Precision $B^* B\pi$ coupling from three flavor lattice QCD

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Lattice 21 July 29, 2021

ALPHA Collaboration

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THE $B^*B\pi$ COUPLING IN THE STATIC APPROXIMATION

- Interactions of heavy-light mesons and soft pseudo-Goldstone bosons in **Heavy Meson χPT** at lowest order determined by single low energy constant $\hat{g}$,

$$L^{\text{int}}_{\text{HM}\chi\text{PT}} = \hat{g}\,\text{Tr}\left(\bar{H}_a H_b \mathcal{A}^{\mu}_{ab}\gamma_\mu\gamma_5\right) .$$

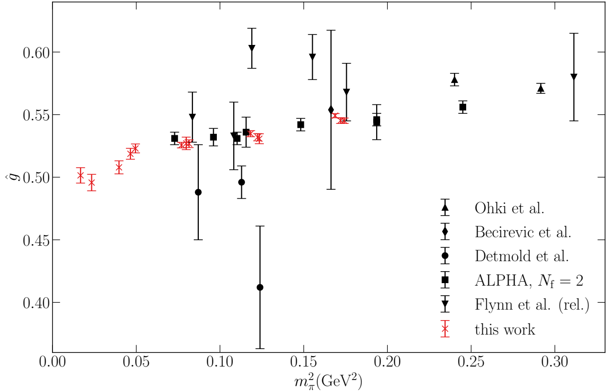
- $\hat{g}$ can be related to the matrix element of the forbidden strong decay $B^* \to B\pi$.

- Compute $\hat{g}$ from matrix element of the light-light axial current

$$\hat{g} = \frac{1}{2}\langle B^0(\mathbf{0})|\hat{A}_k(0)|B^{*+}_k(\mathbf{0})\rangle$$

  with **static heavy quarks** and extrapolate to **chiral light quarks**.

- A precise determination of the coupling $\hat{g}$ is of use in B physics in general and in particular in the ALPHA HQET program.



- Systematic uncertainty in existing work due to far extrapolation to chiral limit ($m_\pi \geq 270\,\mathrm{MeV}$).
- Systematic uncertainties in the extraction of $|B\pi\rangle$ states due to excited state contamination.

- Static quarks introduce significant statistical uncertainties.

Computation of the matrix element

- Have to suppress excited states to extract matrix element at short distances.
- Follow similar strategy as in [ALPHA, 1404.6951]: summed GEVP method.
- Obtain matrix of summed three-point functions

$$D^{3\mathrm{pt}}_{ij}(t) = a^3 \sum_{t_1} \langle (B_i^*)_k\,(t)\,(A_\mathrm{R})_k(t_1)\,B_j^\dagger(0)\rangle\,,$$

  together with the matrix of two-point functions $C^{2\mathrm{pt}}_{ij}(t) = \langle B_i(t) B_j^\dagger(0)\rangle$

- Determine the solution of the GEVP

$$C^{2\mathrm{pt}}(t)v_n(t,t_0) = \lambda_n(t,t_0)C^{2\mathrm{pt}}(t_0)v_n(t,t_0)$$

Computation of the matrix element - GEVP

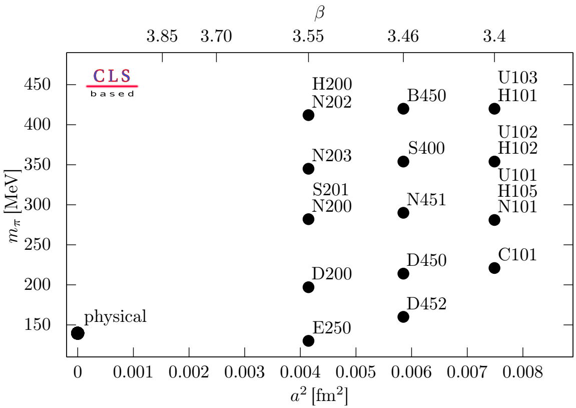
- Extract the effective matrix element via the summed GEVP [ALPHA, 0902.1265], [ALPHA, 1108.3774] with $t_0 = t/2$,

$$M_n^{\mathrm{eff}}(t, t_0) = -\frac{1}{2}\partial_t \frac{\left(v_n(t), \left[\frac{D^{3\mathrm{pt}}(t)}{\lambda_n(t)} - D^{3\mathrm{pt}}(t_0)\right] v_n(t)\right)}{(v_n(t), C^{2\mathrm{pt}}(t_0) v_n(t))} = \hat{g}_{nn} + \mathrm{O}(e^{-(E_{N+1}-E_n)t})$$

- Excited states are suppressed with $E_{N+1} - E_n > 1\,\mathrm{GeV}$ for $n = 1$ and $N > 3$, contamination by multi-pion states not excluded.

- Construction of the variational basis from smeared quark fields:
  - Gaussian smearing with APE smeared gauge links.
  - smearing via 3D scalar and spinor fields [Papinutto et al, 1807.08714].

$2+1$ FLAVOR CLS ENSEMBLES



- Three resolutions on the $\mathrm{Tr}[M_\mathrm{q}] = \mathrm{const.}$ trajectory of the $N_\mathrm{f} = 2+1$ CLS ensembles [Bruno et al, 1411.3982].
- $\mathrm{O}(a)$ clover improved Wilson quarks and tree-level improved Lüscher-Weisz gluons.
- Open or periodic boundary conditions in time direction.

- Excellent coverage of the pion mass dependence down to the physical point.
- Determine finite-volume effects from variation of volumes.

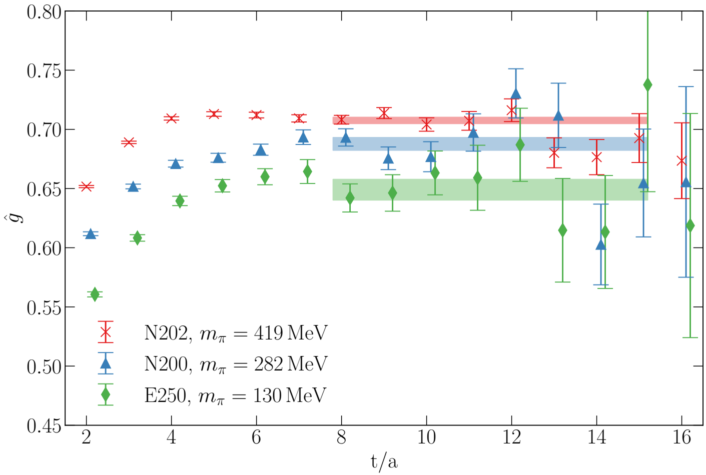
Computation of the matrix element

- Time diluted stochastic sources for the light quarks.
- Sequential propagators for the three-point functions.
- Two HYP static actions to tame the exponential increase of noise-to-signal ratio [ALPHA, hep-lat/0506008] $\rightarrow$ test $\mathrm{O}(a^2)$ effects.
- Determine the renormalized and $\mathrm{O}(a)$ improved matrix element

$$M_{n,R} = Z_\mathrm{A}(1 + b_\mathrm{A} a m_\mathrm{q} + \bar{b}_\mathrm{A} a \mathrm{Tr}\,[M_\mathrm{q}]) M_n^\mathrm{eff}$$

via

  - Renormalization constant Z_A [Dalla Brida et al., 1808.09236].
  - Improvement coefficients $b_\mathrm{A}, \bar{b}_\mathrm{A}$ [Bali et al., 2106.05398], [Bali et al., to appear]
  - Critical hopping parameter $\kappa_\mathrm{crit.}$ [Gérardin et al., 1811.08209]



- Representative extraction of the bare matrix elements at $a \approx 0.064\,\mathrm{fm}$ over the full range of pion masses.
- Fast degradation of the signal.
- GEVP allows extraction at short source-sink distance.
- Impact of t_{min} is monitored in global fit.

Extrapolation to the chiral-continuum limit

- We determine the effective coupling $\hat{g}_{\text{eff}}$ at finite pion mass, lattice spacing and volume. Parameterization based on [Detmold et al, 1108.5594],

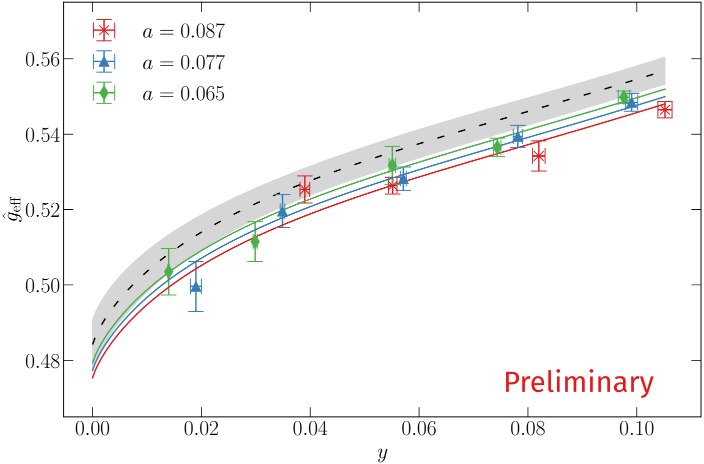
$$\hat{g}_{\text{eff}} \equiv \hat{g}_\chi \Bigg[1 - (1 + 2\hat{g}_\chi^2)\, y \log y + c_1 y + c_2 y^2 + \dots \qquad \textit{chiral dependence}$$

$$+ \hat{g}_\chi^2\, y\, \mathrm{O}\left(\frac{\mathrm{e}^{-m_\pi L}}{(m_\pi L)^{1/2}}\right) + y\, \mathrm{O}\left(\frac{\mathrm{e}^{-m_\pi L}}{(m_\pi L)^{3/2}}\right) + \dots \qquad \textit{finite volume effects}$$

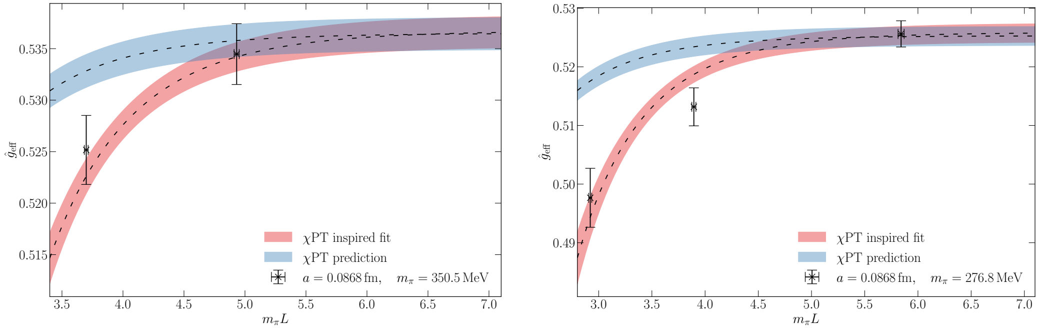
$$+ c_a a^2 + \dots \Bigg] \qquad \textit{cutoff effects}$$

with $y \equiv \frac{m_\pi^2}{8\pi^2 f_\pi^2} \in \{0.013 \dots 0.105\}$, $a \in \{0.065\,\mathrm{fm} \dots 0.087\,\mathrm{fm}\}$, $m_\pi L \in \{2.9 \dots 6.4\}$.

- Perform global fit to extract $\hat{g}_\chi$.



- Describe data over complete range of pion masses.
- Hard to constrain the chiral regime.
- Systematic uncertainties have to be quantified.
- Cutoff and finite-volume (*not in graph*) effects small and described by our fit.



$$\hat{g}_{\mathrm{eff}}^{\mathrm{FV}} = \hat{g}_\chi \left[1 + \hat{g}_\chi^2\, y\, \mathrm{O}\left(\mathrm{e}^{-m_\pi L}, (m_\pi L)^{-\frac{1}{2}}\right) + c_{\mathrm{FV}}\, y\, \mathrm{O}\left(\mathrm{e}^{-m_\pi L}, (m_\pi L)^{-\frac{3}{2}}\right)\right]$$

- Comparison of χPT prediction ($c_{\mathrm{FV}} = 1$) versus fit with fit with parameter c_{FV}.
- No significant influence on chiral-continuum extrapolated result.

Non-perturbative HQET: Status

- We will be able to determine $\hat{g}_\chi$ with much improved precision and better controlled systematics.

- This is the first step in the ALPHA program for B physics with $2+1$ flavors of light quarks.
- Non-perturbative finite-volume matching of QCD and HQET at $\mathrm{O}(1/m_{\mathrm{h}})$.
- Some data in the effective theory already taken.
- Next step: m_{b} and $f_{\mathrm{B}_{(\mathrm{s})}}$. Afterwards: Form factors of $B \to \pi$, $B_{\mathrm{s}} \to K$, $B_{\mathrm{s}} \to D_{\mathrm{s}}$