Logarithmic corrections to a^2 scaling in lattice QCD with Wilson and GW quarks

extension of pure gauge theory [NH, P. Marquard, R. Sommer, 2020] to full lattice QCD

Nikolai Husung with Peter Marquard, Rainer Sommer

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Lattice artifacts in lattice QCD.

Problem: Any quantity $\mathcal{P}$ obtained from lattice data depends on the cut-off a, but we are interested in the continuum value. $\Rightarrow$ **"lattice artifacts" with relevant scale $\mu \sim 1/a$**

In an asymptotically free theory, like QCD, lattice artifacts are of the form

$$\frac{\mathcal{P}(a)}{\mathcal{P}(0)} = 1 + a^{n_{\min}} \underbrace{[\alpha(1/a)]^{\hat{\Gamma}}}_{\substack{=1,\text{ classically} \\ \neq 1,\text{ due to quantum corrections}}} \text{const.} + \mathrm{O}(a^{n_{\min}+1}, a^{n_{\min}} \alpha^{\hat{\Gamma}+1}(1/a), \ldots), \quad \alpha(1/a) \overset{a \searrow 0}{\sim} -\frac{1}{\ln(a\Lambda)}$$

No prior knowledge of $\hat{\Gamma}$, **can be negative and distinctly nonzero**, impacts convergence.

$\hat{\Gamma}$ is directly related to a 1-loop anomalous dimension of a higher dimensional operator.

Lattice artifacts in lattice QCD.

$$\frac{\mathcal{P}(a)}{\mathcal{P}(0)} = 1 + a^{n_\mathrm{min}} \underbrace{[\alpha(1/a)]^{\hat{\Gamma}}} \mathrm{const.} + \mathrm{O}(a^{n_\mathrm{min}+1}, a^{n_\mathrm{min}} \alpha^{\hat{\Gamma}+1}(1/a), \ldots)$$

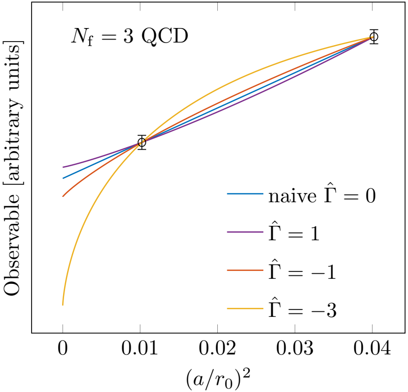
$=1$, classically

$\neq 1$, due to quantum corrections

Warning example:

2d O(3) non-linear sigma model $\hat{\Gamma} = -3$

[Balog et al., 2009, 2010]



$\Rightarrow$ Need to compute $\hat{\Gamma}$ in QCD to gain better control over continuum extrapolation.

Symanzik effective theory.

Describe lattice artifacts in terms of a **continuum** Effective Field Theory [Symanzik, 1980, 1981, 1983a,b]

$$\mathscr{L}_{\mathrm{eff}} = \mathscr{L}_{\mathrm{QCD}} + a^{n_{\mathrm{min}}} \sum_j c_j \mathcal{O}_j + \mathrm{O}(a^{n_{\mathrm{min}}+1})$$

with minimal **on-shell** operator basis $\mathcal{O}_j$ and matching coefficients $c_j(\alpha) = \bar{c}_j + \mathrm{O}(\alpha)$.
$\Rightarrow$ use continuum EOMs to reduce set of operators [Lüscher et al., 1996]

"Naive" expansion in the lattice spacing is sufficient to match the Symanzik effective theory at tree-level ($\bar{c}_j$) to the lattice theory.
$\Rightarrow$ LO lattice artifacts are accessible **without need for lattice perturbation theory** (unless leading $\hat{\Gamma}$ has vanishing tree-level coefficient).

For non-spectral quantities also contributions from discretised local fields must be included.

Operator basis.

Occurring operators $\mathcal{O}_j$ must comply with symmetries of the lattice formulation, e.g. for Wilson's lattice QCD [Wilson, 1974, 1975]

$$S_\mathrm{W} = \frac{2}{g_0^2}\sum_x \mathrm{Re}\,\mathrm{tr}(\mathbb{1} - \mathcal{U}(x)) + a^4 \sum_x \bar{\Psi}(x)\left[\frac{1}{2}\left\{\gamma_\mu(\nabla_\mu^* + \nabla_\mu) - a\nabla_\mu^*\nabla_\mu\right\} + M\right]\Psi(x)$$

- Local SU(N) gauge symmetry,
- $\mathcal{C}$-, $\mathcal{P}$- and $\mathcal{T}$-symmetry,
- broken O(4) symmetry due to reduced rotation symmetry,
- no $\mathrm{SU}(N_\mathrm{f})_\mathrm{L} \times \mathrm{SU}(N_\mathrm{f})_\mathrm{R} \times \mathrm{U}(1)_\mathrm{V}$ flavour symmetry for massless QCD,
- manifolds with boundaries necessitate additional surface terms (e.g. Schrödinger functional).

Chosen lattice discretisation determines realised symmetries and may affect n_min
$\Rightarrow$ require different bases.

Operator basis.

Flavour symmetries:

fermion action	massless	mass-degenerate	massive	$n_{\min}$
Continuum				–
Domain wall	$\mathrm{SU}(N_\mathrm{f})_\mathrm{L} \times \mathrm{SU}(N_\mathrm{f})_\mathrm{R} \times \mathrm{U}(1)_\mathrm{V}$	$\mathrm{SU}(N_\mathrm{f})_\mathrm{V} \times \mathrm{U}(1)_\mathrm{V}$	$\bigtimes_{f=1}^{N_\mathrm{f}} \mathrm{U}(1)_f$	2
Ginsparg-Wilson				2
Wilson	$\mathrm{SU}(N_\mathrm{f})_\mathrm{V} \times \mathrm{U}(1)_\mathrm{V}$	$\mathrm{SU}(N_\mathrm{f})_\mathrm{V} \times \mathrm{U}(1)_\mathrm{V}$	$\bigtimes_{f=1}^{N_\mathrm{f}} \mathrm{U}(1)_f$	1
staggered	$\mathrm{U}(1)_\mathrm{B} \times \mathrm{U}(1)_{\tilde{\mathrm{A}}}$	$\mathrm{U}(1)_\mathrm{B}$	$\mathrm{U}(1)_\mathrm{B}$	2

staggered has flavour changing interactions.

Operator basis.

Wilson-like [Sheikholeslami and Wohlert, 1985]
pure gauge [Weisz, 1983; Lüscher and Weisz, 1985]
$\mathrm{O}(a)$ improved [Sheikholeslami and Wohlert, 1985]

Minimal basis at mass-dimensions 5 and 6

$n_{\mathrm{min}} = 1$	$n_{\mathrm{min}} = 2$	
$i\bar{\Psi}\sigma_{\mu\nu}F_{\mu\nu}\Psi$	$\frac{1}{g_0^2}\mathrm{tr}(D_\mu F_{\nu\rho} D_\mu F_{\nu\rho})$	$g_0^2(\bar{\Psi}\Gamma\Psi)^2$
	$\frac{1}{g_0^2}\sum_\mu \mathrm{tr}(D_\mu F_{\mu\nu} D_\mu F_{\mu\nu})$	$g_0^2(\bar{\Psi}\Gamma T^a\Psi)^2$
	$\sum_\mu \bar{\Psi}\gamma_\mu D_\mu^3\Psi$	$\Gamma \in \{\mathbb{1}, \gamma_5, \gamma_\mu, \gamma_5\gamma_\mu, \sigma_{\mu\nu}\}$

with flavours $\Psi = (\mathrm{u}, \mathrm{d}, \mathrm{s}, \ldots)$ + explicitly mass-dependent operators such as $i\bar{\Psi}M\sigma_{\mu\nu}F_{\mu\nu}\Psi$.

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with flavours $\Psi = (\mathrm{u}, \mathrm{d}, \mathrm{s}, \ldots)$ + explicitly mass-dependent operators such as $i\bar{\Psi}M\sigma_{\mu\nu}F_{\mu\nu}\Psi$.

$\Rightarrow$ e.g. $\mathrm{O}(a)$ improved massless Wilson with $N_\mathrm{f} > 1$: 13 operators

Computing 1-loop anomalous dimensions.

Leading lattice artifacts are then parametrised as

$$\frac{\mathcal{P}(a)}{\mathcal{P}(0)} = 1 - a^{n_{\min}} \sum_j \bar{c}_j^{\mathcal{O}} \delta\mathcal{P}_j^{\mathcal{O}}(1/a) \times [1 + \mathrm{O}(\alpha(1/a)] + \mathrm{O}(a^{n_{\min}+1}).$$

Scale dependence of $\delta\mathcal{P}_j^{\mathcal{O}}(1/a)$ is governed by RGE

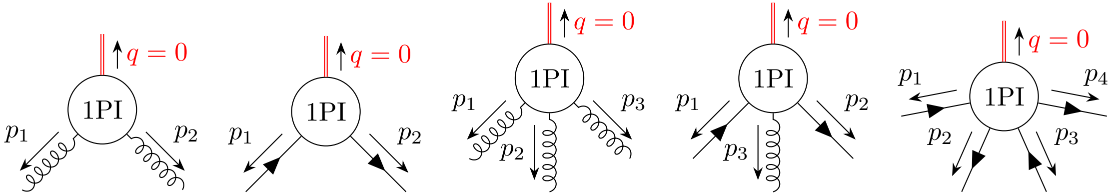
$$\mu^2 \frac{\mathrm{d}\delta\mathcal{P}_i^{\mathcal{O}}(\mu)}{\mathrm{d}\mu^2} = -\left[\gamma_0^{\mathcal{O}}\alpha(\mu) + \mathrm{O}(\alpha^2)\right]_{ij} \delta\mathcal{P}_j^{\mathcal{O}}(\mu).$$

Making a change of basis $\mathcal{O} \to \mathcal{B}$ such that $\gamma_0^{\mathcal{B}} = \mathrm{diag}\left((\gamma_0)_1, \ldots, (\gamma_0)_n\right)$ allows to rewrite

$$\delta\mathcal{P}_j^{\mathcal{B}}(1/a) = [2\beta_0\alpha(1/a)]^{\hat{\gamma}_j} \delta\mathcal{P}_{j;\mathrm{RGI}}^{\mathcal{B}} \times [1 + \mathrm{O}(\alpha(1/a))], \quad \hat{\gamma}_j = \frac{(\gamma_0)_j}{\beta_0}.$$

Computing 1-loop anomalous dimensions.

Renormalise operator basis at 1-loop in $\overline{\mathrm{MS}}$ by computing 1PI graphs with operator insertion $\tilde{\mathcal{O}}(q)$ in background field gauge ['t Hooft, 1975; Abbott, 1981, 1982; Lüscher and Weisz, 1995].



$$\mathcal{O}_{i;\mathrm{R}} = Z_{ij}\mathcal{O}_j, \quad Z = \mathbb{1} + \gamma_0^{\mathcal{O}}\frac{\alpha}{\epsilon} + \mathrm{O}(\alpha^2)$$

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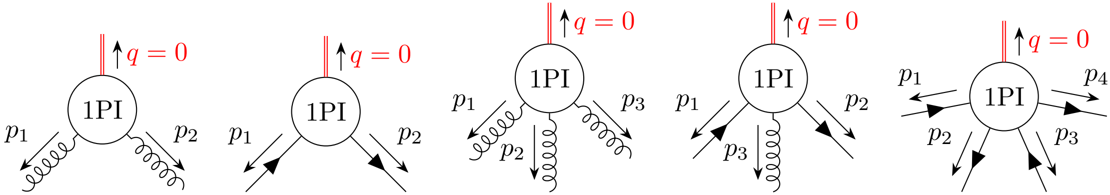


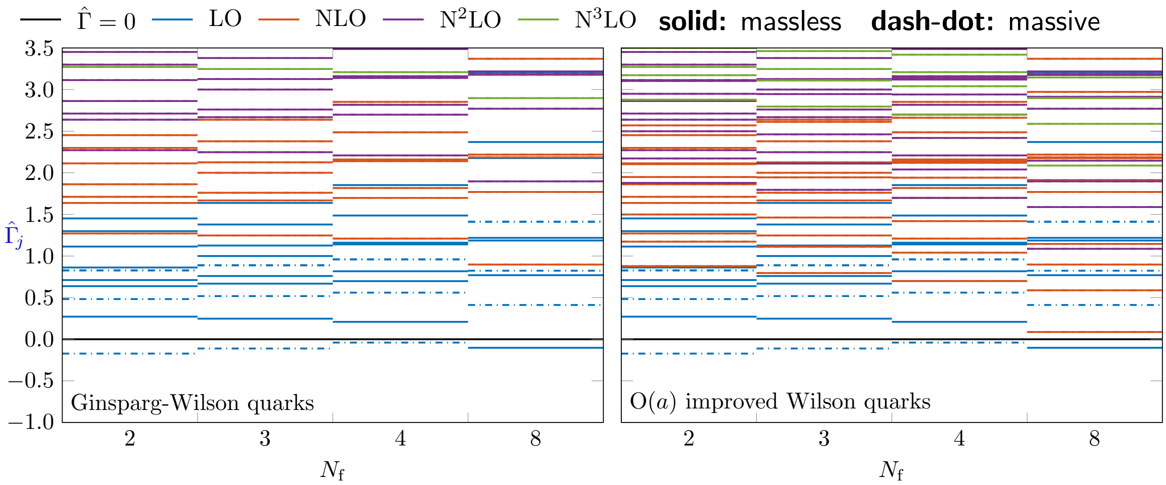
$$\mathcal{O}_{i;\mathrm{R}} = Z_{ij}\mathcal{O}_j, \quad Z = \mathbb{1} + \gamma_0^{\mathcal{O}}\frac{\alpha}{\epsilon} + \mathrm{O}(\alpha^2)$$

Taking into account (TL) matching $c_j^{\mathcal{B}} = \bar{c}_j + \mathrm{O}(\alpha)$ for a general plaquette+rectangle lattice gauge action eventually yields up to $\mathrm{O}(a^3)$ terms

$$\frac{\mathcal{P}(a)}{\mathcal{P}(0)} - 1 = a^2 \sum_j c_j^{\mathcal{B}} [2\beta_0 \alpha(1/a)]^{\hat{\gamma}_j} \delta\mathcal{P}^{\mathcal{B}}_{j;\mathrm{RGI}} = a^2 \sum_j d_j [\alpha(1/a)]^{\hat{\Gamma}_j}, \quad \hat{\Gamma}_j = \hat{\gamma}_j + n_j, n_j \in \mathbb{N}$$

Leading powers in the coupling.

$$\frac{\mathcal{P}(a)}{\mathcal{P}(0)} = 1 + \sum_j d_j a^2 [\alpha(1/a)]^{\hat{\Gamma}_j} + \mathrm{O}(a^3)$$



Conclusion.

- No $a^2[\alpha(1/a)]^{-3}$ behaviour like for the O(3) model nor naive a^2, but e.g. for $N_\mathrm{f} = 3$

$$\frac{\mathcal{P}(a)}{\mathcal{P}(0)} = 1 + d_1 a^2 [\alpha(1/a)]^{\hat{\Gamma}_\mathrm{min}} \left(1 + d_2 [\alpha(1/a)]^{\Delta\hat{\Gamma}} + \ldots\right),$$

	massless	massive
$\hat{\Gamma}_\mathrm{min}$	0.25	−0.11
$\Delta\hat{\Gamma}$	0.42	0.36

- Coefficients d_i incorporate both an RGI quantity and a matching coefficient. Hierarchy in matching coefficients should be taken into account as well.
- For typical $N_\mathrm{f} = 2, 3, 4$ leading anomalous dimensions of contributions from considered massless lattice QCD actions improve convergence as $a \searrow 0$.
- Presence of 4-fermion operators gives a dense spectrum for $\hat{\gamma}$, i.e. no clearly dominating contributions. Can lead to complicated lattice artifacts with cancellations and pile ups.
- Slightly negative anomalous dimensions for massive operators at $N_\mathrm{f} = 2, 3, 4$ but still close to zero. For large quark masses these are then the dominant sources of lattice artifacts.

Outlook.

> Leading asymptotic behaviour is now known and should be incorporated into continuum extrapolations e.g.:
>
> - through use of dominant $\hat{\Gamma}$ in extrapolations,
> - or estimate uncertainty in truncation to leading anomalous dimension as $\hat{\Gamma}_{\mathrm{average}}$,
> - or vary $\hat{\Gamma}$ in the range of 1-loop anomalous dimensions,
> - ...
>
> Best practice must still be worked out.

> **Possible directions for future research:**
>
> - extracting the spectra for (partially) quenched and mixed actions,
> - Gradient flow for full QCD requires the inclusion of two additional operators,
> - staggered quarks require additional operators in the minimal basis due to flavour changing interactions,
> - corrections to electro-weak flavour currents + electromagnetic current,
> - leading matching coefficients of some actions still need to be worked out,
> - ...

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