Istituto Nazionale di Fisica Nucleare

LATTICE 21

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Quark mass RG-running for Nf=3 QCD in a χSF setup

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ALPHA Collaboration

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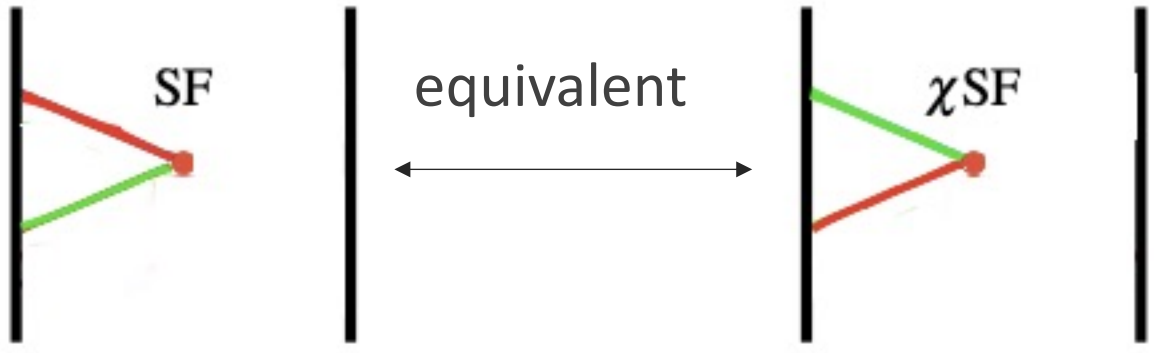
Main goal:

$$M^{RGI} = \frac{M^{RGI}}{\bar{m}(\mu_{pt})} \frac{\bar{m}(\mu_{pt})}{\bar{m}(\mu_0/2)} \frac{\bar{m}(\mu_0/2)}{\bar{m}(\mu_{had})} \bar{m}(\mu_{had})$$

in a χSF setup: Sint, arXiv 1008.4857

Chiral non-singlet trasformation of fermion fields

Massless continuum QCD with SF boundaries



Massless continuum QCD with χSF boundaries

How does a comparison to SF look like?

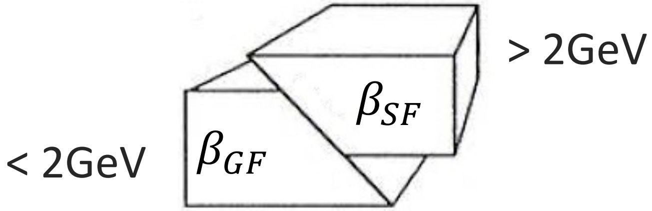
We have a direct precursor to look at! (arXiv 1802.05243)

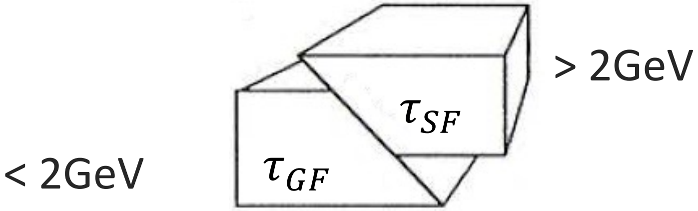
Non–perturbative quark mass renormalisation and running in $N_f = 3$ QCD

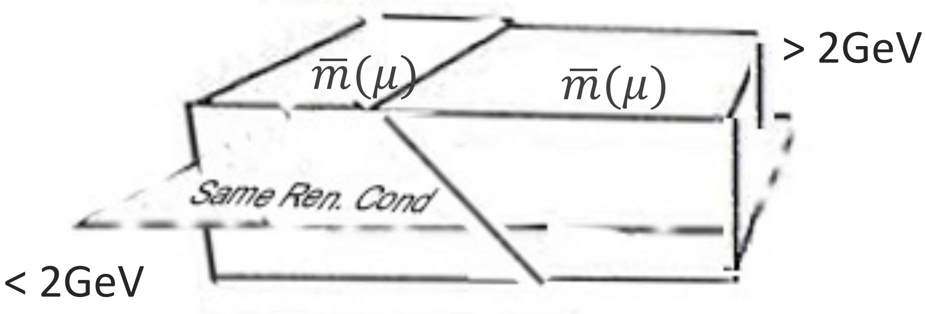
Isabel Campos, Patrick Fritzsch, Carlos Pena, David Preti, Alberto Ramos, Anastassios Vladikas

Both:

- Massless QCD, Nf=3
- $\Lambda_{QCD} \lesssim \mu \lesssim \frac{\mu_0}{2} \sim 2\ GeV$: GF-coupling ; $\frac{\mu_0}{2} \lesssim \mu \lesssim M_W$: SF-coupling







- →… But same renormalisation condition for the mass!
- $O(a^2)$-effects in the bulk, $O({g_0}^4 a)$-effects at time boundaries
- Same configurations (same SF regularisation for the sea quarks)

But…

New setup

χSF boundary conditions

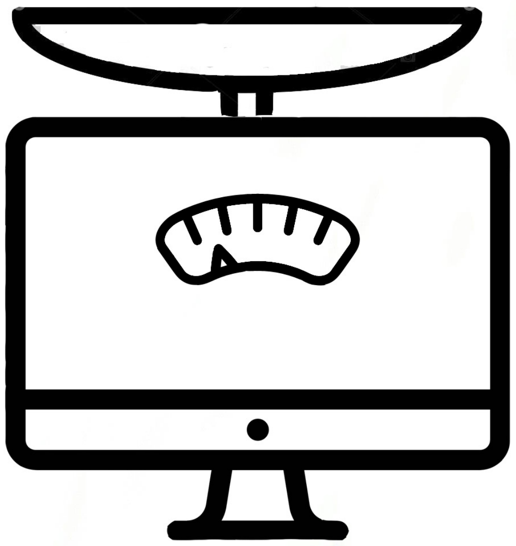
$\rightarrow$ mixed action approach

1802.05243 setup

SF boundary conditions

Symanzik counterterms needed to improve the action:

time boundaries: $\begin{cases} d_s \\ c_t \end{cases}$

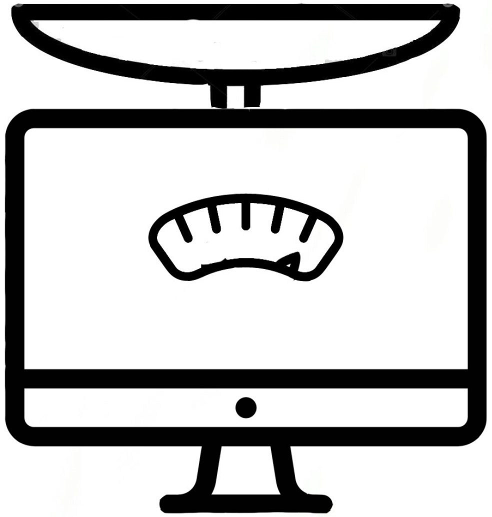


$\rightarrow$ Automatic $O(a)$ improvement!!

bulk: c_{SW}

time boundaries: $\begin{cases} c_t \\ \tilde{c}_t \end{cases}$

counterterms for the operators: c_T,c_A...



- In practice, we used also c_{SW} : same bulk action for sea and valence quarks (not for O(a) imp.)

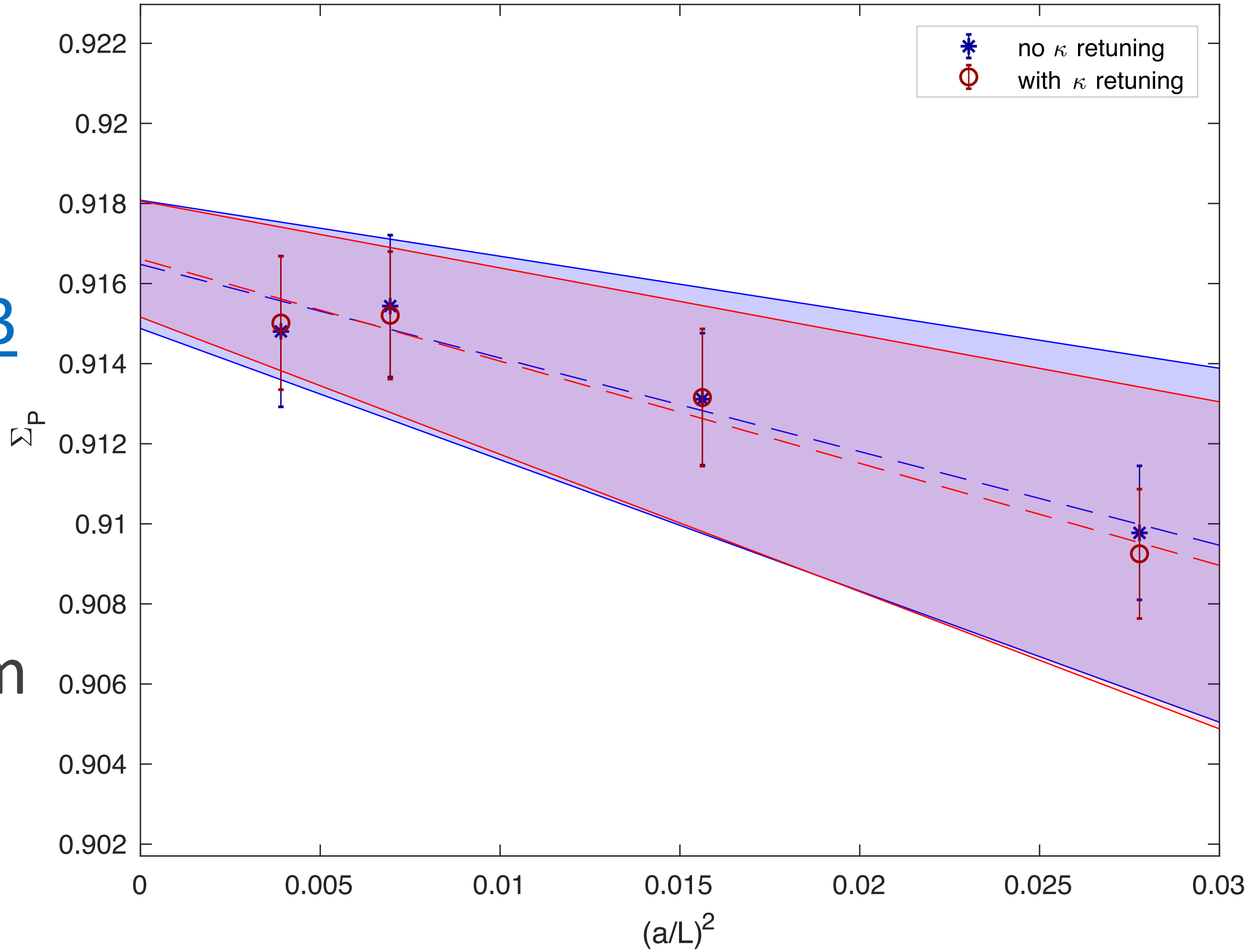
- Price: need extra renormalisation parameter z_f to restore parity

- tuning of z_f $\leftrightarrow g_A^{ud}(x_0) = 0$

- Chiral limit: take k_c^{SF} from 1802.05243

- A check :

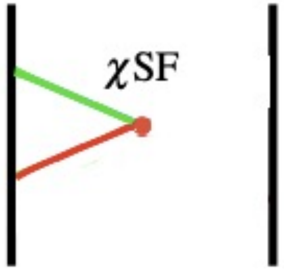
- If you tune $k_c^{\chi SF}$ together with z_f from $\tilde{\partial}_0 g_A^{ud}(x_0)$=0 ,
  results are unaffected



Renormalisation conditions:

$$Z_P^{\chi SF}\left(g_0^2,\frac{L}{a}\right)\frac{g_P^{ud}(\frac{T}{2})}{\sqrt{g_1^{ud}}}=\left[\frac{g_P^{ud}(\frac{T}{2})}{\sqrt{g_1^{ud}}}\right]^{tree\ level}$$

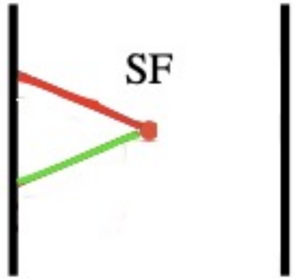
$$Z_P^{SF}\left(g_0^2,\frac{L}{a}\right)\frac{f_P(\frac{T}{2})}{\sqrt{f_1}}=\left[\frac{f_P(\frac{T}{2})}{\sqrt{f_1}}\right]^{tree\ level}$$



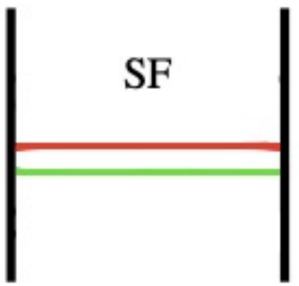
g_P^{ud}

g_1^{ud}

Chiral non-singlet trasformation of fermion fields



f_P



f_1

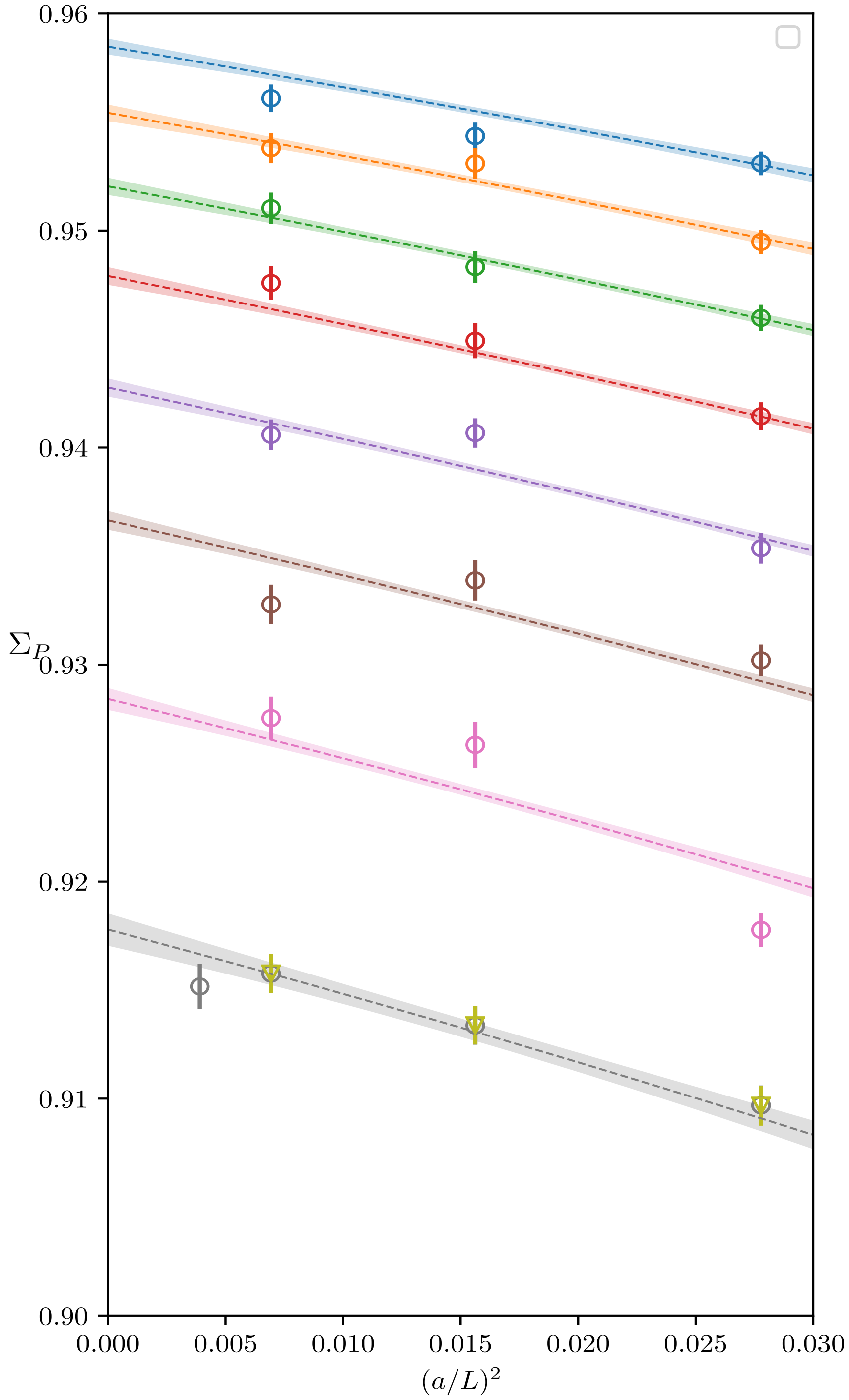
Σ_P^Y with Y= χSF, SF have the same continuum limit $\sigma_P(u)$

$$\Sigma_P^Y(g_0^2,\frac{a}{L})=\frac{Z_P^Y(g_0^2,\frac{2L}{a})}{Z_P^Y(g_0^2,\frac{L}{a})}$$

High energy: setup and data

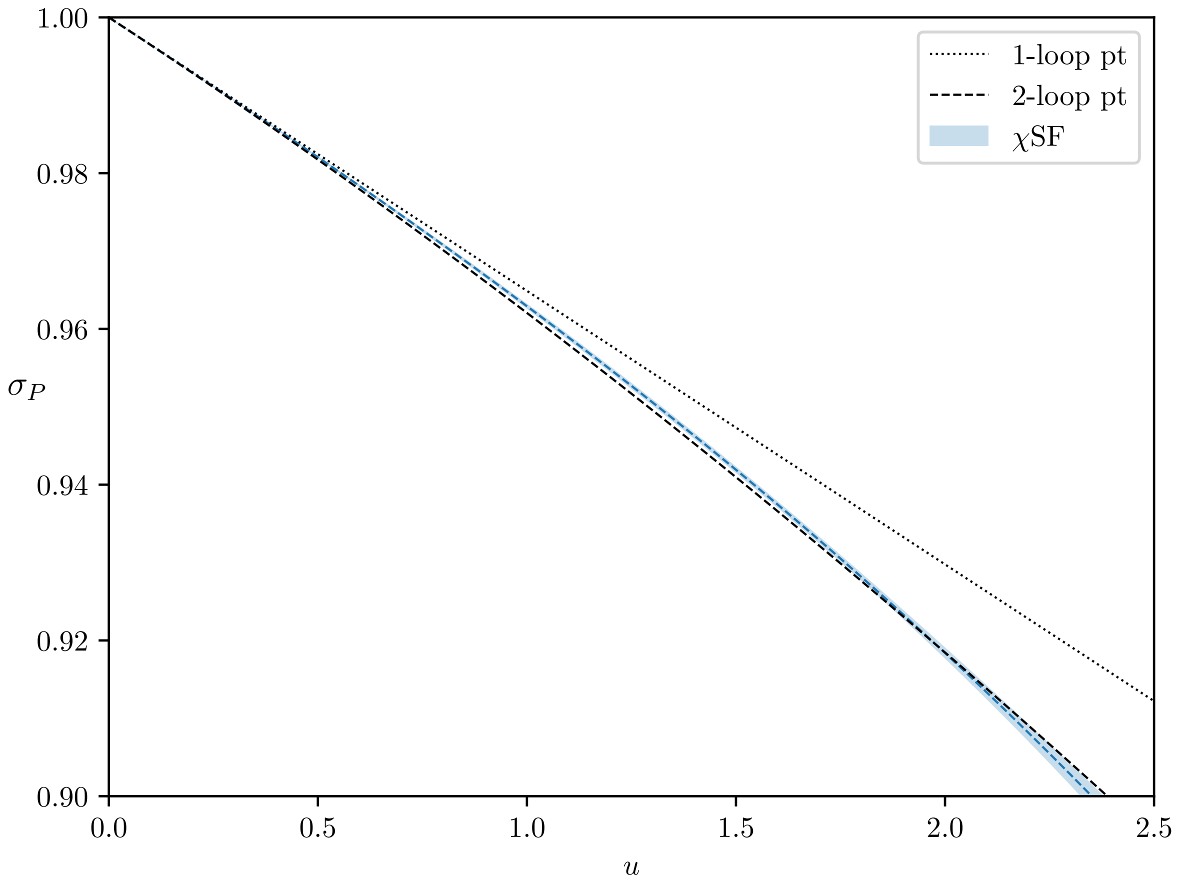
- $\frac{\mu_0}{2} \sim 2\ GeV \lesssim \mu \lesssim M_W$
- Simulations at 8 values of u, at 3 lattice spacings L/a=6,8,12 (also 16 at the switching scale $\mu_0/2$)
- Plaquette gauge action, O(a) non-perturbatively improved fermion action
- We extract $\sigma_P(u)$ from a global fit in u and a/L

$$\Sigma_P\left(u, \frac{a}{L}\right) = \sigma_P(u) + \rho_P(u)\left(\frac{a}{L}\right)^2$$



High energy : results

- Good agreement with perturbation theory…



- ….and also with 1802.05243 !

Pt theory+ $R^{(k)}$

$$R^{(k)} = \frac{\bar{m}(2^k\mu_0)}{\bar{m}(\mu_0/2)} = \prod_{n=0}^{k} \sigma_P(u_n)$$

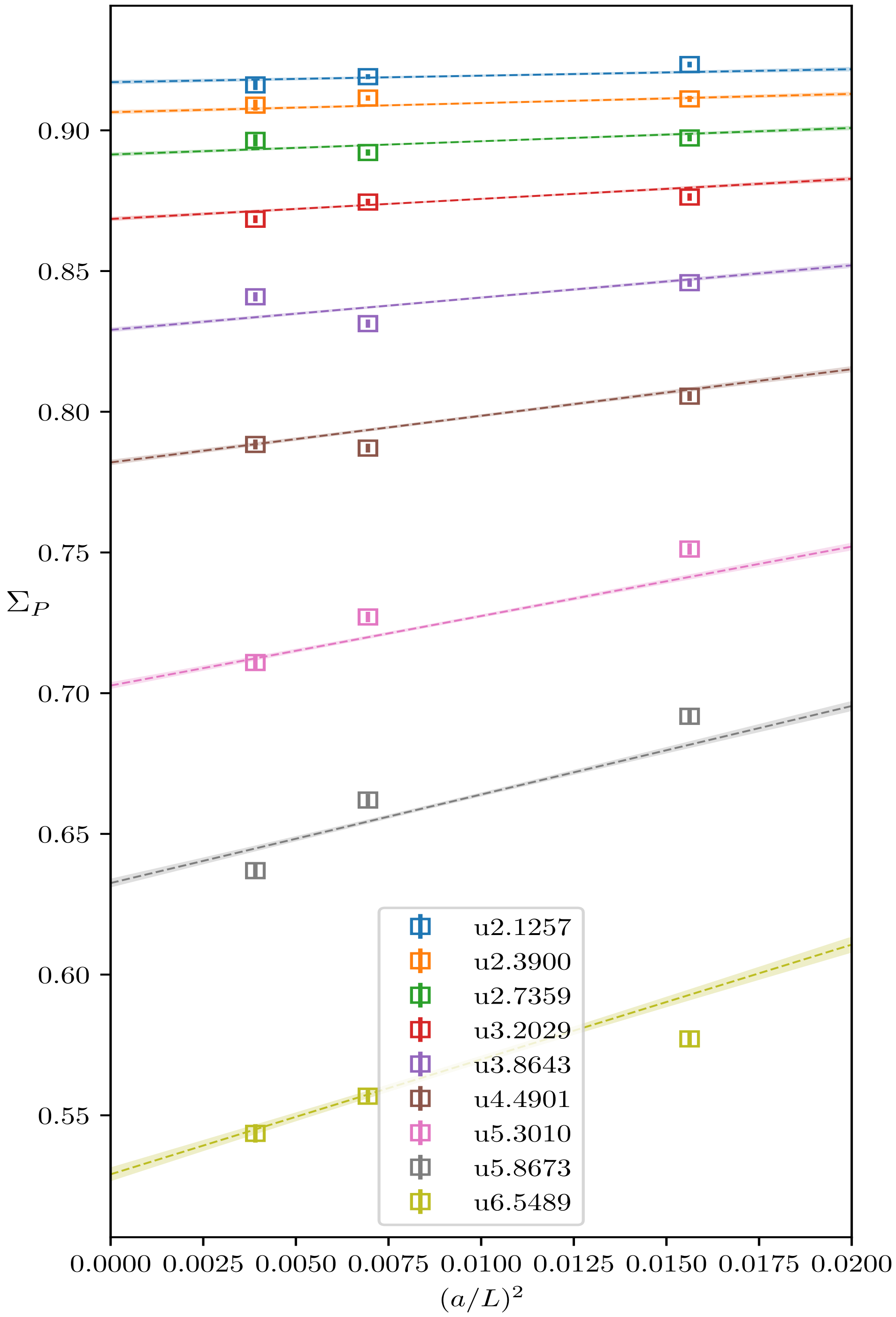
$$M^{RGI} = \frac{M^{RGI}}{\bar{m}(\mu_{pt})} \frac{\bar{m}(\mu_{pt})}{\bar{m}(\mu_0/2)} \frac{\bar{m}(\mu_0/2)}{\bar{m}(\mu_{had})} \bar{m}(\mu_{had})$$

$$\frac{M^{RGI}}{\bar{m}(\mu_0/2)}$$

χSF	SF
1.7596(74)	1.7505(89)

Low energy: setup and data

- $\Lambda_{QCD} \lesssim \mu \lesssim \frac{\mu_0}{2} \sim 2\ GeV$
- Simulations at 7 values of u, at 3 lattice spacings L/a=8,12,16
- Lüscher-Weisz gauge action, O(a) non-perturbatively improved fermion action
- Work in progress! Preliminary results



Low energy: results

$$M^{RGI} = \frac{M^{RGI}}{\overline{m}(\mu_{pt})} \frac{\overline{m}(\mu_{pt})}{\overline{m}(\mu_0/2)} \frac{\overline{\boldsymbol{m}}(\boldsymbol{\mu_0/2})}{\overline{\boldsymbol{m}}(\boldsymbol{\mu_{had}})} \overline{m}(\mu_{had})$$

- $\mu_{had}/\left(\frac{\mu_0}{2}\right)$ not necessarily a factor of 2 $\rightarrow$ use of $\tau(\bar{g})$ could be more handy

- We extract $\tau(\bar{g})/\beta(\bar{g})$ from a global fit in u and a/L

$$\Sigma_P\left(u, \frac{a}{L}\right) = \sigma_P(u) + \rho_P(u)\left(\frac{a}{L}\right)^2 \text{, with } \sigma_P(u) = exp\left\{-\int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dx \frac{\tau(x)}{\beta(x)}\right\}$$

- $R^{(had)} = \frac{\overline{m}(\mu_0/2)}{\overline{m}(\mu_{had})} = exp\left\{-\int_{\sqrt{\sigma(u_0)}}^{\sqrt{u_{had}}} dx \frac{\tau(x)}{\beta(x)}\right\}$

χSF	SF
0.5203(61)	0.5226(43)

Conclusion

- Satisfactory comparison with 1802.05243

  ⟶ a non-trivial check for universality of the 2 theories!

- Future: four fermions operators. O(a) improvement of χSF really useful here!

In the meantime: apply the analysis to other bilinear operators…

For example the tensor operator!

Stay tuned to Giulia Maria de Divitiis talk!

(next talk)

Operator renormalization & improvement for $N_f = 3$ QCD in a χSF setup

Some side dishes…

How do we choose fits?

I would like…	To go directly to the running of the masses	To reconstruct the anomalous dimension $\tau(\bar{g})$
Full control of the extrapolations	• $\sigma_P(u) = \lim_{a\to 0} \Sigma_P\ (g_0^2, \frac{a}{L})$ u by u fit • $R^{(k)} = \prod_{n=0}^{k} \sigma_P(u_n)$	• $\sigma_P(u) = \lim_{a\to 0} \Sigma_P\ (g_0^2, \frac{a}{L})$ $\lim_{a\to 0} \Sigma_P\ (g_0^2, \frac{a}{L})$ u by u fit • $\sigma_P(u) = exp\left\{- \int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dx\, \frac{\tau(x)}{\beta(x)}\right\}$ • $R^{(k)} = exp\left\{- \int_{\sqrt{u_k}}^{\sqrt{\sigma(u_0)}} dx\, \frac{\tau(x)}{\beta(x)}\right\}$
To exclude some data points/ $\neq$ u for $\neq$ a/L	• $\Sigma_P\left(u, \frac{a}{L}\right) = \sigma_P(u) + \rho_P(u)(\frac{a}{L})^2$ global fit in u, a/L • $R^{(k)} = \prod_{n=0}^{k} \sigma_P(u_n)$	• $\Sigma_P\left(u, \frac{a}{L}\right) = \sigma_P(u) + \rho_P(u)(\frac{a}{L})^2$, with $\sigma_P(u)$ as above (global fit in u, a/L) • $R^{(k)} = exp\left\{- \int_{\sqrt{u_k}}^{\sqrt{\sigma(u_0)}} dx\, \frac{\tau(x)}{\beta(x)}\right\}$

- High energy: we choose
- Low energy: we choose

Were there some more checks on χSF before this one?

A quick (partial) review:

- First tests on automatic O(a) improvement (quenched setup) →
- Perturbation theory plugged in →
- Tests centred on Σ_P →
- Dynamical fermions! Z_A and Z_V with high precision →

Some references

- arXiv 1012.2500
- arXiv 1603.00046
- arXiv 1208.4591/.4661
- arXiv 1808.09236

What is χSF exactly? How do you derive it from SF?

- A very quick answer: it is a renormalization scheme that derives from SF scheme through a chiral non-singlet trasformation of the fermions

$$\psi = R\left(\frac{\pi}{2}\right)\psi' \qquad \bar{\psi} = \bar{\psi}' R\left(\frac{\pi}{2}\right) \qquad \text{with } R(\alpha) = e^{i\frac{\alpha}{2}\gamma_5\tau^3}$$

- It allow us to exploit the mechanism of automatic O(a) improvement:

$$< O_{even} > = < O_{even} >^{cont} + O(a^2)$$

- In SF, we can't:

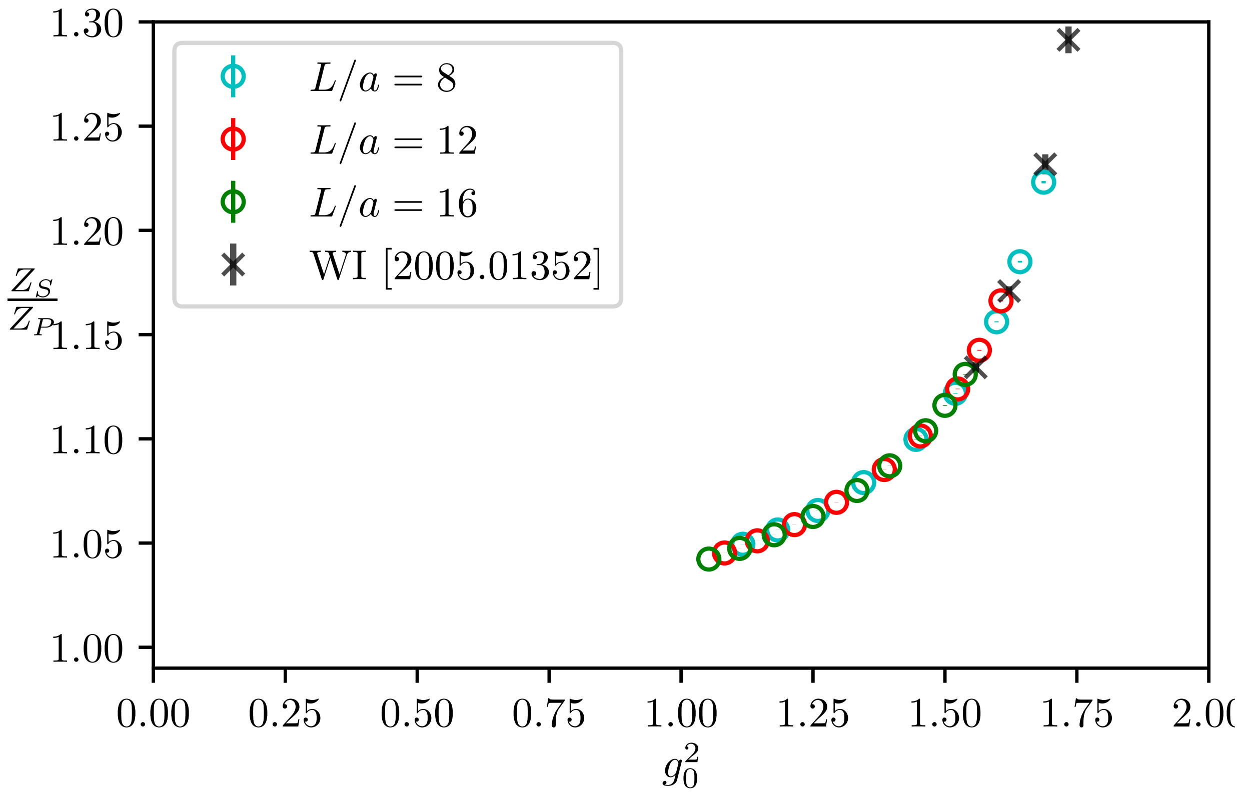
What is z_f? Or: what happens to χSF on lattice?

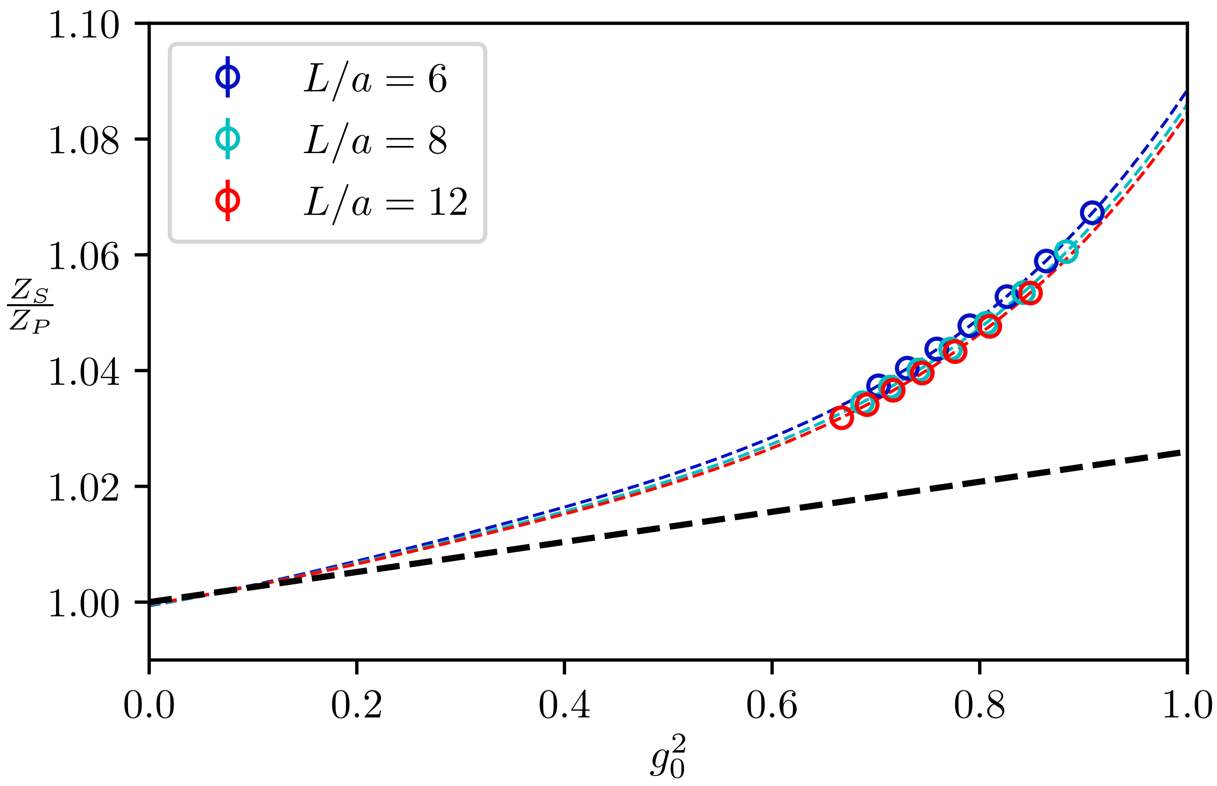
- The redefinition of the fermions modifies the symmetries as well
- New chirally-rotated parity!

- Now we are on the lattice: Wilson fermions

- → we need to restore P_5 !
- That is the role of the boundary counterterm multiplied by z_f
- Correspondance between SF and χSF is restored on the lattice

Some more things you'll find in the proceeding/paper-1





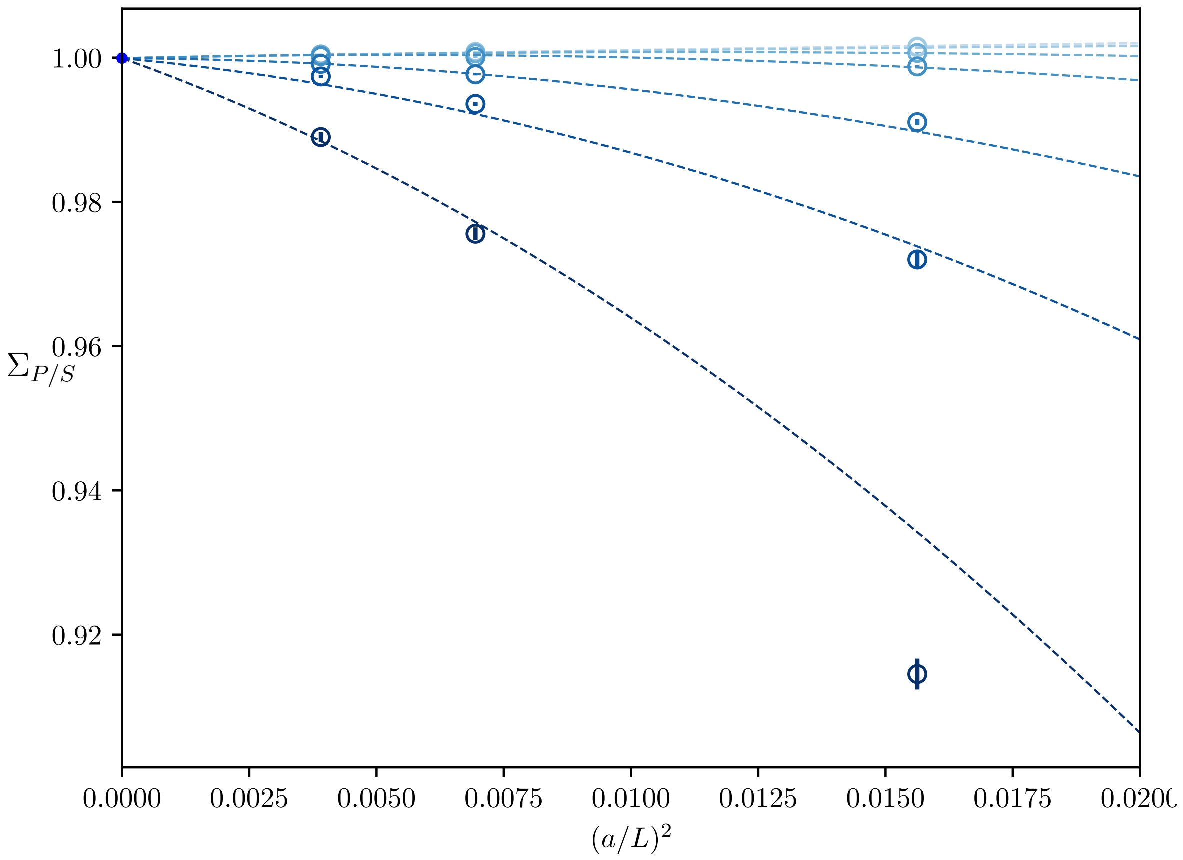
Z_S / Z_P in low energy regime.

WI: a previous result based on a Ward Identity method

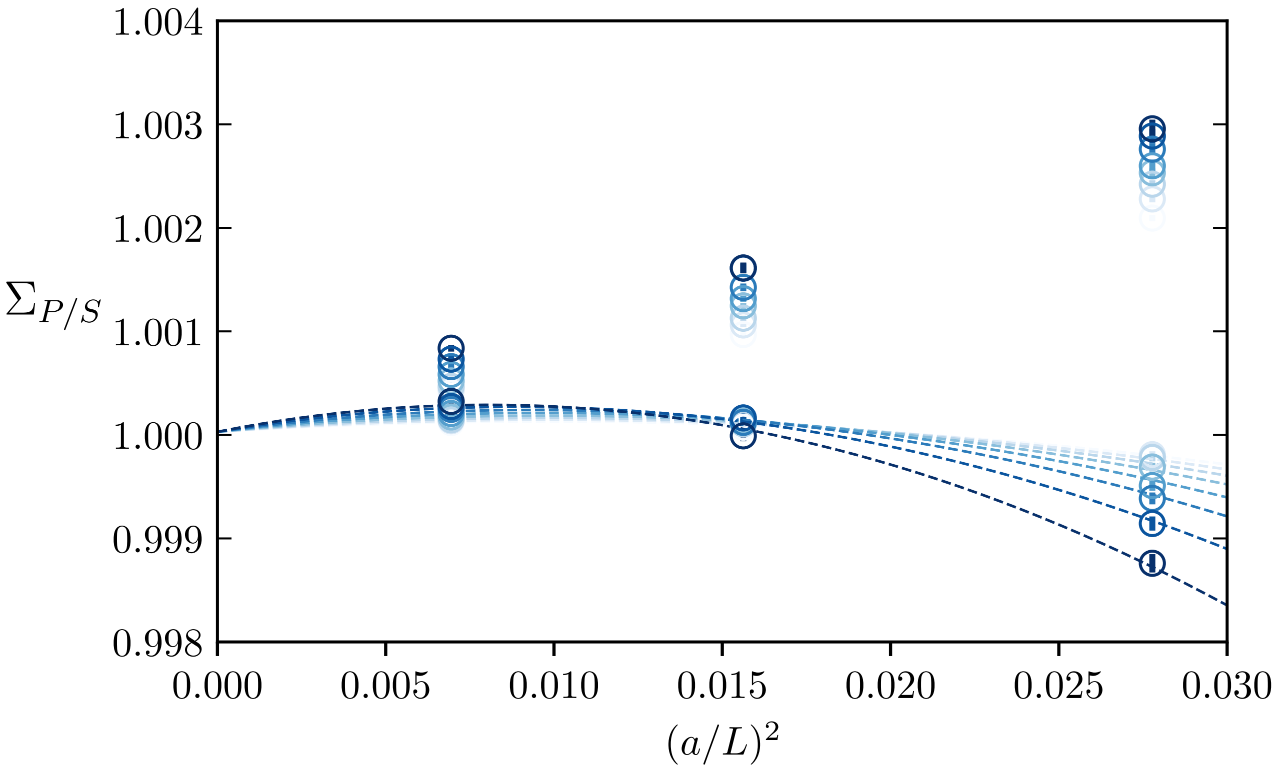
Z_S / Z_P in high energy regime.

Dotted line: perturbative results

Some more things you'll find in the proceeding/paper-2



$\Sigma_P\ /\Sigma_S$ in low energy regime.



$\Sigma_P\ /\Sigma_S$ in high energy regime.

On curves: data after subtraction of 1-loop terms