Operator renormalization & improvement for $N_f = 3$ QCD in a χSF setup

ALPHA Collaboration

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Flavor-non-singlet tensor current

"Tensor currents are the only quark bilinear operators lacking a non-perturbative determination of their renormalisation group (RG) running between hadronic and electroweak scales ... "

[C Pena and D Preti [ALPHA], [arXiv:1706.06674 [hep-lat]], Eur.Phys.J.C **78** (2018) 7, 575]

[L Chimirri *et al.* [ALPHA], [arXiv:1910.06759 [hep-lat]], PoS LATTICE2019 (2020) 212]

Flavor-non-singlet tensor current

$$T^a_{\mu\nu}(x) = i\bar{\psi}(x)\,\sigma_{\mu\nu}\,\tfrac{1}{2}\tau^a\,\psi(x),$$
$$\sigma_{\mu\nu} \equiv \tfrac{i}{2}[\gamma_\mu, \gamma_\nu]$$

- theoretical interest
- phenomenological interest for effective Hamiltonian amplitudes (rare heavy meson decays, neutron beta decays, BSM ...)

$$\mathcal{A} = \langle f|\mathcal{H}_{eff}|i\rangle = C_W(\mu)\langle f|\mathcal{O}_{ren}(\mu)|i\rangle$$
$$\mathcal{O} \sim (\bar{l}\sigma_{\mu\nu}e)(\bar{q}\sigma_{\mu\nu}u),\ G_{\mu\nu}(\bar{q}_i\sigma_{\mu\nu}q_j)\ldots$$

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RG flow

- Renormalized couplings and operators satisfy the RG flow equations. In mass-independent scheme:

$$\mu\frac{\partial}{\partial\mu}g_{\mathrm{R}}(\mu) = \beta(g_{\mathrm{R}}(\mu), \cancel{m_{\mathrm{R}}(\mu)})$$

$$\mu\frac{\partial}{\partial\mu}m_{\mathrm{R}}(\mu) = \tau(g_{\mathrm{R}}(\mu), \cancel{m_{\mathrm{R}}(\mu)})\, m_{\mathrm{R}}(\mu)$$

$$\mu\frac{\partial}{\partial\mu}T_{\mathrm{R}}(\mu) = \gamma(g_{\mathrm{R}}(\mu), \cancel{m_{\mathrm{R}}(\mu)})\, T_{\mathrm{R}}(\mu)$$

- Velocities β, τ and γ have perturbative expansion

$$\beta(g_{\mathrm{R}}) \overset{g_{\mathrm{R}}\to 0}{\sim} -{g_{\mathrm{R}}}^3(b_0 + b_1{g_{\mathrm{R}}}^2 + b_2{g_{\mathrm{R}}}^4 + \mathcal{O}({g_{\mathrm{R}}}^6))$$

$$\tau(g_{\mathrm{R}}) \overset{g_{\mathrm{R}}\to 0}{\sim} -{g_{\mathrm{R}}}^2(d_0 + d_1{g_{\mathrm{R}}}^2 + d_2{g_{\mathrm{R}}}^4 + \mathcal{O}({g_{\mathrm{R}}}^6))$$

$$\gamma(g_{\mathrm{R}}) \overset{g_{\mathrm{R}}\to 0}{\sim} -{g_{\mathrm{R}}}^2(\gamma_0 + \gamma_1{g_{\mathrm{R}}}^2 + \gamma_2{g_{\mathrm{R}}}^4 + \mathcal{O}({g_{\mathrm{R}}}^6))$$

with universal coefficients b_0, b_1, d_0, γ_0.

RGI quantities

- Integration yields to dimensionful finite constants, the renormalization group invariants (RGI)

$$\Lambda = \mu \left[b_0 {g_{\mathrm{R}}}^2(\mu)\right]^{-\frac{b_1}{2b_0^2}} \exp\left\{-\int_0^{g_{\mathrm{R}}(\mu)} dg \left[\frac{1}{\beta(g)} + \frac{1}{b_0 g^3} - \frac{b_1}{b_0^2 g}\right] - \frac{1}{2b_0 {g_{\mathrm{R}}}^2(\mu)}\right\}$$

$$M = m_{\mathrm{R}}(\mu) \left[2b_0 {g_{\mathrm{R}}}^2(\mu)\right]^{-\frac{d_0}{2b_0}} \exp\left\{-\int_0^{g_{\mathrm{R}}(\mu)} dg \left[\frac{\tau(g)}{\beta(g)} - \frac{d_0}{b_0 g}\right]\right\}$$

$$T = T_{\mathrm{R}}(\mu) \left[\frac{{g_{\mathrm{R}}}^2(\mu)}{4\pi}\right]^{-\frac{\gamma_0}{2b_0}} \exp\left\{-\int_0^{g_{\mathrm{R}}(\mu)} dg \left[\frac{\gamma(g)}{\beta(g)} - \frac{\gamma_0}{b_0 g}\right]\right\}$$

- Splitting of the integral

$$T = \underbrace{\frac{T}{T_{\mathrm{R}}(\mu_{\mathrm{pt}})}}_{PT} \underbrace{\frac{T_{\mathrm{R}}(\mu_{\mathrm{pt}})}{T_{\mathrm{R}}(\mu_0/2)}}_{SF} \underbrace{\frac{T_{\mathrm{R}}(\mu_0/2)}{T_{\mathrm{R}}(\mu_{\mathrm{had}})}}_{GF} T_{\mathrm{R}}(\mu_{\mathrm{had}})$$

Step Scaling Functions

Integration of RG equations between two scales.
Factor $\mu_1/\mu_2 = 2$ chosen

$$\begin{cases} u \equiv {g_{\rm R}}^2(\mu) \\ \sigma(u) \equiv {g_{\rm R}}^2(\mu/2) \end{cases} \qquad 2 = \exp\left\{ - \int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dg \, \frac{1}{\beta(g)} \right\}$$

$$\sigma_T(u) = \exp\left\{ - \int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dg \, \frac{\gamma(g)}{\beta(g)} \right\}$$

Scale(μ) evolution as Finite-size(L) scaling

$$\mu = \frac{1}{L}, \quad u \equiv {g_{\rm R}}^2(L)$$

$$\sigma(u) = \lim_{a\to 0} \Sigma(u, a/L) \qquad \Sigma(u, a/L) = {g_{\rm R}}^2(2L)$$

$$\sigma_T(u) = \lim_{a\to 0} \Sigma_T(u, a/L) \qquad \Sigma_T(u, a/L) = \frac{Z_T(g_0^2, a/2L)}{Z_T(g_0^2, a/L)}$$

"Actual" RG flow: simulation details

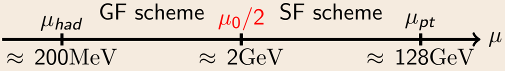
- Gauge configurations

[I Campos *et al.* [ALPHA], [arXiv:1802.05243 [hep-lat]],Eur.Phys.J.C **78**(2018) 5, 387]

SF boundary conditions, `openQCD`, $N_f = 3$ massless Wilson-clover fermions, non-perturbative $c_{\rm SW}$, `RHMC` reweighting factors, SF-bc choices $C = C' = 0$ and $\theta = 0.5$

Scheme switching:

- ${g_{\rm R}}^2_{SF}$: Schrödinger functional scheme
  plaquette gauge action



- ${g_{\rm R}}^2_{GF}$: Gradient flow scheme
  tree-level improved Lüscher–Weisz gauge action

- Valence quarks

χSF boundary conditions, non-perturbative z_f

[S Sint, [arXiv:1008.4857 [hep-lat]], Nucl. Phys. B **847** (2011) 491]

[M Dalla Brida, S Sint and P Vilaseca, [arXiv:1603.00046 [hep-lat]], JHEP **1608** (2016) 102]

[M Dalla Brida *et al.*, [arXiv:1808.09236 [hep-lat]], Eur. Phys. J. C **79** (2019) 1, 23]

[M Dalla Brida, M Papinutto and P Vilaseca, [arXiv:1605.09053 [hep-lat]], PoS LATTICE2015 (2016) 252]

χSF Chirally Rotated Schrödinger Functional

- Universality with the SF (same continuum limit)

$$R = \exp\left(i\frac{\alpha}{2}\gamma_5\tau^3\right)\Big|_{\alpha=\pi/2} \qquad \begin{cases}\psi & \to \psi' = R\psi \\ \bar\psi & \to \bar\psi' = \bar\psi R\end{cases}$$

$$P_\pm \equiv \tfrac{1}{2}(1 \pm \gamma_0) \to \boxed{Q_\pm \equiv \tfrac{1}{2}(1 \pm i\gamma_0\gamma_5\tau^3)}$$

- Determination of m_{cr} and z_f quite independent. Our choice:

$$\begin{cases} m = \dfrac{\tilde\partial_0 f_{\mathrm{A},I}^{ud}(x_0)}{2f_{\mathrm{P}}^{ud}(x_0)}\Big|_{x_0=L/2} = 0 & m_{cr} \text{ tuning } \textit{inherited} \\ g_{\mathrm{A}}^{ud}(x_0)\Big|_{x_0=L/2} = 0 & z_f \text{ tuning} \end{cases}$$

$$S_f = a^4 \sum_{x_0=0}^{T}\sum_{\mathbf{x}} \bar\psi(x)(\mathcal{D}_W + \delta\mathcal{D}_W + m_0)\psi(x)$$

$$\delta\mathcal{D}_W\psi(x) = (\delta_{x_0,0} + \delta_{x_0,T})\left[(z_f - 1) + (d_s - 1)\, a\mathrm{D}_s\right]\psi(x)$$

[I Campos *et al.*, [arXiv:1910.01898 [hep-lat]], PoS LATTICE2019 (2019) 202]
[Ludovica Pirelli's talk, this conference]

- Automatic $O(a)$ improvement

$$g_{even} = g_{even}^{\mathrm{continuum}} + O(a^2)$$
$$g_{odd} = O(a)$$

Renormalization condition

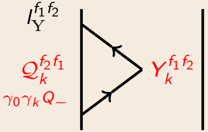
- SF boundary conditions

Improvement

$$T^{\mathrm{I}}_{\mu\nu} = T_{\mu\nu} + a\, c_{\mathrm{T}}(g_0^2)\, (\tilde{\partial}_\mu V_\nu - \tilde{\partial}_\nu V_\mu)$$

Renormalization condition

$$Z_{\mathrm{T}}(g_0, a/L) \frac{k^{\mathrm{I}}_{\mathrm{T}}(L/2)}{\sqrt{k_1}} = \frac{k_{\mathrm{T}}(L/2)}{\sqrt{k_1}}\Bigg|_{\text{Tree Level}}$$

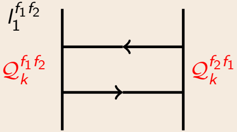


- χSF boundary conditions

$O(a)$ Improvement not needed

$$T^{\mathrm{I}}_{\mu\nu} = T_{\mu\nu} + a\, c_{\mathrm{T}}(g_0^2)\, (\tilde{\partial}_\mu V_\nu - \tilde{\partial}_\nu V_\mu)$$

Renormalization condition



$$Z_{\mathrm{T}}(g_0, a/L) \frac{l^{ud}_{\mathrm{T}}(L/2)}{\sqrt{l^{ud}_1}} = \frac{l^{ud}_{\mathrm{T}}(L/2)}{\sqrt{l^{ud}_1}}\Bigg|_{\text{Tree Level}}$$

"electric" tensor T_{0k}

$$Z_{\mathrm{T}}(g_0, a/L) \frac{l^{ud}_{\tilde{\mathrm{T}}}(L/2)}{\sqrt{l^{ud}_1}} = \frac{l^{ud}_{\tilde{\mathrm{T}}}(L/2)}{\sqrt{l^{ud}_1}}\Bigg|_{\text{Tree Level}}$$

"magnetic" tensor $\tilde{T}_{0k} = -\frac{1}{2}\varepsilon_{0kij} T_{ij}$

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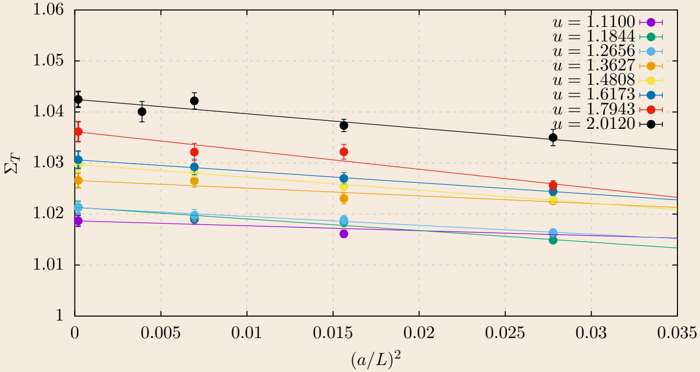
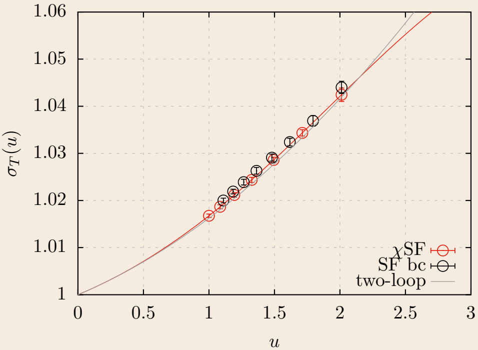
Continuum limit (High Energy region)

$$\Sigma_{\mathrm{T}}(u, a/L) = \frac{Z_T(g_0^2, a/2L)}{Z_T(g_0^2, a/L)}$$
$$= \sigma_{\mathrm{T}}(u) + \rho_{\mathrm{T}}(u) \left(\frac{a}{L}\right)^2$$



SSF in the continuum (High Energy region)

$$\sigma_T(u) = \lim_{a\to 0} \Sigma_T(u, a/L)$$

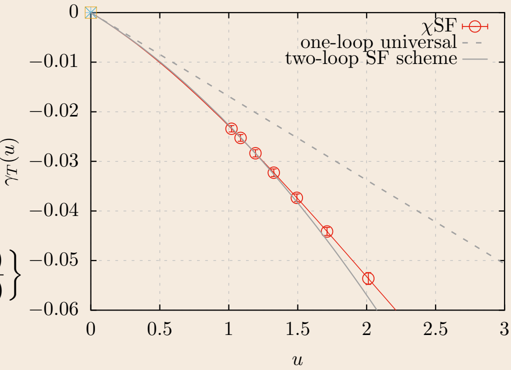


SF data (black circles) from slides by Fabian Joswig @Lattice 2019

[L Chimirri *et al.* [ALPHA], [arXiv:1910.06759 [hep-lat]], PoS LATTICE2019 (2020) 212]

Anomalous dimension (High Energy region)

$$\sigma_T(u) = \exp\left\{ - \int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dg \, \frac{\gamma_{SF}(g)}{\beta_{SF}(g)} \right\}$$



$$\mu \frac{\partial}{\partial \mu} T_{\mathrm{R}}(\mu) = \gamma(g_{\mathrm{R}}(\mu)) \, T_{\mathrm{R}}(\mu)$$

$$\gamma(g_{\mathrm{R}}) \overset{g_{\mathrm{R}}\to 0}{\sim} -g_{\mathrm{R}}{}^2(\gamma_0 + \gamma_1 g_{\mathrm{R}}{}^2 + \gamma_2 g_{\mathrm{R}}{}^4 + \mathcal{O}(g_{\mathrm{R}}{}^6)) \qquad \begin{cases} \gamma_0 = \frac{2C_F}{(4\pi)^2} \\ \gamma_1 = 0.0063609(8) - 0.00018863(5) \times N_f \end{cases}$$

two-loop computation in:

[C Pena and D Preti [ALPHA], [arXiv:1706.06674 [hep-lat]], Eur.Phys.J.C **78** (2018) 7, 575]

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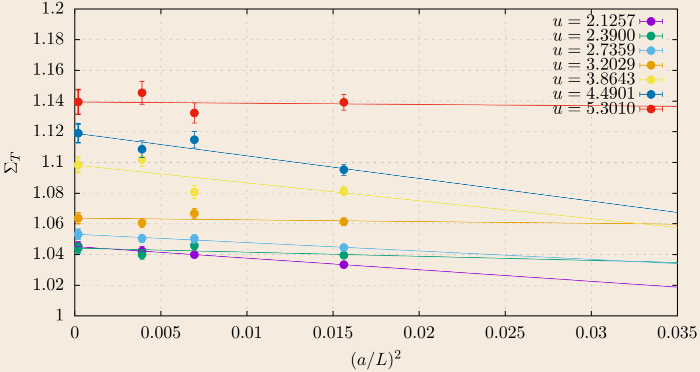
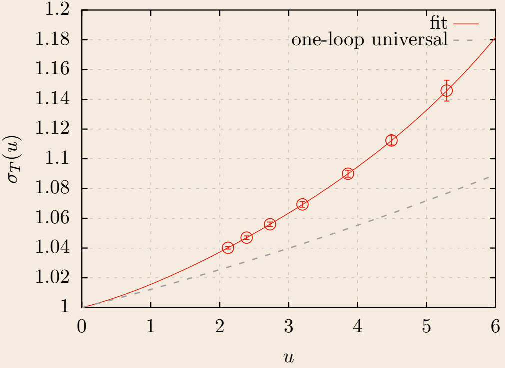
Continuum limit (Low Energy region)

$$\Sigma_{\rm T}(u, a/L) = \frac{Z_T(g_0^2, a/2L)}{Z_T(g_0^2, a/L)}$$
$$= \sigma_{\rm T}(u) + \rho_{\rm T}(u)\left(\frac{a}{L}\right)^2$$



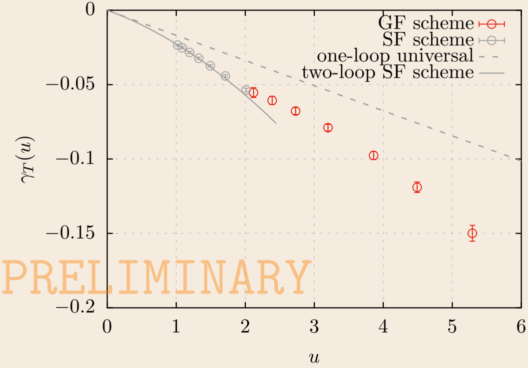
SSF in the continuum (Low Energy region)

$$\sigma_T(u) = \lim_{a \to 0} \Sigma_T(u, a/L)$$



Anomalous dimension (Low Energy region)

$$\sigma_T(u) = \exp\left\{ - \int_{\sqrt{u}}^{\sqrt{\sigma(u)}} dg \, \frac{\gamma_{GF}(g)}{\beta_{GF}(g)} \right\}$$



Conclusion & Outlooks

- Agreement with PT at high energy
- Universality with SF data
- Absence of mixing (coefficient c_T not needed)
- $\Longrightarrow$ Definitions with different normalizations
- $\Longrightarrow$ Four fermions analysis

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Correlation functions

flavours $f_1 f_2 = u, d, u', d'$

bulk operator $X = V_0, A_0, S, P \quad Y_k = V_k, A_k, T_{k0}, \widetilde{T}_{k0}$

SF

$$f_{\mathrm{X}}(x_0) = -\frac{1}{2}\left\langle X^{f_1 f_2}(x)\mathcal{O}_5^{f_2 f_1}\right\rangle$$

$$k_{\mathrm{Y}}(x_0) = -\frac{1}{6}\sum_{k=1}^{3}\left\langle Y_k^{f_1 f_2}(x)\mathcal{O}_k^{f_2 f_1}\right\rangle$$

$$f_1 = -\frac{1}{2}\left\langle \mathcal{O}_5^{f_1 f_2}\mathcal{O}_5^{\prime f_2 f_1}\right\rangle$$

$$k_1 = -\frac{1}{6}\sum_{k=1}^{3}\left\langle \mathcal{O}_k^{f_1 f_2}\mathcal{O}_k^{\prime f_2 f_1}\right\rangle$$

$$\mathcal{O}_5^{f_1 f_2} = a^6 \sum_{\mathrm{y,z}} \bar{\zeta}_{f_1}(\mathrm{y}) P_+ \gamma_5 \zeta_{f_2}(\mathrm{z})$$

$\vdots$

$$P_\pm \equiv \frac{1}{2}(1 \pm \gamma_0)$$

χSF

$$g_{\mathrm{X}}(x_0) = -\frac{1}{2}\left\langle X^{f_1 f_2}(x)\mathcal{Q}_5^{f_2 f_1}\right\rangle$$

$$l_{\mathrm{Y}}(x_0) = -\frac{1}{6}\sum_{k=1}^{3}\left\langle Y_k^{f_1 f_2}(x)\mathcal{Q}_k^{f_2 f_1}\right\rangle$$

$$g_1 = -\frac{1}{2}\left\langle \mathcal{Q}_5^{f_1 f_2}\mathcal{Q}_5^{\prime f_2 f_1}\right\rangle$$

$$l_1 = -\frac{1}{6}\sum_{k=1}^{3}\left\langle \mathcal{Q}_k^{f_1 f_2}\mathcal{Q}_k^{\prime f_2 f_1}\right\rangle$$

$$\mathcal{Q}_5^{uu'} = a^6 \sum_{\mathrm{y,z}} \bar{\zeta}_u(\mathrm{y}) \gamma_0 \gamma_5 Q_- \zeta_{u'}(\mathrm{z})$$

$\vdots$

$$Q_\pm \equiv \frac{1}{2}(1 \pm i\gamma_0\gamma_5)$$

Universality relations

$l_{even} = l_{even}^{\text{continuum}} + O(a^2)$	Automatic $O(a)$ improvement
$l_{odd} = 0 + O(a)$	Cutoff effect linear in a

SF-χSF continuum relations:

$$f_{\rm A} = g_{\rm A}^{uu'} = g_{\rm A}^{dd'} = -ig_{\rm V}^{ud} = ig_{\rm V}^{du}$$
$$0 = f_{\rm V} = g_{\rm V}^{uu'} = g_{\rm V}^{dd'} = -ig_{\rm A}^{ud} = ig_{\rm A}^{du}$$
$$f_{\rm P} = ig_{\rm S}^{uu'} = -ig_{\rm S}^{dd'} = g_{\rm P}^{ud} = g_{\rm P}^{du}$$
$$0 = f_{\rm S} = ig_{\rm P}^{uu'} = -ig_{\rm P}^{dd'} = g_{\rm S}^{ud} = g_{\rm S}^{du}$$

$$k_{\rm V} = l_{\rm V}^{uu'} = l_{\rm V}^{dd'} = -il_{\rm A}^{ud} = il_{\rm A}^{du}$$
$$0 = k_{\rm A} = l_{\rm A}^{uu'} = l_{\rm A}^{dd'} = -il_{\rm V}^{ud} = il_{\rm V}^{du}$$
$$k_{\rm T} = il_{\widetilde{\rm T}}^{uu'} = -il_{\widetilde{\rm T}}^{dd'} = l_{\rm T}^{ud} = l_{\rm T}^{du}$$
$$0 = k_{\widetilde{\rm T}} = il_{\rm T}^{uu'} = -il_{\rm T}^{dd'} = l_{\widetilde{\rm T}}^{ud} = l_{\widetilde{\rm T}}^{du}$$

$$f_1 = g_1^{uu'} = g_1^{dd'} = g_1^{ud} = g_1^{du}$$
$$k_1 = l_1^{uu'} = l_1^{dd'} = l_1^{ud} = l_1^{du}$$

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SF and GF schemes

- SF scheme

$$g_{\mathrm{R}}{}^{2}_{SF} = k \left(\frac{\partial \Gamma}{\partial \eta} \right)^{-1}$$

$$\beta_{SF}(x) = -x^3 \sum_{n=0}^{3} b_n x^{2n}$$

- GF scheme

$$g_{\mathrm{R}}{}^{2}_{GF} = g_{\mathrm{R}}{}^{2}_{GF}(c = 0.3, x_0 = T/2)\,, \qquad \mu = \frac{1}{cL} = \frac{1}{\sqrt{8t}}$$

$$g_{\mathrm{R}}{}^{2}_{GF}(c, x_0) = \frac{t^2}{\mathcal{N}} \sum_{\mathbf{x}} \left[\tfrac{5}{3} P_{(ij)}(x,t) - \tfrac{1}{12} R^{2x1}_{(ij)}(x,t) \right]^{ZN-flow}_{cL=\sqrt{8t}}$$

$$\beta_{GF}(x) = -\frac{x^3}{\sum_{n=0}^{2} p_n x^{2n}}$$

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