Determination of the light, strange and charm quark masses using twisted mass fermions

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Extended Twisted Mass Collaboration

ETMC: C. A., S. Bacchio, G. Bergner, M. Constantinou, M. Di Carlo, P. Dimopoulos, J. Finkenrath, E. Fiorenza, R. Frezzotti, M. Garofalo, K. Hadjiyiannakou, B. Kostrzewa, G. Koutsou, K. Jansen, V. Lubicz, M. Mangin-Brinet, F. Manigrasso, G. Martinelli, E. Papadiofantous, F. Pittler, G.C. Rossi, F. Sanfilippo, S. Simula, C. Tarantino, A. Todaro, C. Urbach, U. Wenger, arXiv:**2104.13408**,

LATTICE 21
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STIMULATE
European Joint Doctorates

Approach

- Perform analysis of $N_f=2+1+1$ twisted mass fermions gauge ensembles simulated
  - at three values of the lattice spacing
  - with a range of light quark masses in the range of 135 MeV to 350 MeV
  - strange and charm quark mass tuned to approximately their physical value

- Employ $N_f=4$ twisted mass fermion gauge ensembles for the computation of the renormalisation constants - see talk by M. Di Carlo, Friday 6:15 am

- Extract the quark masses using two setups
  - the meson sector
  - the baryon sector

- Main improvements as compared to our previous work
  - two ensembles at the physical point
  - more accurate determination of renormalisation constants
  - extraction using baryon sector

Lattice ensembles

N_f=2+1+1 twisted mass fermions with a clover term

Generated by the European Twisted Mass Collaboration (ETMC)

Abdel-Rehim *et al.* (ETMC) PRD 95, 094515 (2017), arXiv: 1507.05068
C. A. *et al.* (ETMC) PRD 98, 054518 (20118), arXiv:1807.00495
C. A. *et al.* (ETMC), arXiv: 2104.06747

Ensemble	$L^3 \times T$	MDUs	$a\mu_\ell$	af_π	$m_\pi L$	m_N/m_π	m_π [MeV]
A-ensembles: $\beta = 1.726,\ c_{SW} = 1.74,\ a\mu_\sigma = 0.1408,\ a\mu_\delta = 0.1521,\ w_0/a = 1.8352\ (35)$							
cA211.53.24	$24^3 \times 48$	5026	0.00530	0.07106 (36)	3.99	–	346.4 (1.6)
cA211.40.24	$24^3 \times 48$	5298	0.00400	0.06809 (30)	3.47	–	301.6 (2.1)
cA211.30.32	$32^3 \times 64$	10234	0.00300	0.06674 (15)	4.01	4.049 (14)	261.1 (1.1)
cA211.12.48	$48^3 \times 96$	2936	0.00120	0.06133 (33)	3.85	5.685 (28)	167.1 (0.8)
B-ensembles: $\beta = 1.778,\ c_{SW} = 1.69,\ a\mu_\sigma = 0.1246864,\ a\mu_\delta = 0.1315052,\ w_0/a = 2.1299\ (16)$							
cB211.25.32	$32^3 \times 64$	3959	0.00250	0.05652 (38)	3.35	4.104 (36)	253.3 (1.4)
cB211.25.48	$48^3 \times 96$	5246	0.00250	0.05726 (12)	5.02	4.124 (17)	253.0 (1.0)
cB211.14.64	$64^3 \times 128$	6187	0.00140	0.05477 (12)	5.02	5.119 (36)	189.8 (0.7)
cB211.072.64	$64^3 \times 128$	3161	0.00072	0.05267 (14)	3.62	6.760 (30)	136.8 (0.6)
C-ensembles: $\beta = 1.836,\ c_{SW} = 1.6452,\ a\mu_\sigma = 0.106586,\ a\mu_\delta = 0.107146,\ w_0/a = 2.5045\ (17)$							
cC211.20.48	$48^3 \times 96$	2000	0.00200	0.04892 (13)	4.13	4.244 (25)	245.73 (98)
cC211.06.80	$80^3 \times 160$	3207	0.00060	0.04504 (10)	3.78	6.916 (19)	134.3 (0.5)

Not used in the baryon sector (cA211.53.24, cA211.40.24)

Lattice spacing from pion sector

- Use Osterwalder-Seiler strange and charm valence quarks —> tune using i) meson and ii) baryon masses
- Lattice spacing determine from i) f_π and ii) the nucleon mass m_N in the so symmetric limit

Pseudoscalar renormalisation constant Z_P

For details see talk by M. Di Carlo, Friday 6:15 am

In the twisted mass fermion (TMF) formulation at maximal twist the renormalised mass is given by $m_f = \frac{\mu_f}{Z_P}$

bare twisted mass parameter of quark f

- We employ the non-perturbative, mass independent RI'-MOM renormalisation scheme defined at the chiral limit —> need to generated N_f=4 gauge ensembles close to the chiral limit and at the same lattice spacing as the three sets of N_f=2+1+1 ensembles

G. Martinelli, C. Pittori, C. T. Sachrajda, M. Testa, and A. Vladikas, NPB445 (1995) arXiv:help/lat/9411010

$\beta = 1.726$ $24^3 \times 48$			$\beta = 1.778$ $24^3 \times 48$			$\beta = 1.836$ $32^3 \times 64$		
$a\mu_{\rm sea}$	am_{PS}	am_{PCAC}	$a\mu_{\rm sea}$	am_{PS}	am_{PCAC}	$a\mu_{\rm sea}$	am_{PS}	am_{PCAC}
0.0060	0.1689(15)	$-4.1(1.4)\times10^{-4}$	0.0075	0.1748(15)	$-2.3(0.8)\times10^{-5}$	0.0050	0.1276(14)	$-4.3(3.1)\times10^{-5}$
0.0080	0.1905(11)	$-4.3(1.1)\times10^{-5}$	0.0088	0.1871(18)	$-8.6(8.0)\times10^{-5}$	0.0065	0.1447(14)	$+5.9(2.1)\times10^{-5}$
0.0100	0.2155(12)	$+1.5(1.3)\times10^{-4}$	0.0100	0.2006(18)	$-1.6(0.8)\times10^{-4}$	0.0080	0.1585(14)	$+1.6(0.3)\times10^{-4}$
0.0115	0.2289(12)	$+1.7(1.1)\times10^{-4}$	0.0115	0.2158(11)	$+0.2(9.5)\times10^{-5}$	0.0095	0.1744(12)	$+2.0(0.3)\times10^{-4}$

maximal twist

- For the subtraction of the pion pole, a partially quenched approach is employed using additional values of the valence quarks at fixed sea quark mass
- We use two fitting Ansaetze for removing order $(ap)^2$ discretisation effects and two p^2-ranges
  —> resulting Z_P all consistent - average over all of them
- Convert to $\overline{\rm MS}$ and evolve to 2 GeV or 3 GeV

Scale setting

Meson sector: see also talk by P. Dimopoulos Thursday, July 29th, 05:30

- Use iso-symmetric values of pion mass and decay constant

C.A. *et al.* (2021), arXiv:2104.06747

$$m_\pi^{isoQCD} = 135.0(2)\ \text{MeV} \quad \text{and} \quad f_\pi^{isoQCD} = 130.4(2)\ \text{MeV}$$

FLAG report, S. Aoki *et al.*, Eur Phys. J. 77 (2017), arXiV:1607.00299

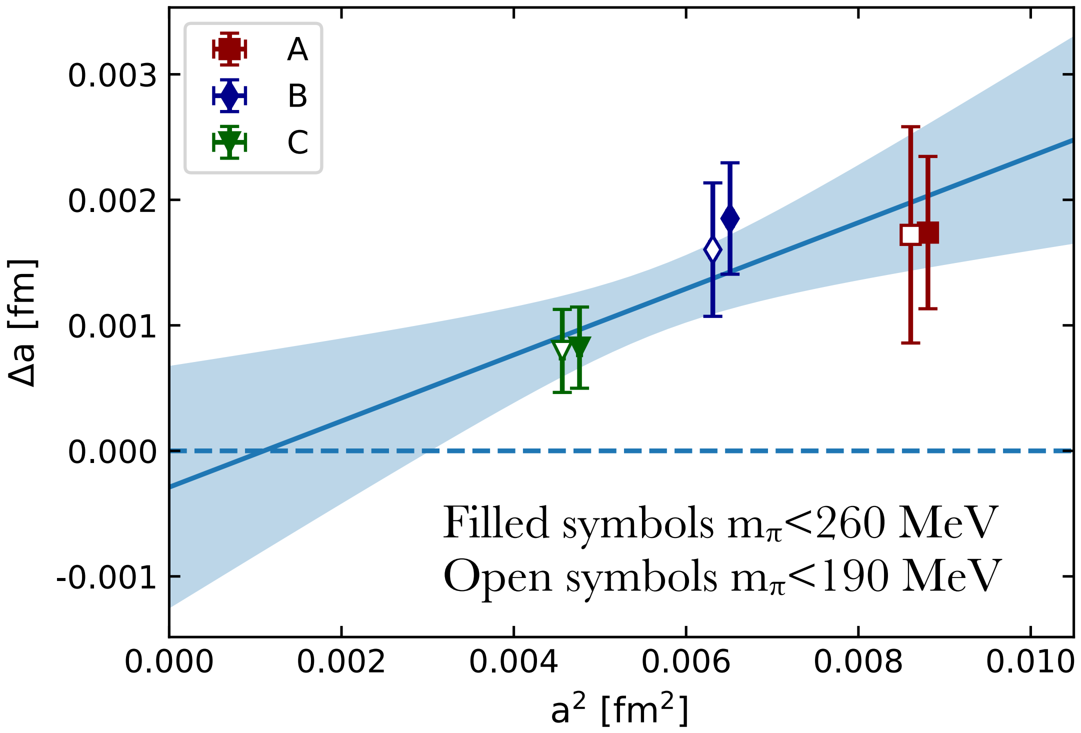
- NLO SU(2) chiral perturbation theory, correct for volume effects and take continuum limit of m_π and f_π in units of w_0
- Determine w_0 in the continuum limit and use w_0/a to extract a

Baryon sector:

- Use iso-symmetric values of pion and nucleon mass $m_N^{isoQCD} = 0.9389\,\text{GeV}$
- SU(2) chiral perturbation theory to one-loop

$$(a_i m_N) = a_i m_N^0 - 4c_1 \frac{(a_i m_\pi)^2}{a_i} - \frac{3g_A^2}{16\pi f_\pi^2}\frac{(a_i m_\pi)^3}{a_i^2}$$

Fixed by physical value of the nucleon mass



The lattice spacings from meson and baryon sector agree in the continuum limit

Determination of quark masses in the meson sector (I)

We use the lattice spacings determined in the meson sector

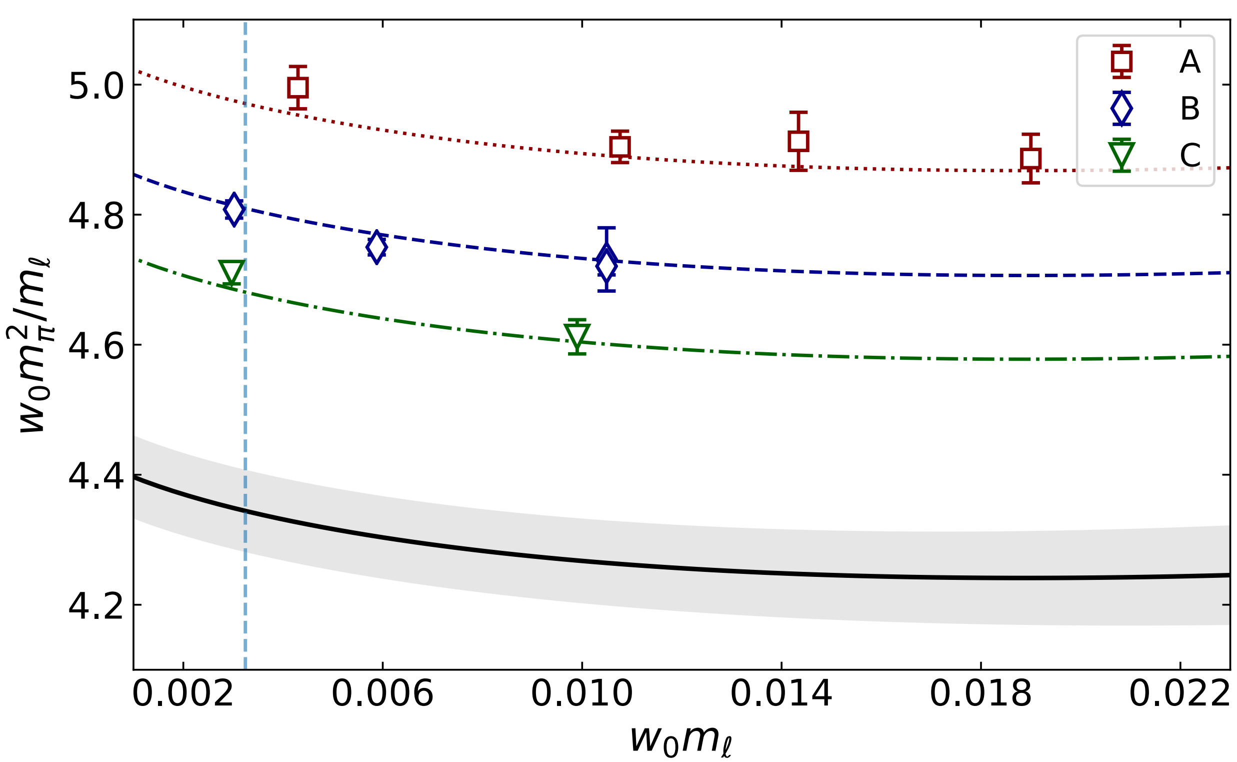
Light (u/d) quark mass

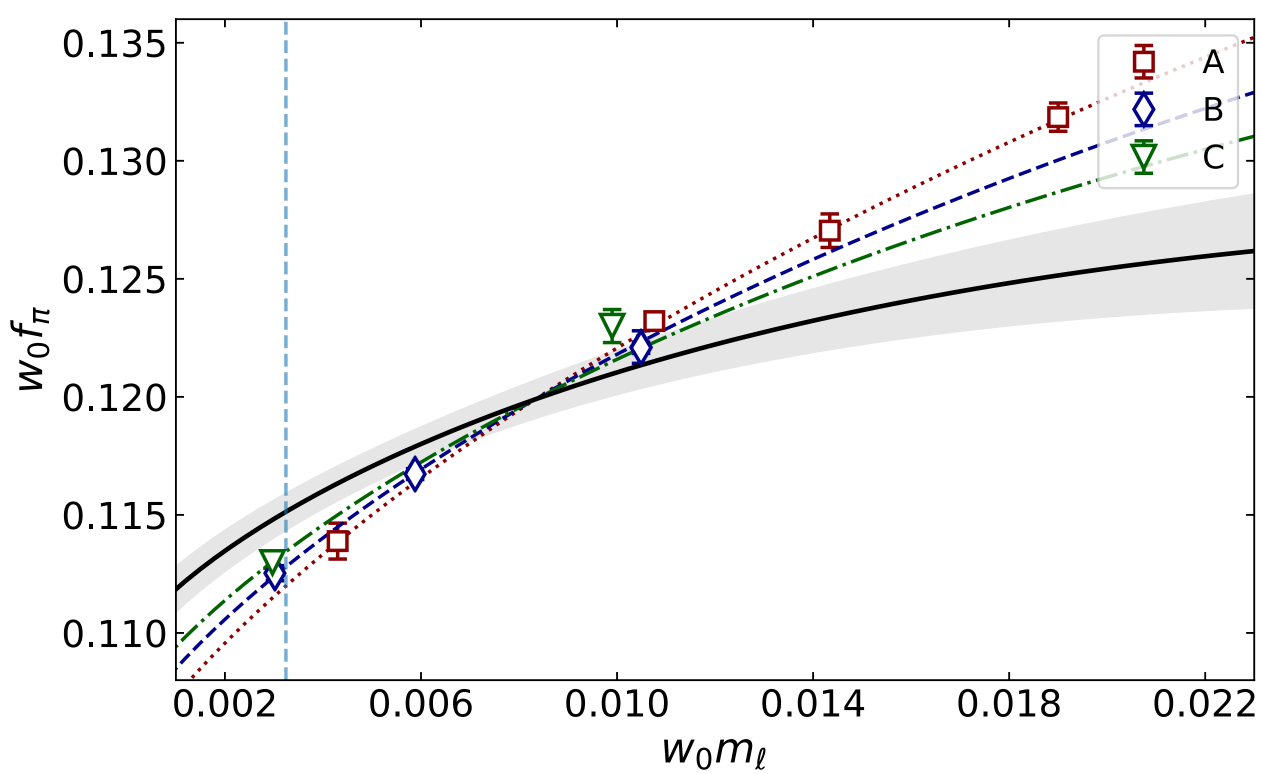
$$(m_\pi w_0)^2 = 2(Bw_0)(m_\ell w_0)\left[1 + \xi_\ell \log \xi_\ell + P_1 \xi_\ell + P_2\, a^2/w_0^2\right] K_{M^2}^{FSE}$$

$$(f_\pi w_0) = (f w_0)\left[1 - 2\xi_\ell \log \xi_\ell + P_3 \xi_\ell + P_4\, a^2/w_0^2 + a^2 m_\ell P_5\right] K_f^{FSE}$$

Finite volume corrections

$$\xi_\ell = \frac{2Bm_\ell}{(4\pi f)^2}, \quad P_1 = -\bar{\ell}_3 - 2\log\left(m_\pi^{\text{isoQCD}}/(4\pi f)\right), \quad P_3 = 2\bar{\ell}_4 + 4\log\left(m_\pi^{\text{isoQCD}}/(4\pi f)\right)$$





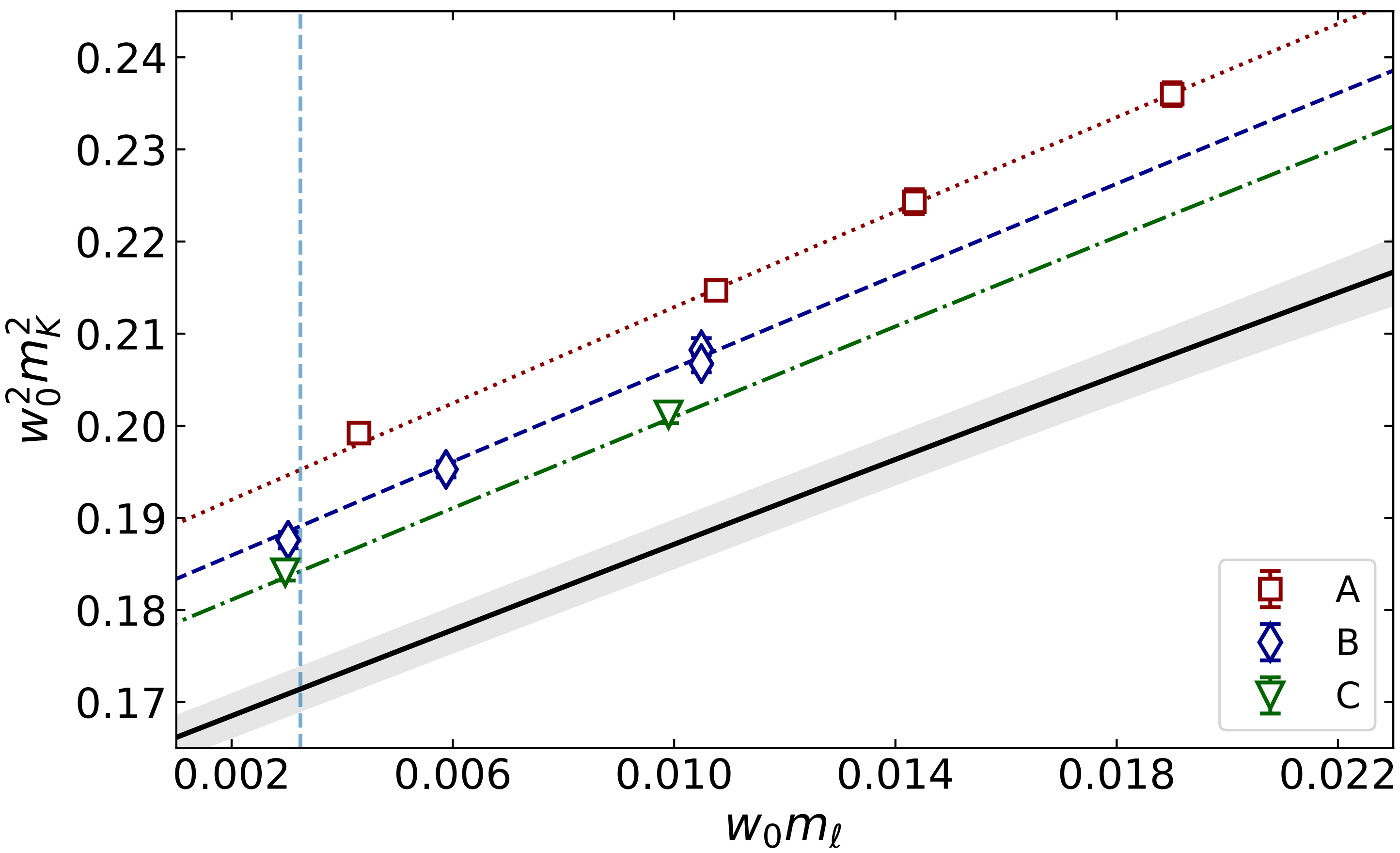
Determination of quark masses in the meson sector (II)

Strange quark mass

- Use as input m_K^{isoQCD}=494.2(3) MeV

- Use three reference values of m_s and NLO ChPT

$$m_K^2 = a + b m_s w_0 \qquad (m_K w_0)^2 = P_0(m_\ell w_0 + m_s w_0)\left[1 + P_1 m_\ell w_0 + P_2 m_\ell^2 w_0^2 + P_3\, a^2/w_0^2\right]$$



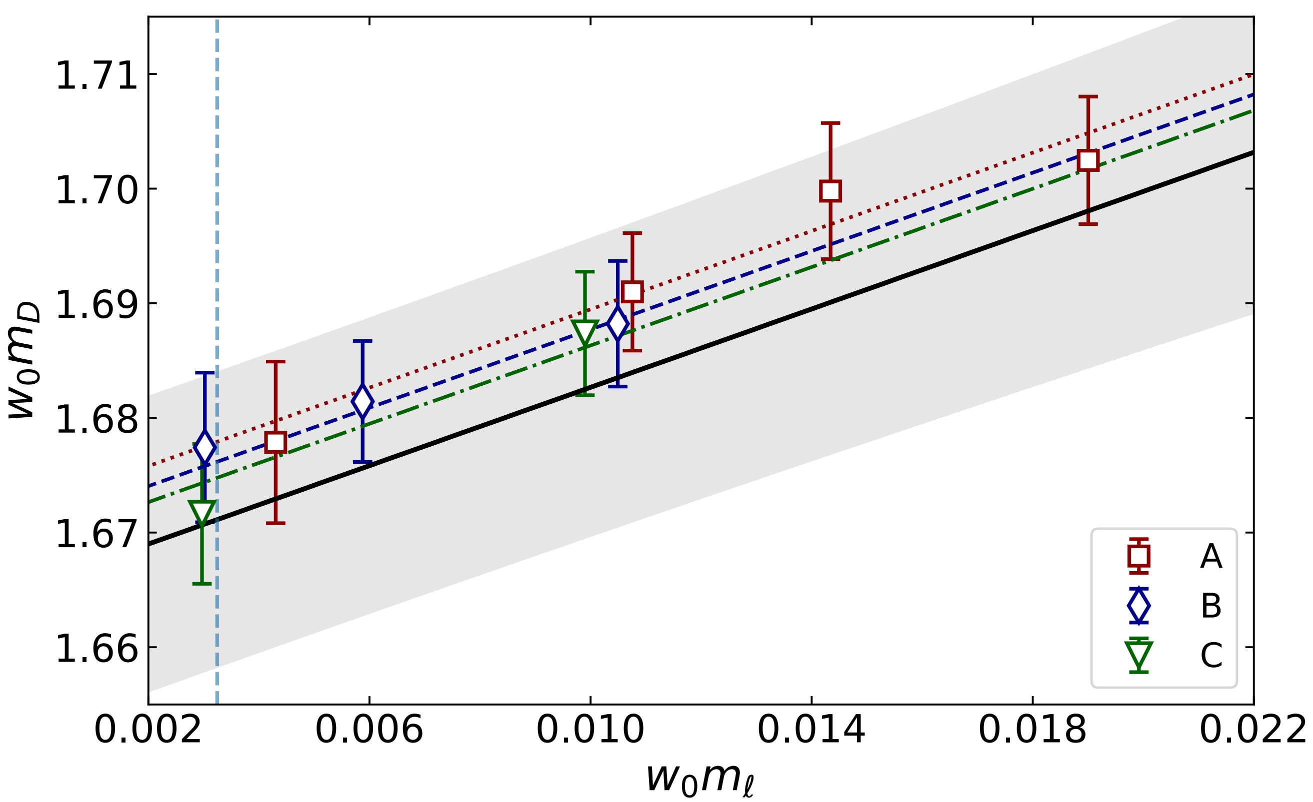
Determination of quark masses in the meson sector (III)

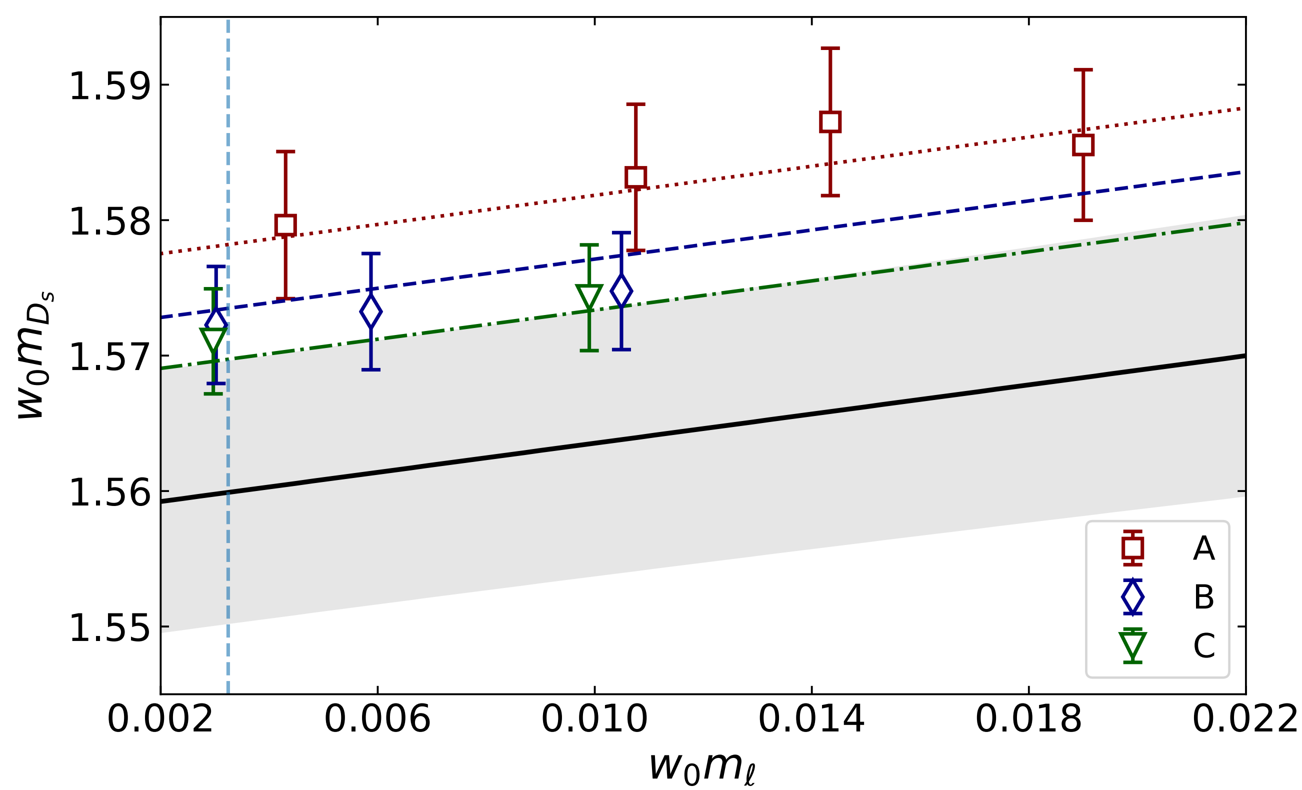
Charm quark mass

- Use as input $m_D^{\rm isoQCD} = 1867.0(4)$ MeV , $m_{D_s}^{\rm isoQCD} = 1969.0(4)$MeV .

- Use three reference values of m_c and the polynomials

$$m_{D_s} = a + b m_c w_0 \qquad m_D = P_0 + P_1 m_\ell w_0 + P_2 a^2/w_0^2 \qquad m_{D_s} = P_0^s + P_1^s m_\ell w_0 + P_2^s a^2/w_0^2$$





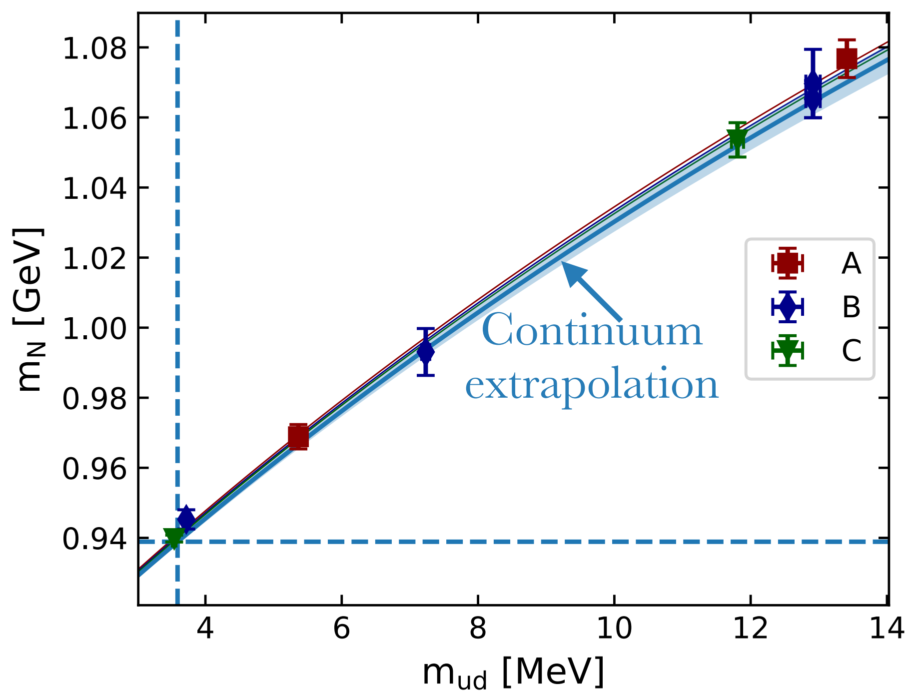
Determination of quark masses in the baryon sector (I)

We use the lattice spacings determined from the nucleon mass

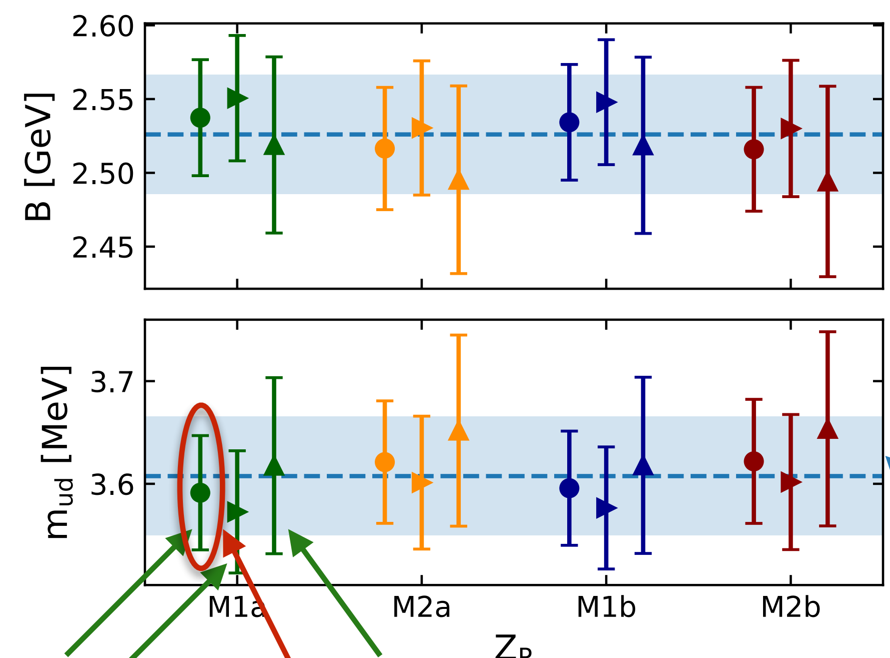
Light (u/d) quark mass

To one-loop order $m_\pi^2 = 2Bm_{ud}(1 + c_2 a^2)$ i.e. two fit parameters B and c_2

$$\longrightarrow \quad m_N(m_{ud}) = m_N^0 - 4c_1\left(2Bm_{ud}(1+c_2a^2)\right) - \frac{3g_A^2}{16\pi f_\pi^2}\left(2Bm_{ud}(1+c_2a^2)\right)^{3/2}$$



Dependence on the extraction of Z_P



Average over the values using the different Z_P

Fit using m_π<260 MeV

Fit using m_π<190 MeV

Increase lower fit range by one for extraction of m_N

take deviation from the mean of the other two as systematic error

Determination of quark masses in the baryon sector (II)

Strange quark mass

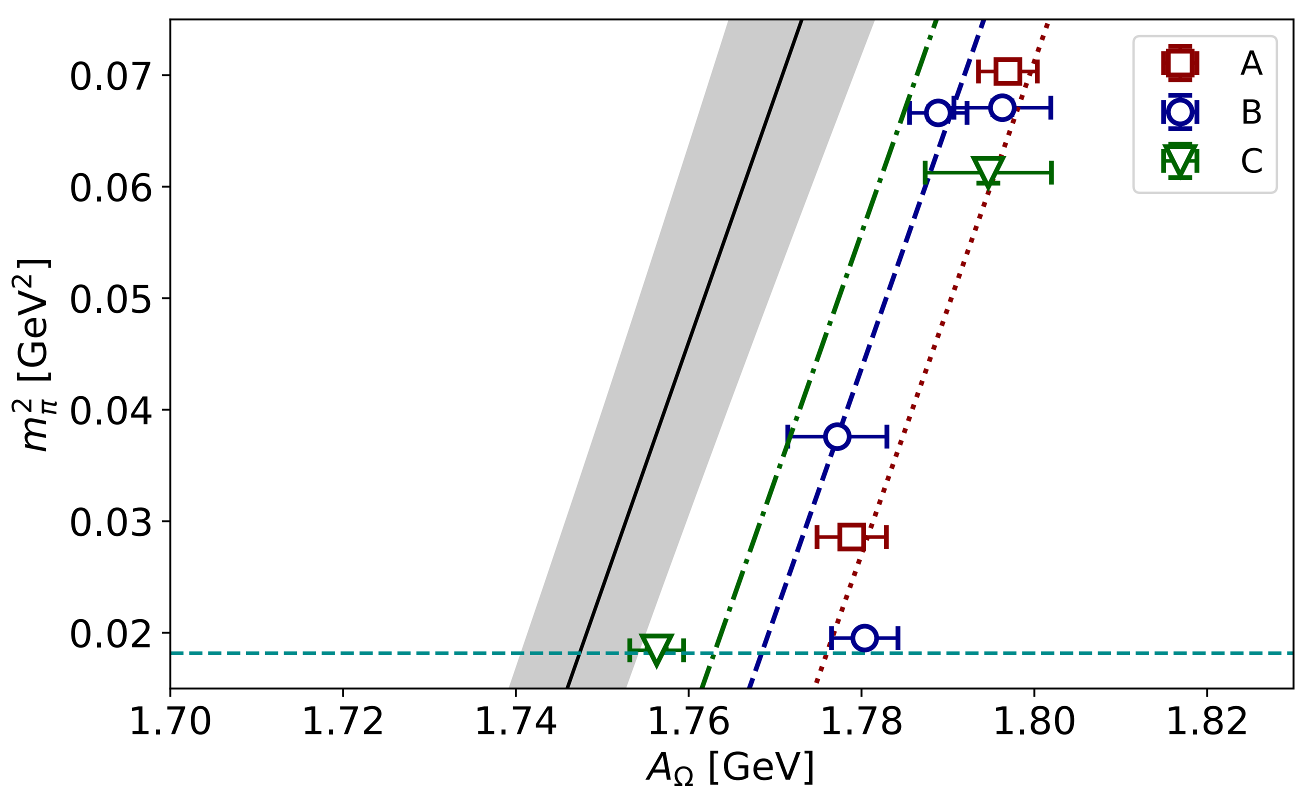
- Use as input the physical value of th e Ω^- mass, $m_\Omega^{(phys.)}$ 1672.5(3) MeV and parameterise linearly its dependence on m_s around a reference mass $\tilde{m}_s$

$$m_\Omega = A_\Omega + B_\Omega\,(m_s - \tilde{m}_s)$$

- Employ two fully consistent approaches:
  I. Expand directly the parameters A_Ω and B_Ω and perform extrapolation in m_π and continuum limit

$$A_\Omega(a, m_\pi^2) = c_1 + c_2 m_\pi^2 + c_3 a^2\,, \quad B_\Omega(a, m_\pi^2) = c_1' + c_2' m_\pi^2$$

$$\rightarrow \quad m_s = \tilde{m}_s + \frac{m_\Omega^{(phys.)} - A_\Omega(0, m_\pi^{(phys.)})}{Z_P\, B_\Omega(0, m_\pi^{(phys.)})}$$



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$$m_\Omega = A_\Omega + B_\Omega\,(m_s - \tilde{m}_s)$$

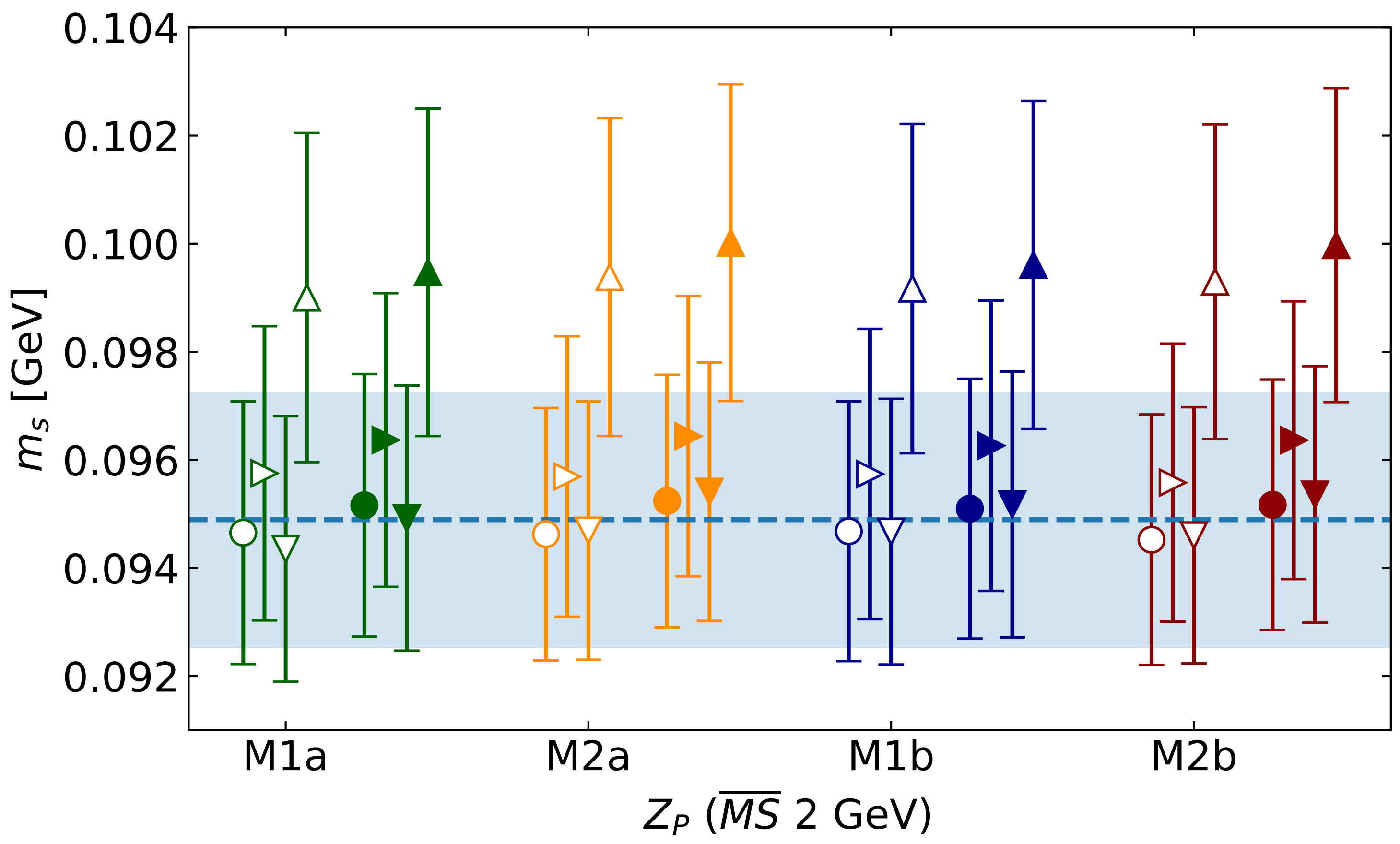
- Employ two fully consistent approaches:

  II. adopt an iterative strategy:

  - start by fixing a value of the renormalized strange quark mass m_s in physical units for all ensembles and extrapolate to the continuum limit and physical point using

$$m_\Omega = m_\Omega^{(0)} - 4c_\Omega^{(1)} m_\pi^2 + d_\Omega^{(2)} a^2$$

  - iterate changing the value of m_s until the resulting value of the mass of Ω at the physical point and continuum limit matches the physical value



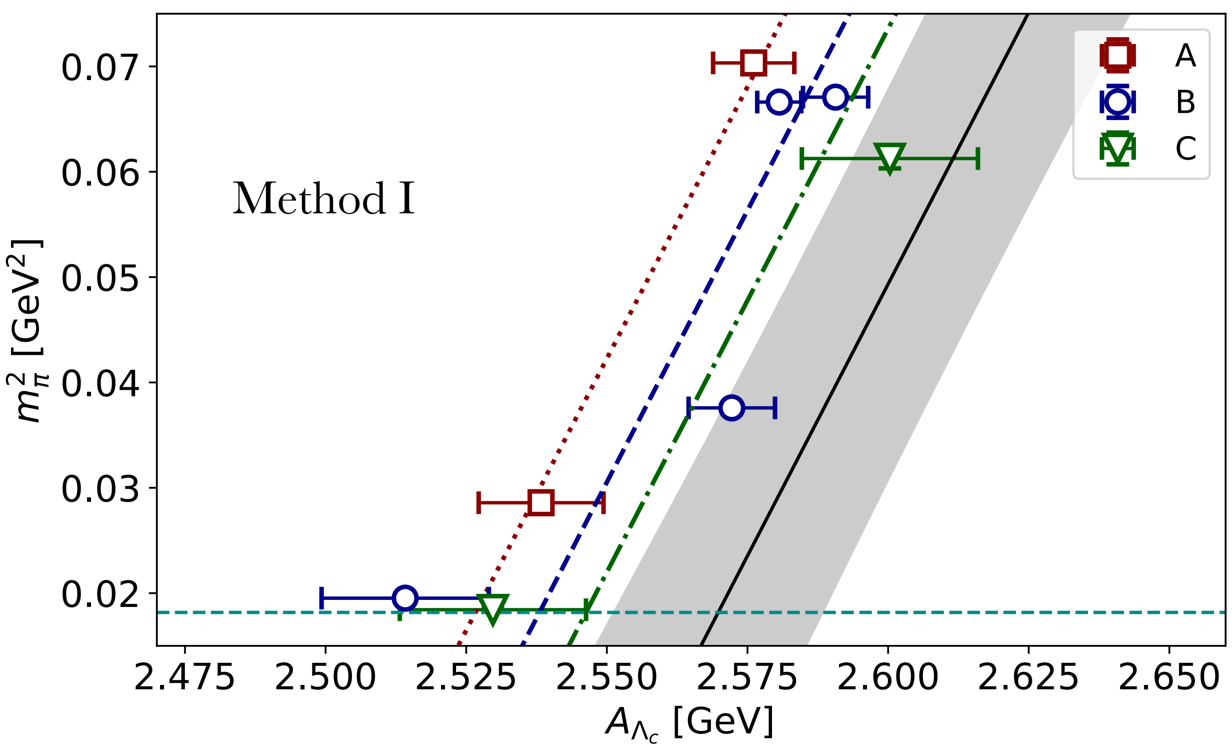
Determination of quark masses in the baryon sector (III)

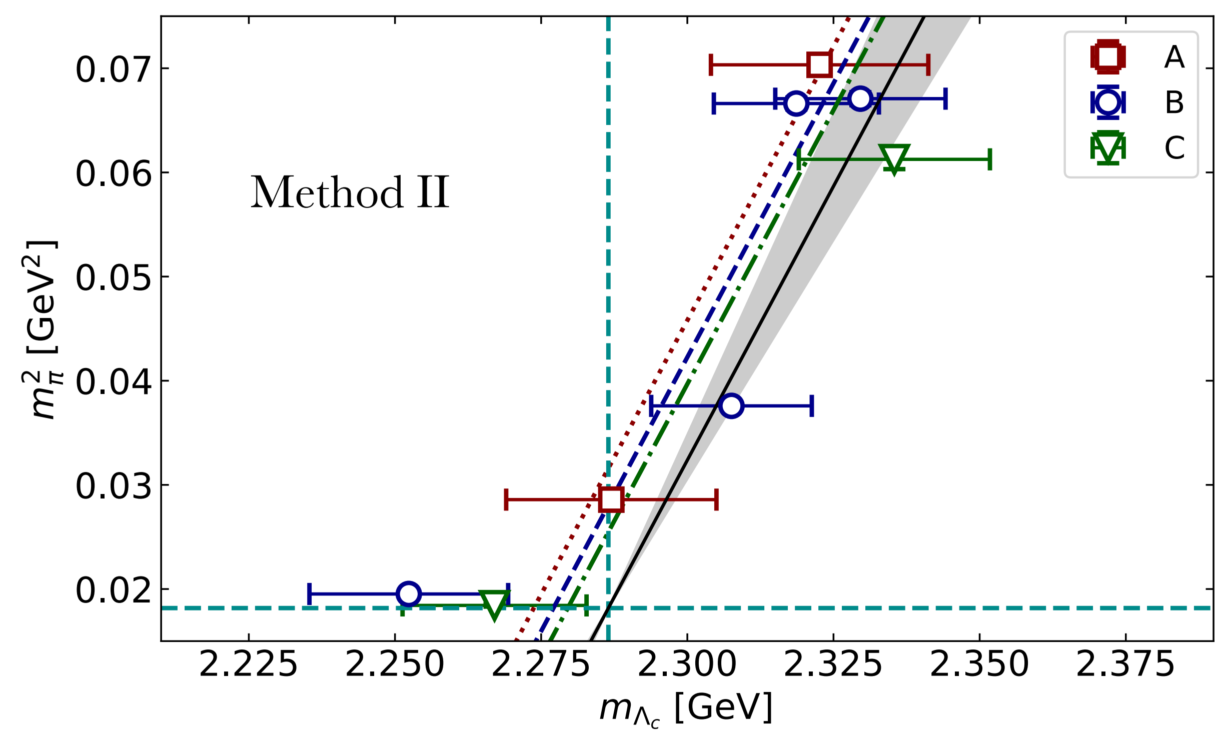
Charm quark mass

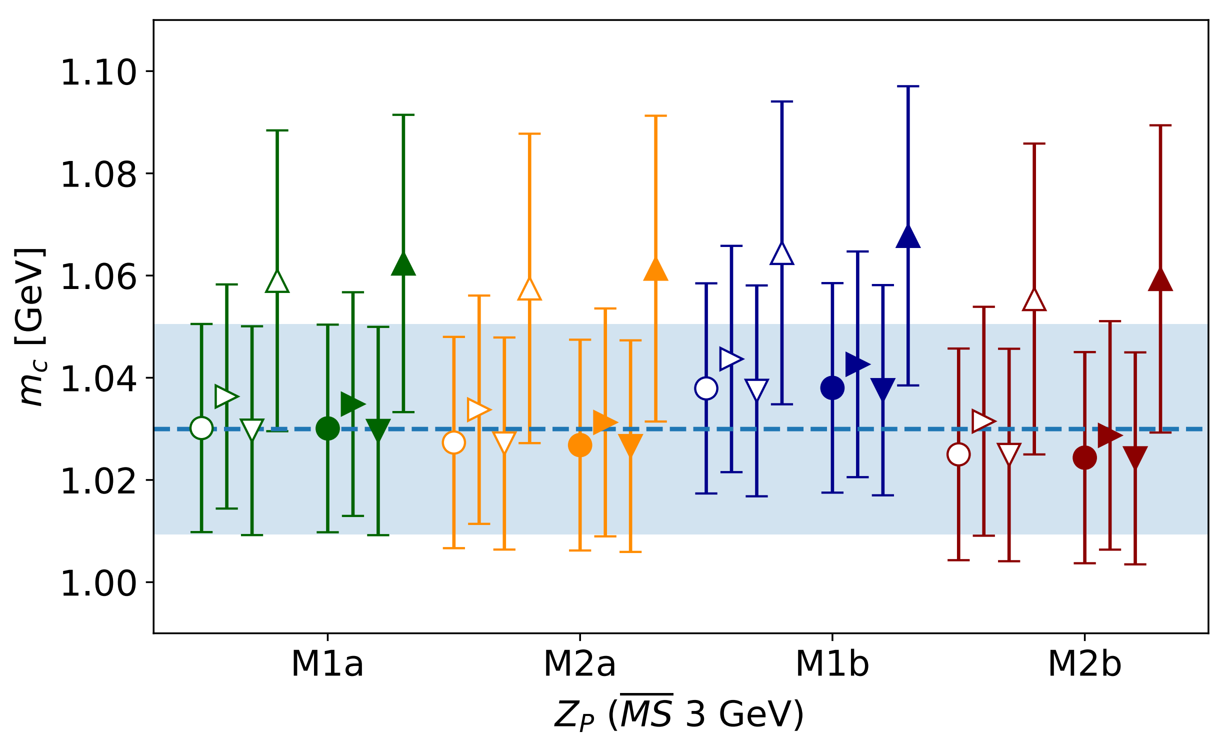
- Use a similar approach to the one used for m_s with input the physical value of the mass of Λ_c
  $m_{\Lambda_c}^{(phys.)}$ =2286.5(1) MeV

$$m_{\Lambda_c} = A_{\Lambda_c} + B_{\Lambda_c}\,(m_c - \tilde{m}_c)$$

$$m_{\Lambda_c} = m_{\Lambda_c}^{(0)} + c_{\Lambda_c}^{(1)} m_\pi^2 + d_{\Lambda_c}^{(2)} a^2$$







Fully consistent results from methods I (open symbols) & II (filled symbols)

Results

- We take a weighted average over values from different Z_P
- In the meson sector we average also over results from using different scale settings (w_0, t_0/w_0, $\sqrt{t_0}$), ensembles for which m_π<190 MeV (setting P_3=0), and using only two of the ensembles providing the χ^2/dof<2.5. For the charm we also average the results using D and D_s

  To compute the mean and error we use

$$\overline{x} = \sum_{i=1}^{N} w_i \; \overline{x}_i \; , \; \sigma^2_{\rm stat} = \sum_{i=1}^{N} w_i \; \sigma_i^2 \; , \; \sigma^2_{\rm syst} = \sum_{i=1}^{N} w_i \; (\overline{x}_i - \overline{x})^2 \; , \qquad w_i = \frac{1/\sigma_i^2}{\sum_{j=1}^{N} 1/\sigma_j^2}$$

- In the baryon sector for the strange and charm quark masses we average over the results obtained from methods I and II. The systematic error is due to the chiral extrapolation and the fit range in extracting the input mass

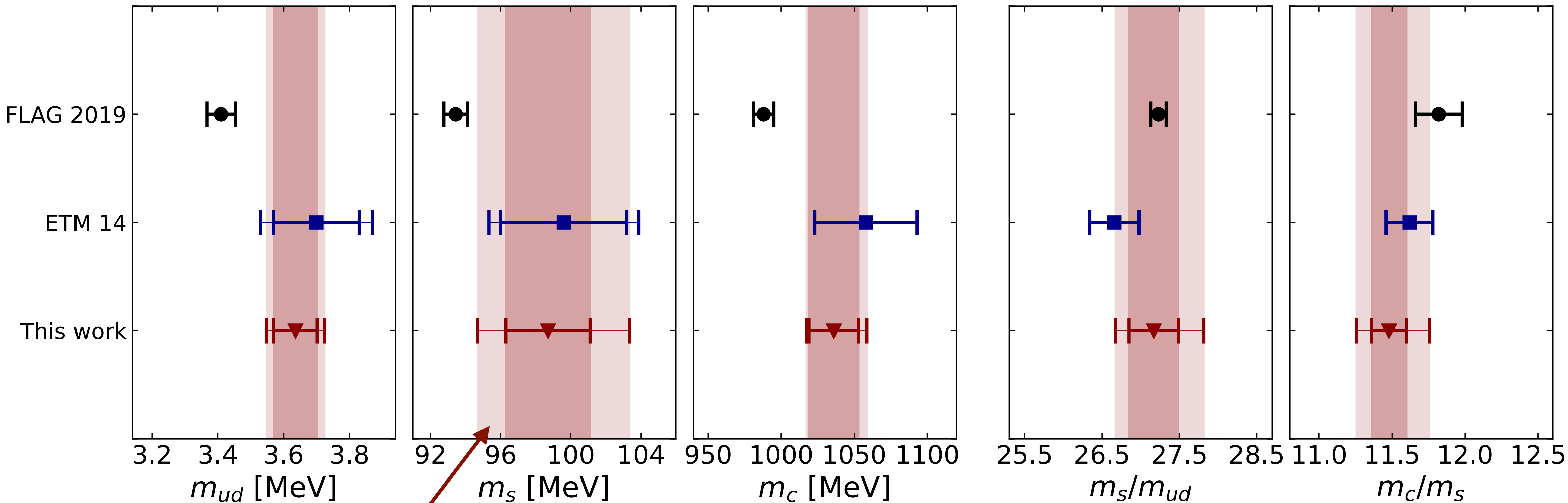
	m_{ud} [MeV]	m_s [MeV]	m_c [MeV]	m_s/m_{ud}	m_c/m_s
Meson sector	3.689(80)(66)	101.0(1.9)(1.4)	1039(15)(8)	27.30(24)(14)	11.43(9)(10)
Baryon sector	$3.608(58)(^{+32}_{-19})$	$94.9(2.4)(^{+4.1}_{-1.0})$	$1030(21)(^{+22}_{-5})$	$26.30(61)(^{+1.17}_{-0.33})$	$12.04(31)(^{+58}_{-15})$

- Biggest tension between the meson and baryon determinations is seen for m_s for which we have a large systematic error in the baryon sector many due to the chiral extrapolation

Conclusions

❋ Final values

Final results	$3.636(66)(^{+60}_{-57})$	$98.7(2.4)(^{+4.0}_{-3.2})$	$1036(17)(^{+15}_{-8})$	$27.17(32)(^{+56}_{-38})$	$11.48(12)(^{+25}_{-19})$
FLAG 2019	3.410(43)	93.44(68)	988(7)	27.23(10)	11.82(16)



Large error due to the tension between the determinations in the meson and baryon sectors

❋ Next step: Add a third lattice spacing at the physical point

Would allow a continuum extrapolation directly at the physical point
Avoiding chiral extrapolation is important for the baryon sector