Determination of m_c from $N_f = 2 + 1$ QCD with Wilson fermions

Sjoerd Bouma

Friedrich-Alexander Universität Erlangen-Nürnberg
University of Regensburg

RQCD collaborators: Gunnar Bali, Sara Collins, Wolfgang Söldner

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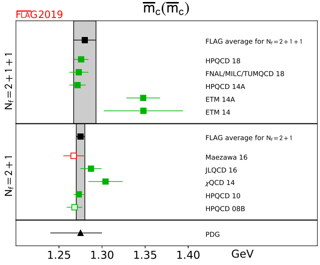
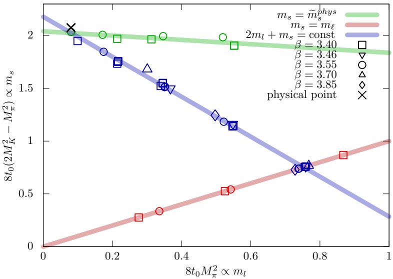
Motivation

- Quark masses are fundamental parameters of the standard model
- Input for many phenomenological predictions, including for BSM physics.
- Not directly measurable (confinement) - values depend on renormalization scheme.
- Charm observables difficult to simulate on the lattice - in between relativistic and non-relativistic regimes; am_c large for many lattice spacings currently used.
- We use 5 lattice spacings down to 0.04 fm to control discretization effects



Ensembles

- CLS-generated ensembles $N_f = 2+1$ Wilson-Clover $\mathcal{O}(a)$ improved fermions.
- 5 lattice spacings ~ 0.085 to 0.04 fm
- Pion masses from 420 MeV down to the physical point
- Three different chiral trajectories:
  - $m_s = m_s^{\rm phys}$
  - $m_s = m_\ell$
  - $\mathrm{Tr}[M_q] = 2m_\ell + m_s = \mathrm{constant}$



- $\Rightarrow$ we can adjust for any 'mistuning' in the fit.
- For each ensemble, simulated 2 heavy quark masses around $m_c^{\rm phys}$ and interpolate the PCAC mass to $\sqrt{8t_0}m_{\overline{D}} := \sqrt{8t_0^{\rm phys}}m_{\overline{D}}^{\rm phys}$, where $\sqrt{8t_0^{\rm phys}} = 0.413(5)$ [arXiv:1608.08900] and $m_{\overline{D}} = 2m_D + m_{D_s}$.

Coordinated Lattice Simulations (CLS): Berlin, CERN, Mainz, UA Madrid, Milano Bicocca, Münster, Odense, Regensburg, Rome I and II, Wuppertal, DESY-Zeuthen, Kraków.

Motivation

- Using the lattice PCAC relation:

$$am_{ij} = \frac{\partial_0 C_{A_0P} + c_A a\partial_0^2 C_{PP}}{2C_{PP}},$$

and the Vector-Ward Identity (VWI) quark masses:

$$am_{q,ij} = \left(\frac{1}{4\kappa_i} + \frac{1}{4\kappa_j} - \frac{1}{2\kappa_{\mathrm{crit}}}\right),$$

we can determine the renormalization-group independent (RGI) mass:

$$m_{ij}^{\mathrm{RGI}} = Z_M m_{ij}\left[1 + (b_A - b_P)am_{q,ij} + (\tilde{b}_A - \tilde{b}_P)a\mathrm{Tr}[M_q]\right] + \mathcal{O}(a^2),$$

where, following arXiv:1906.03445, arXiv:1502.04999, arXiv:1802.05243 :

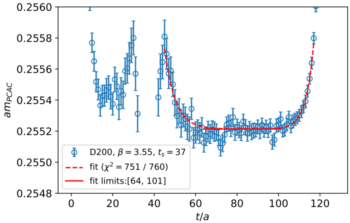
$$Z_M = \frac{M}{m(\mu_{\mathrm{had}})}\frac{Z_A(g_0^2)}{Z_P(g_0^2, a\mu_{\mathrm{had}})},$$

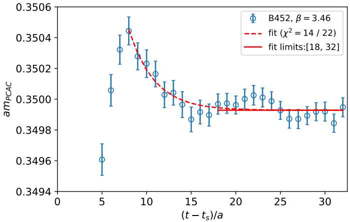
$$\mathrm{Tr}[M_q] \approx 2m_\ell + m_s,$$

$$C_{OO'}(t) = \langle O(t)O'(0)\rangle,$$

$$\partial_0 f(t) = \frac{f(t+a) - f(t-a)}{2a}.$$

PCAC masses





- PCAC masses determined by fitting to a constant in a 'plateau' region.
- Ansatz for boundary effects and contact terms:

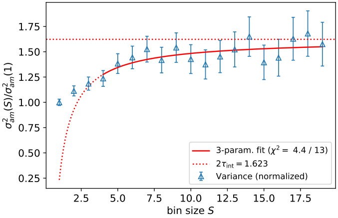
$$am_{\mathrm{PCAC}}(t) \approx am_{\mathrm{PCAC}} + c_1 e^{-b_1 t} + c_2 e^{-b_2 (T_{bd} - t)}.$$

- Plateau defined as the region where

$$4 \cdot \left(c_1 \cdot \exp^{-b_1 t} + c_2 e^{-b_2 (T_{bd} - t)} \right) \leq \Delta^{\mathrm{stat}} am_{\mathrm{PCAC}}(t).$$

Error extrapolation

- Have to deal with autocorrelations in lattice and Monte-Carlo time
- Strategy: estimate errors through a binned jackknife procedure
- Bin in Monte-Carlo time and obtain jackknife error on m_{PCAC} at each bin size S.



- Then extrapolate to infinite bin size using the formula:

$$\frac{\sigma^2[S]}{\sigma^2[1]} \approx 2\tau_{\mathrm{int}} \left(1 - \frac{c_A}{S} + \frac{d_A}{S} e^{-S/\tau_{\mathrm{int}}}\right).$$

For the chiral-continuum extrapolation, the following parametrizations were used:

$$
\begin{aligned}
f_{\chi\mathrm{PT}}(\overline{\mathbb{M}}^2, \delta\mathbb{M}^2, \mathbb{m}_D) = p_0 &+ p_1\overline{\mathbb{M}}^2 + p_2\delta\mathbb{M}^2 + p_7\delta\mathbb{m}_D \\
&+ p_3\overline{\mathbb{M}}^4 + p_4\delta\mathbb{M}^4 \\
&+ p_8\mathbb{m}_D\overline{\mathbb{M}}^2 + p_9\mathbb{m}_D\delta\mathbb{M}^2 + p_{10}\delta\mathbb{m}_D^2 + p_{11}\overline{\mathbb{M}}^2\delta\mathbb{M}^2 + p_{14}\delta\mathbb{m}_D^3,
\end{aligned}
$$

and

$$
\begin{aligned}
f_{\mathrm{latt}}(a^2/t_0^*, \overline{\mathbb{M}}^2, \delta\mathbb{M}^2, \mathbb{m}_D) = \frac{a^2}{t_0^*}\Big[&p_{15} + p_{16}\overline{\mathbb{M}}^2 + p_{17}\delta\mathbb{M}^2 + p_{20}\delta\mathbb{m}_D \\
&+ p_{24}\delta\mathbb{m}_D^2 + p_{27}\mathbb{m}_D\overline{\mathbb{M}}^2 + p_{28}\mathbb{m}_D\delta\mathbb{M}^2\Big] \\
+ \left(\frac{a^2}{t_0^*}\right)^k &\Big[p_{18} + p_{21}\delta\mathbb{m}_D + p_{25}\overline{\mathbb{M}}^2 + p_{26}\delta\mathbb{M}^2\Big],
\end{aligned}
$$

where

$$
\mathbb{m} = \sqrt{8t_0}m \quad , \qquad \overline{\mathbb{M}}^2 = \frac{2\mathbb{m}_K^2 + \mathbb{m}_\pi^2}{3},
$$

$$
\delta\mathbb{m}_D = \mathbb{m}_D - \mathbb{m}_D^{\mathrm{phys}}, \qquad \delta\mathbb{M}^2 = 2(\mathbb{m}_K^2 - \mathbb{m}_\pi^2).
$$

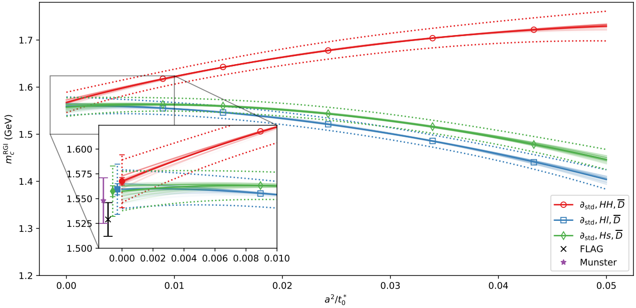
Chiral-continuum extrapolation

- We use a generalized chi-squared fit, in which the π, K and D masses are included as priors, and their correlations with the PCAC masses are also taken into account.
- Fit parametrizations were varied by including/excluding different fit parameters.
- Different combinations of chiral + lattice terms were tried:

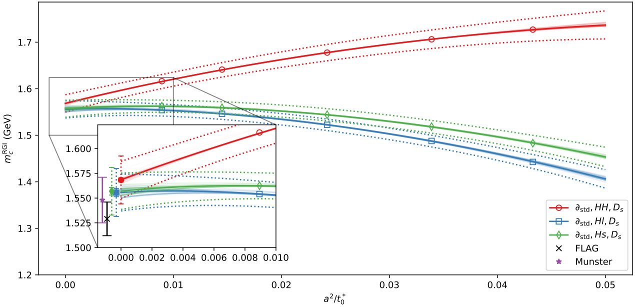
non-linear, cubic	$m^{\mathrm{RGI}} = f_{\chi\mathrm{PT}} \times (1 + f_{\mathrm{latt}}(k = 3/2))$
non-linear, quartic	$m^{\mathrm{RGI}} = f_{\chi\mathrm{PT}} \times (1 + f_{\mathrm{latt}}(k = 2))$
linear, cubic	$m^{\mathrm{RGI}} = f_{\chi\mathrm{PT}} + f_{\mathrm{latt}}(k = 3/2)$
linear, quartic	$m^{\mathrm{RGI}} = f_{\chi\mathrm{PT}} + f_{\mathrm{latt}}(k = 2)$

- The flavour composition in the PCAC current was varied between $HH = cc$, Hl and Hs;
- Two definitions of the physical point were used: either $m_{\overline{D}} = 2m_D + m_{D_s} = m_{\overline{D}}^{\mathrm{phys}}$ or $m_{D_s} = m_{D_s}^{\mathrm{phys}}$;
- Finally, two different definitions of the discrete derivative were employed ($\partial_{\mathrm{std}} f(t) = \frac{1}{2}\left(f(t+1) - f(t-1)\right)$ and $\partial_{\mathrm{fit}} f(t) = \frac{1}{2} \log\left(\frac{f(t+1)}{f(t-1)}\right) f(t)$.
- In total, around ~ 100 different fits were tried for each choice of flavour combination, derivative and D-meson. These were then combined by weighting them according to their AIC in order to estimate the systematic error.

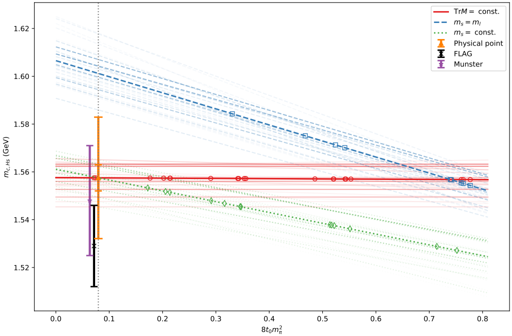
Preliminary results



- Reasonable fit quality ($\chi^2_{\rm red} \sim 1$) for many fits
- Good agreement between the different definitions
- Final value slightly higher than FLAG ($\sim 1\sigma$); compatible with Munster (which used a subset of the same data, but a different method).



- Similar results when using D_s instead of $\overline{D}$
- Slightly larger gap between HH and Hs, Hl results. Discretisation effects for HH are larger than those for Hl, Hs, particularly at the coarsest lattice spacing.



- Chiral extrapolation for $(\partial_{\rm std}, Hs, \overline{D})$
- Chiral dependence subdominant compared to continuum extrapolation; seems to be under good control.

Error budget

Error budget (contribution to σ^2_{tot})	
Statistical (PCAC masses, m_π, m_D, t_0)	9%
$\mathcal{O}(a)$ improvement	19%
Renormalization	11%
Scale setting (t_0^{phys})	21%
Renormalization scale	35%
$N_f = 3 \to 4$ conversion	1%
Fit parametrization	5%

- As our preliminary result, we quote the result for $\partial_{\mathrm{std}}, Hs, \overline{D}$, converted to the 4-flavour scheme:

$$m_c^{RGI}(N_f = 4) = 1.557(19)^{\mathrm{stat}}(14)^{\mathrm{scale}}(5)^{\mathrm{sys}}(2)^{\mathrm{conv}} = 1.557(24)\ \mathrm{GeV}$$

- The overall error is dominated by the errors on the renormalization scale, t_0^{phys}, and the $\mathcal{O}(a)$ improvement coefficients.
- Result for m_c/m_s on the way!