K and $D_{(s)}$-meson leptonic decay constants with physical light, strange and charm quarks by ETMC

Petros Dimopoulos Roberto Frezzotti
Marco Garofalo Silvano Simula

on behalf of ETMC

The 38th International Symposium on Lattice Field Theory

LATTICE21 July 29th, 2021

Outline

- Lattice action & ensembles details
- Computation of PS-decay constans
  Analysis & error budget details
- Results for f_K, f_K/f_π, $f_{D_{(s)}}$, f_{D_s}/f_D
- Determination of CKM ME $|V_{us}|$, $|V_{cd(s)}|$
- Conclusions & Outlook

ETMC Lattice action - Properties

- **Gluons**: Iwasaki
- **Sea**: Wilson-tm (tuned at maximal twist) + clover term;
  - $N_f = 2 + 1 + 1$ tuned at physical quark mass values;
  - maximal twist ensures automatic $\mathcal{O}(a)$-improvement for all phys. observables;
  - clover term yields reduced $\mathcal{O}(a^2)$ effects.
- **Valence**: Osterwalder-Seiler action for strange and charm;
  - non-unitary setup in order to avoid undesired $\mathcal{O}(a^2)$ mixing between strange and charm if the sea action were used;
  - unitarity recovered in the continuum limit.

[Frezzotti, Rossi JHEP 2004 & NPB Proc. Suppl. 2014]
[ETMC PRD 2018, 2104.06747 & 2104.13408]

Ensembles details

β	Ens.	(L, T)	M_π (MeV)	# meas.	w_0/a
1.726 ($a = 0.09471(39)$ fm)	cA211.12.48	(48,96)	167	322	1.8355(35)
	cA211.30.32	(32,64)	261	1237	
	cA211.40.24	(24,48)	302	662	
	cA211.53.24	(24,48)	346	628	
1.778 ($a = 0.08161(30)$ fm)	cB211.072.64	(64,128)	137	374	2.1300(16)
	cB211.14.64	(64,128)	190	437	
	cB211.25.48	(48,96)	253	314	
	cB211.25.32	(32,64)	253	400	
1.836 ($a = 0.06942(26)$ fm)	cC211.06.80	(80,160)	134	401	2.5045(17)
	cC211.20.48	(48,96)	246	890	

- scale setting: f_π
- $L(M_\pi^{min}) = \{4.5, 5.2, 5.5\}$
- $M_\pi^{min} L = \{3.9, 3.6, 3.8\}$

[*On ETMC simulations, talk by J. Finkenrath, Fri 30/7, 07:15 (session: Algorithms)*]

PS-meson lepton decay constants computation (1)

- Thanks to WI valid in Wilson-Mtm fermion folmulation no RC needed for ps-decay constant determination:

$$f_{\rm ps} = (m^f + m^{f'}) \frac{\langle 0 | P^{ff'} | {\rm ps} \rangle}{M_{\rm ps}^{ff'} \sinh(M_{\rm ps}^{ff'})}$$

- Employ correlation functions of the type

$$C_{PP}(t) = (1/L^3) \sum_{\vec{x},\vec{y}} \langle 0 | P^{ff'}(\vec{x}, t) P^{\dagger ff'}(\vec{y}, 0) | 0 \rangle,$$

with $P^{ff'}(x) = \bar{q}^f(x) \gamma_5 q^{f'}(x)$, $\{f, f'\} = \{\ell, s, c\}$;

- Use of local sources in the π and K cases and also smeared sources in the D and $D_{(s)}$ cases.

PS-meson lepton decay constants computation (2)

- Combined chiral & continuum fits;
- Interpolations of the ps-decay constants at the physical $m_{u/d}$ (or short extrapolation for the case of the coarse lattice) and well controlled interpolations at m_s and m_c for the two heavier quarks;
- Physical quark mass values $m_{u/d}$, m_s and m_c have been determined in ETMC 2104.13408 [scale setting from f_π];

[*On quark masses determination, talk by C. Alexandrou, Wed 28/7, 13:00 (session: SM parameters)*]

PS-meson lepton decay constants computation (3)

- **Chiral extrapolation**: use of polynomial (linear and quadratic) fit ansätze in m_ℓ and (HM)ChPT formulas.

- **Control of discretisation & other systematics**:
  (**a**) Extra analyses where data of two out of three lattice spacings are employed.

  (**b**) Use of scaling variables: (i) w_0 and (ii) alternative ones provided by an appropriate pseudoscalar mass (for the $D_{(s)}$ case);

  [*on gradient flow scale setting, talk by B. Kostrzewa Fri 30/7, 06:00 (session: Hadron Spectroscopy).*]

  (**c**) Four RI-MOM determinations for the quark mass renormalisation which differ by $\mathcal{O}(a^2)$ effects;

  [*RCs determination, talk by M. Di Carlo, Fri 30/7, 06:30 (session: SM parameters)*]

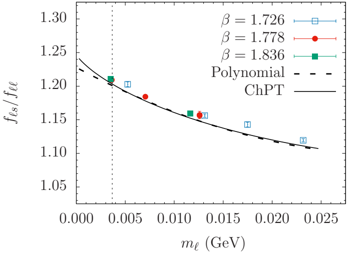
f_K/f_π, f_K

f_K/f_π

- Linear **Interpolation** of $f_{s\ell}$ to $m_s^{\text{phys.}}$ using data with s-like quark mass close to the physical one.
- *Combined* **chiral** and **continuum** fit for $(f_{\ell s}/f_{\ell\ell})$:

SU(2) ChPT: $P_0 \left(1 + (5/4)\xi_\ell log\xi_\ell + P_1\xi_\ell + P_2(a/w_0)^2\right) \mathrm{K}^{\mathrm{FSE}}_{f_K/f_\pi}, \quad \xi_\ell = (2B_0 m_\ell)/(4\pi f_0)^2$

Polynomial: $Q_0 \left(1 + Q_1 m_\ell + Q_2 m_\ell^2 + Q_3(a/w_0)^2\right) \mathrm{K}^{\mathrm{FSE}}_{f_K/f_\pi}$



⋆ $f_{\ell s}/f_{\ell\ell} \xrightarrow[m_\ell \to m_{u/d}]{a\to 0} (f_K/f_{pi})^{\text{isoQCD}}$.

⋆ $\chi^2/(\text{d.o.f.}) \lesssim 1$ for the fits.

⋆ f_K/f_π: compatible results from the two kinds of chiral fit ansatz.

⋆ $(f_K/f_\pi)^{\text{isoQCD}} = 1.2023\ (41)$ $(38)_{(\text{stat+fit})}(3)_{\text{quark mass RC}}(11)_{\text{chiral}}(8)_{\text{discr.}}(5)_{\text{FSE}}$ (0.34 %)

⋆ Compatible with $(f_K/f_\pi)^{\text{isoQCD}} = 1.1995\ (44)$ from analysis on the same ensembles but in terms of ps-masses [ETMC 2104.06747].

⋆ Taking into account estimate for IB correction: [Di Carlo et al. PRD 2019]

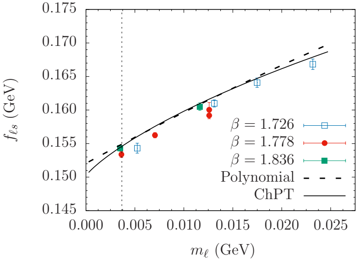
$f_{K^\pm}/f_{\pi^\pm} = (f_K/f_\pi)^{\text{isoQCD}}\sqrt{1+\delta_{R_{K\pi}}} \implies f_{K^\pm}/f_{\pi^\pm} = 1.1984\ (41)$

f_K

- Linear **Interpolation** of $f_{s\ell}$ to $m_s^{\text{phys.}}$ using data with s-like quark mass close to the physical one.
- *Combined* **chiral** and **continuum** fit for $f_{\ell s}$:

SU(2) ChPT: $P_0'\left(1-(3/4)\xi_\ell log\xi_\ell + P_1'\xi_\ell + P_2'(a/w_0)^2\right) \mathrm{K}^{\text{FSE}}_{f_K/f_\pi}, \quad \xi_\ell = (2B_0 m_\ell)/(4\pi f_0)^2$

Polynomial: $Q_0'\left(1 + Q_1' m_\ell + Q_2' m_\ell^2 + Q_3'(a/w_0)^2\right) \mathrm{K}^{\text{FSE}}_{f_K/f_\pi}$



⋆ $f_{\ell s} \xrightarrow[m_\ell \to m_{u/d}]{a\to 0} f_K^{\text{isoQCD}}$.

⋆ $\chi^2/(\text{d.o.f.}) \sim [0.5, 1.5]$ for the fits.

⋆ f_K: compatible results from the two kinds of chiral fit ansatz.

(*Preliminary*) results and error budget:

⋆ $f_K^{\text{isoQCD}} = 155.3\ (1.7)\ \ (0.9)_{\text{(stat+fit)}}(0.1)_{\text{quark mass RC}}(0.2)_{\text{chiral}}(1.4)_{\text{discr.}}(0.2)_{\text{FSE}}$ MeV (1.1 %)

⋆ Using the f_K/f_π result:

$f_K^{\text{isoQCD}} = (f_K/f_\pi)^{\text{isoQCD}} f_\pi^{\text{(expt.)}} = 156.8(0.6)$ MeV (0.4 %)

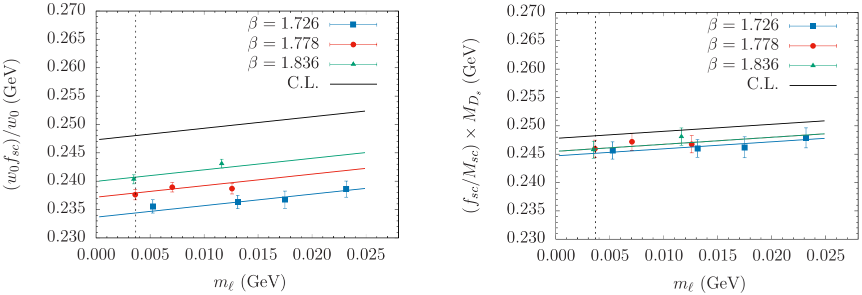
and

$f_{K^\pm} = (f_{K^\pm}/f_{\pi^\pm}) f_\pi^{\text{(expt.)}} = 156.3(0.6)$ MeV (0.4 %)

f_{D_s}, f_D, f_{Ds}/f_D

f_{D_s}

- Linear **Interpolation** of f_{sc} to $m_s^{\text{phys.}}$ and $m_c^{\text{phys.}}$ using data of *s*- and *c*-like quark mass values close to the physical ones.
- *Combined* **chiral** and **continuum** fit for f_{sc} $\rightarrow$ **linear and quadratic** polynomial fits wrt m_ℓ



⋆ Check two methods as for the scaling variable.
Both methods lead to compatible results in the C.L. differing by $\sim 0.1\%$; good quality fits.

⋆ Method with scaling variable M_{sc} yields *much suppressed* cut off effects.

- Result and error budget (*preliminary*) for f_{D_s}

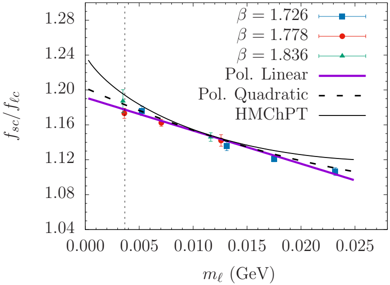
$f_{D_s} = 248.9\ (2.0)\ \ (1.6)_{\text{(stat+fit)}}(0.5)_{\text{quark mass RC}}(0.2)_{\text{chiral}}(1.0)_{\text{discr.}}$ MeV (0.8 %)

f_{D_s}/f_D

- *Combined* **chiral** and **continuum** fit for $(f_{sc}/f_{\ell c})$:

Polynomial (linear and quadratic) in m_ℓ: $Q_0\left(1 + Q_1 m_\ell + [Q_2 m_\ell^2] + Q_3(a/w_0)^2\right)$

HMChPT: $P_0\left(1 + \frac{3}{4}(1+3\hat{g}^2)\xi_\ell log(\xi_\ell) + P_1 m_\ell + P_2(a/w_0)^2\right) \quad (\hat{g} = 0.61(7))$



⋆ Discretisation effects for decay constant ratio are much suppressed.

⋆ Not convincing fit quality for the HMChPT ansatz.

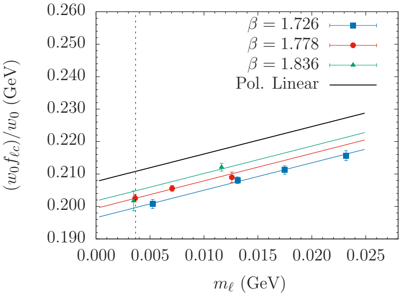
⋆ Having data at and close to the physical point polynomial chiral fit ansätze are preferable.

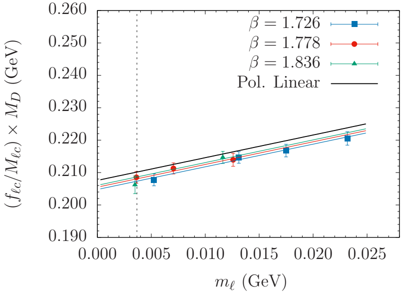
⋆ Result and error budget (*preliminary*) for the ratio f_{D_s}/f_D:

$f_{D_s}/f_D = 1.1838\ (115)\ \ (90)_{(\text{stat+fit})}(25)_{\text{quark mass RC}}(38)_{\text{chiral}}(57)_{\text{discr.}}\ (1.0\ \%)$

f_D

- Two methods of computation: **direct** fitting method for f_D employing two different scaling variables & the **indirect** one $f_D = f_{D_s}/(f_{D_s}/f_D)$.
- Use of *Combined* **chiral** and **continuum** fit method.





⋆ Check two methods as for the scaling variable.
Both methods lead to compatible results in the C.L. differing by $\sim 0.2\%$; good quality fits.

⋆ Method with scaling variable $M_{\ell c}$ yields *much suppressed* cut off effects.

- Result and error budget (*preliminary*) for f_D combining direct and indirect methods results:

$f_D = 210.1\ (2.4)\ \ (2.2)_{\text{(stat+fit)}}(0.1)_{\text{quark mass RC}}(0.4)_{\text{chiral}}(0.8)_{\text{discr.}}$ MeV (1.1 %)

Summary and comparison of results

Quantity	**ETMC 21** ($N_f = 2+1+1$)	**ETMC 14** ($N_f = 2+1+1$)	**FLAG 19** ($N_f = 2+1+1$)	**FLAG 19** ($N_f = 2+1$)
$(f_K/f_\pi)^{\text{isoQCD}}$	1.2023(41)	1.188(15)	-	
$f_{K^\pm}/f_{\pi^\pm}$	1.1984(41)	1.184(16)	1.1932(19)	1.1917(37)
f_K^{isoQCD} (MeV)	155.3(1.7)	155.0(1.9)	-	-
$f_K^{\text{isoQCD}} = (f_K/f_\pi)^{\text{isoQCD}} f_\pi^{(\text{expt})}$ (MeV)	156.8(0.6)	154.9(1.9)	-	-
$f_{K^\pm} = (f_{K^\pm}/f_{\pi^\pm}) f_\pi^{(\text{expt})}$ (MeV)	156.3(0.6)	154.4(2.0)	155.7(0.3)	155.7(0.7)
f_{D_s} (MeV)	248.9(2.0)	247.2(4.1)	249.9(0.5)	248.0(1.6)
f_{D_s}/f_D	1.1838(115)	1.192(22)	1.1783(16)	1.1740(70)
f_D (MeV)	210.1(2.4)	207.4(3.8)	212.0(0.7)	209.0(2.4)
$(\frac{f_{D_s}}{f_D})/(\frac{f_K}{f_\pi})$	0.995(13)	1.003(14)	-	-

- Good comparison of **ETMC 21** results with the **FLAG 19** averages.
- Precision of **ETMC 21** results from simulations with four dynamical quark masses set to their physical values is much improved wrt older **ETMC 14** results from simulations far from the physical pion mass ($\sim$ 210 MeV).
- Improvement on both statistical and systematic sources of uncertainty has now achieved.

CKM ME: $|V_{us}|$, $|V_{cd}|$, $|V_{cs}|$

$|V_{us}|$, $|V_{cd}|$, $|V_{cs}|$ & 1st and 2nd row unitarity checks

Experiment (PDG 20)	Theory (ETMC 21)	CKM ME
$\left\lvert\frac{V_{us}}{V_{ud}}\right\rvert \frac{f_{K^\pm}}{f_{\pi^\pm}} = 0.27600(37)$ $\lvert V_{ud}\rvert = 0.97370(14)$	$\frac{f_{K^\pm}}{f_{\pi^\pm}} = 1.1984(41)$	$\left\lvert\frac{V_{us}}{V_{ud}}\right\rvert = 0.2303(8)_{\rm th}(3)_{\rm expt}[8]$ $\lvert V_{us}\rvert = 0.2242(8)_{\rm th}(3)_{\rm expt}[8]$
$f_D\lvert V_{cd}\rvert = 46.2(1.2)$ MeV	$f_D = 210.1(2.4)$ MeV	$\lvert V_{cd}\rvert = 0.2199(25)_{\rm th}(57)_{\rm expt}[62]$
$f_{D_s}\lvert V_{cs}\rvert = 245.7(4.6)$ MeV	$f_{D_s} = 248.9(2.0)$ MeV	$\lvert V_{cs}\rvert = 0.9871(79)_{\rm th}(185)_{\rm expt}[201]$

- 1st row unitarity:

$$|V_{ud}|^2 + |V_{us}|^2 + \overbrace{|V_{ub}|^2}^{=10^{-6}} - 1 = -1.62(0.34)_{\rm th}(0.31)_{\rm expt}[0.46] \times 10^{-3}$$

- 2nd row unitarity:

$$|V_{cd}|^2 + |V_{cs}|^2 + \overbrace{|V_{cb}|^2}^{=10^{-4}} - 1 = +2.3(1.6)_{\rm th}(3.7)_{\rm expt}[4.0] \times 10^{-2}$$

Conclusions and Outlook

- Precision determinations for the K and $D_{(s)}$ leptonic decay constants in the isoQCD limit by ETMC, employing simulations with $N_f = 2 + 1 + 1$ dynamical quarks with their masses set to the physical values.
- All values presented here are greatly improved wrt the older ETMC determinations with simulations not at the physical point.
  They compare well with the FLAG averages and may serve to get more precise world determinations for the leptonic decay constants up to the charm region.
- Simulations at a (fourth) superfine lattice spacing are under way by ETMC aiming at even higher total precision.
- ETMC $N_f = 2 + 1 + 1$ gauge configurations will be also employed for the evaluation of leptonic constants with full QED+QCD effects taken into account in the framework of methods and analyses presented in [D. Giusti et al. PRL 2018, Di Carlo et al. PRD 2019].

Conclusions and Outlook

- Precision determinations for the K and $D_{(s)}$ leptonic decay constants in the isoQCD limit by ETMC, employing simulations with $N_f = 2 + 1 + 1$ dynamical quarks with their masses set to the physical values.
- All values presented here are greatly improved wrt the older ETMC determinations with simulations not at the physical point.
  They compare well with the FLAG averages and may serve to get more precise world determinations for the leptonic decay constants up to the charm region.
- Simulations at a (fourth) superfine lattice spacing are under way by ETMC aiming at even higher total precision.
- ETMC $N_f = 2 + 1 + 1$ gauge configurations will be also employed for the evaluation of leptonic constants with full QED+QCD effects taken into account in the framework of methods and analyses presented in [D. Giusti et al. PRL 2018, Di Carlo et al. PRD 2019].

Thank you for your attention!

Extra slides

Lattice action

- $S = S_{sea} + S_{val} = S_g^{\rm Iwa} + S_{tm}^{\ell} + S_{tm}^{h} + S_{val}^{f}$

$$S_{tm}^{\ell} = \sum_x \bar{\chi}_\ell(x) \left[D_W(U) + \frac{i}{4} c_{\rm SW} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}(U) + m_\ell + i\mu_\ell \tau^3 \gamma^5 \right] \chi_\ell(x)\,,$$

$$S_{tm}^{h} = \sum_x \bar{\chi}_h(x) \left[D_W(U) + \frac{i}{4} c_{\rm SW} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}(U) + m_h - \mu_\delta \tau_1 + i\mu_\sigma \tau^3 \gamma^5 \right] \chi_h(x)\,,$$

with $m_\ell, m_h \to m_{cr}$ (maximal twist), $c_{SW} \cong 1 + 0.113(3)\dfrac{g_0^2}{P}$ (1-loop tadpole boosted)

$$S_{val}^{f} = \sum_x \bar{q}_f(x) \left(D_W^{cr}(U) + \frac{i}{4} c_{\rm SW} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}(U) + \mu_f \right) q_f(x)\,, f = s, c$$

- Tuning to maximal twist guarantees automatic $\mathcal{O}(a)$-improvement. Inclusion of clover term yields reduced $\mathcal{O}(a^2)$ effects.
- Strange and charm sea quark mass parameters ($a\mu_\sigma$, $a\mu_\delta$) have been adjusted so as to reproduce the phenomenological conditions $m_c/m_s = 11.8$ and $M_{D_s}/f_{D_s} = 7.9$.

[Frezzotti, Rossi JHEP 2004 & NPB Proc. Suppl. 2014]

[ETMC PRD 2018, 2104.06747 & 2104.13408]

Determination of syst. uncertainties & error analysis

Quantity	Number of fits
f_K/f_π, f_K	$4_{(\text{discr.})} \times 4_{(\text{q.mass RC})} \times 2_{(\text{chiral})} \quad (= 32)$
f_{D_s}	$4_{(\text{discr.})} \times 4_{(\text{q.mass RC})} \times 2_{(\text{chiral})} \times 2_{(\text{scal. var.})} \quad (= 64)$
f_{D_s}/f_D	$4_{(\text{discr.})} \times 4_{(\text{q.mass RC})} \times 2_{(\text{chiral})} \quad (= 32)$
f_D	$4_{(\text{discr.})} \times 4_{(\text{q.mass RC})} \times 2_{(\text{chiral})} \times (2_{(\text{scal. var.})} + 1_{(\text{indir. method})}) \quad (= 96)$

- **Method of final error estimates:**

Let for N analyses the probability distribution $f(x) = \sum_{i=1}^{N} w_i \; f_i(x)$, where w_i some weights $f_i(x)$ are the probability distributions corresponding to the individual analyses ($i = 1, 2, \ldots, N$). Let $\overline{x}_i$ and σ_i the mean value and standard deviation of the distribution $f_i(x)$.
We can get:

$$\overline{x} \pm \sigma_{\rm stat} \pm \sigma_{\rm syst}$$

where $\overline{x} = \sum_{i=1}^{N} w_i \; \overline{x}_i$, $\quad \sigma^2_{\rm stat} = \sum_{i=1}^{N} w_i \; \sigma_i^2$, $\quad \sigma^2_{\rm syst} = \sum_{i=1}^{N} w_i \; (\overline{x}_i - \overline{x})^2$
with $w_i = \frac{1}{\sigma_i^2} \cdot \frac{1}{\sum_{j=1}^{N} 1/\sigma_j^2}$