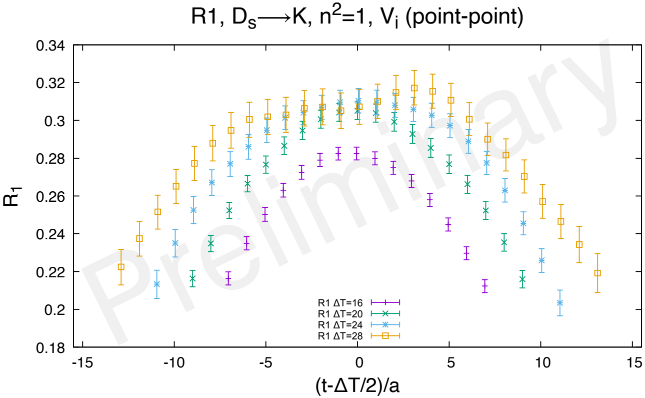


$\Longrightarrow$ Increase statistics and carry out simultaneous fits to multiple ΔT

Non-zero momentum

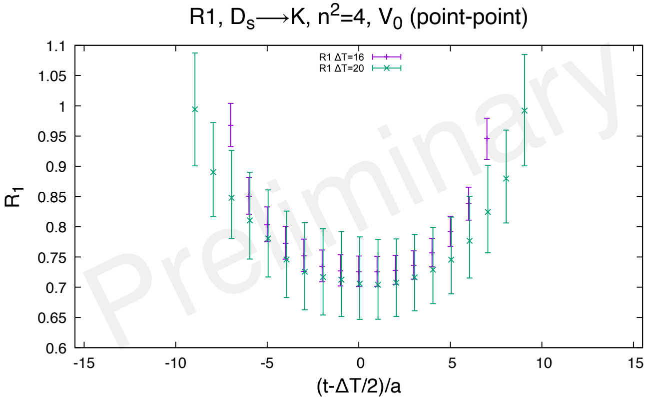
- Increased data collection will be needed
- $\sqrt{N}$ scaling alone prohibitively costly
- Noise for larger ΔT motivates excited-state modelling at smaller ΔT
- We are seeking the right balance
  - Reduced statistical error at smaller ΔT
  - increased systematics arising from data with higher excited state contamination





We see a signal over the entire physical q^2 range

$D_s \to K$ at maximum momentum transfer (temporal)



Observations

- As expected, the signal is very noisy
- $\Delta T \geq 24$ have been removed from the diagram because noise swamps signal

We are investigating

- Whether at higher momenta we are better off producing data over a smaller range of ΔT in finer increments
- Will produce more data to investigate $\Delta T = 16, 17, 18, 19, 20$ simultaneous fit

Wall separation choices for $p^2 \neq 0$ may differ

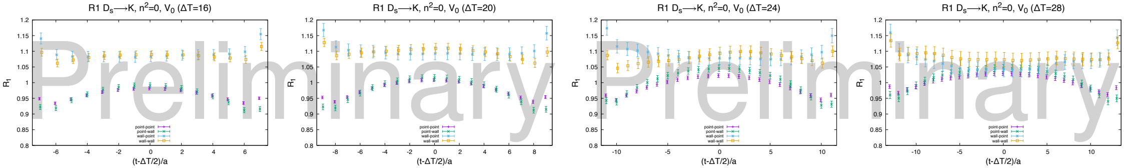
Devising a fitting strategy

Analysis and fitting strategies

- Decay channels show excited-state contamination
- Simultaneous fits using multiple ΔT and operators (approach from above and below)
- Quantify tradeoffs of systematics vs stat error
- Extract q^2 dependence of form factors

Data production

- Complete data over all ensembles
- After optimal strategy has been fully determined on this ensemble
- Increase statistics



Summary and outlook

Achieved to date

- Point-wall diagonalisation study promising for two-point functions
- Did not translate to three-point functions – wall-sources are close to optimal
- Data produced for $D_{(s)} \to K/\pi$ semileptonic decays on first ensemble
- Analysis in progress

Outlook

- The target result is the q^2-dependence of the $D_{(s)} \to K/\pi$ form factors
- Covering the entire physical q^2 range
- Current data indicates percent-scale errors are achievable
- We expect to be able to address systematics with simultaneous fits of multiple ΔT and operators

This work used the DiRAC Extreme Scaling HPC Service (`https://www.dirac.ac.uk`)

Data produced using Grid [1] and Hadrons [2]

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Ratio cancellation – R_1

$$R_1^{\mu}\left(t_i,t,t_f,\mathbf{p}_i,\mathbf{p}_f\right)=2\sqrt{E_iE_f}\sqrt{\frac{C^{\mu}_{P_iP_f}\left(t_i,t,t_f,\mathbf{p}_i,\mathbf{p}_f\right)C^{\mu}_{P_fP_i}\left(t_i,t,t_f,\mathbf{p}_f,\mathbf{p}_i\right)}{\tilde{C}_{P_i}\left(t_f-t_i,\mathbf{p}_i\right)\tilde{C}_{P_f}\left(t_f-t_i,\mathbf{p}_f\right)}} \tag{1}$$

i.e.

$$R_1=\cancel{\sqrt{4E_iE_f}}\sqrt{\frac{Z_V\frac{Z_i^*Z_f}{\cancel{4E_iE_f}}Z_V\frac{Z_f^*Z_i}{4E_fE_i}}{\frac{|Z_i|^2}{2E_i}\frac{|Z_f|^2}{2E_f}}}\left\langle P_f\left(\mathbf{p}_f\right)\right|V_\mu\left(\mathbf{q}^2\right)\left|P_i\left(\mathbf{p}_i\right)\right\rangle \tag{2}$$

$$\sqrt{\frac{e^{-E_i(t-t_i)}e^{-E_f(t_f-t)}e^{-E_f(t-t_i)}e^{-E_i(t_f-t)}}{e^{-E_i(t_f-t_i)}e^{-E_f(t_f-t_i)}}} \tag{3}$$

$$=Z_V\left\langle P_f\left(\mathbf{p}_f\right)\right|V_\mu\left(\mathbf{q}^2\right)\left|P_i\left(\mathbf{p}_i\right)\right\rangle\sqrt{\frac{e^{-E_f(t_f-t_i)}e^{-E_i(t_f-t_i)}}{e^{-E_i(t_f-t_i)}e^{-E_f(t_f-t_i)}}} \tag{4}$$

i.e. R_1 yields the renormalised matrix element when built with renormalised 3-pt correlators

$$R_1=Z_V\left\langle P_f\left(\mathbf{p}_f\right)\right|V_\mu\left(\mathbf{q}\right)\left|P_i\left(\mathbf{p}_i\right)\right\rangle \tag{5}$$

More usefully, we can construct the ratio from bare correlators and multiply by Z_V at the end

Ratio cancellation – R_2

$$R_2^{\mu}\left(t, \mathbf{p}_{P_f}, \mathbf{p}_{P_i}\right) = 2\sqrt{E_i E_f}\sqrt{\frac{C^{\mu}_{P_i P_f}\left(t_i, t, t_f, \mathbf{p}_i, \mathbf{p}_f\right) C^{\mu}_{P_f P_i}\left(t_i, t, t_f, \mathbf{p}_f, \mathbf{p}_i\right)}{C^{0}_{P_i P_i}\left(t_i, t, t_f, \mathbf{p}_i, \mathbf{p}_i\right) C^{0}_{P_f P_f}\left(t_i, t, t_f, \mathbf{p}_f, \mathbf{p}_f\right)}} \tag{6}$$

i.e.

$$R_2 = \sqrt{\frac{Z_V \frac{Z_i^* Z_f}{4E_i E_f} Z_V \frac{Z_f^* Z_i}{4E_f E_i}}{\frac{|Z_i|^2}{4E_i^2}\frac{|Z_f|^2}{4E_f^2}}}\sqrt{\frac{\mathrm{e}^{-E_i(t-t_i)}\,\cancel{\mathrm{e}^{-E_f(t_f-t)}}\,\mathrm{e}^{-E_f(t-t_i)}\,\cancel{\mathrm{e}^{-E_i(t_f-t)}}}{\mathrm{e}^{-E_i(t-t_i)}\,\cancel{\mathrm{e}^{-E_i(t_f-t)}}\,\mathrm{e}^{-E_f(t-t_i)}\,\cancel{\mathrm{e}^{-E_f(t_f-t)}}}} \tag{7}$$

$$\left(\frac{\langle P_f(\mathbf{p}_f)|V_\mu(\mathbf{q})|P_i(\mathbf{p}_i)\rangle\sqrt{4\,E_i E_f}}{\sqrt{Z_V\langle P_i(\mathbf{p}_i)|V_\mu(\mathbf{0})|P_i(\mathbf{p}_i)\rangle Z_V\langle P_f(\mathbf{p}_f)|V_\mu(\mathbf{0})|P_f(\mathbf{p}_f)\rangle}}\right) \tag{8}$$

Bearing in mind (11), all the extraneous factors cancel, leaving

$$R_2 = Z_V\langle P_f(\mathbf{p}_f)|V_\mu(\mathbf{q})|P_i(\mathbf{p}_i)\rangle \tag{9}$$

NB: Whether we construct the ratio from the bare correlators or not, we do not need to multiply by Z_V at the end

Extracting Z_V

$$Z_V \langle P_f(\mathbf{p}_f)| V_\mu \left(\mathbf{q}^2\right) | P_i(\mathbf{p}_i)\rangle = f_+^{P_i P_f}\left(q^2\right)(p_i + p_f)_\mu + f_-^{P_i P_f}\left(q^2\right)(p_i - p_f)_\mu \tag{10}$$

Charge conservation ($f_+^{P_i P_i} = 1$), $q^2 = 0$ and rest frame $\implies$ $p_i + p_f = (2E_i, \mathbf{0})$ and $p_i - p_f = 0$ give

$$Z_V = \frac{2E_i}{\langle P_i(\mathbf{0})| V_0(\mathbf{0}) | P_i(\mathbf{0})\rangle} \tag{11}$$

$$\tilde{C}_i(t, \mathbf{p}) \equiv C_i(t, \mathbf{p}_i) - \frac{1}{2} C_i\left(\frac{T}{2}, \mathbf{p}\right) \mathrm{e}^{-E_i(T/2 - t)} \tag{12}$$

$$\tilde{C}_i(t, \mathbf{p}) \simeq \frac{|Z_i|^2}{2E_i} \mathrm{e}^{-E_i t} \qquad \text{(away from the midpoint – } \times 2 \text{ at midpoint)} \tag{13}$$

We use this to extract Z_V by taking a ratio of (13) over the three-point correlator

$$\frac{\tilde{C}_{P_i}(t_f - t_i, \mathbf{0})}{C^{(0)}_{P_i P_i \text{bare}}(t_i, t, t_f, \mathbf{0}, \mathbf{0})} = \frac{\frac{|Z_i|^2}{2E_i} \mathrm{e}^{-E_i(t_f - t_i)}}{\frac{|Z_i|^2}{4E_i^2} \langle P_i(\mathbf{0})| V_0(\mathbf{0}) | P_i(\mathbf{0})\rangle \mathrm{e}^{-E_i(t - t_i + t_f - t)}} \tag{14}$$

$$= \frac{2E_i}{\langle P_i(\mathbf{0})| V_0(\mathbf{0}) | P_i(\mathbf{0})\rangle} = Z_V \tag{15}$$

$D_s \to K$ at maximum momentum transfer (spatial)

- Consistent with $n^2 = 4$ temporal