Spectral sum from Euclidean lattice correlators and determination of renormalization constants

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RIKEN R-CCS

Programs for Junior Scientists

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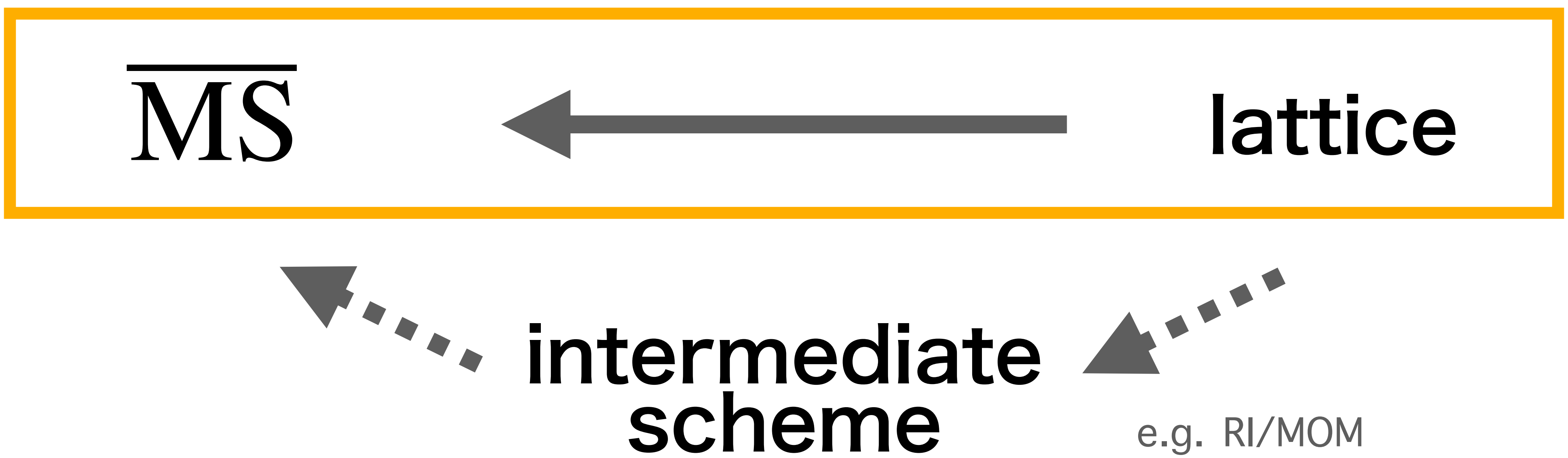
Outline

- A new method to renormalize lattice operators
- Based on the technique to compute the weighted spectrum, or the Borel transform in the QCD sum rule.

1. motivation
2. computation of the weighted spectrum
3. result for the vector current operator
4. summary

Lattice operator renormalization

Renormalization can be performed through a matching



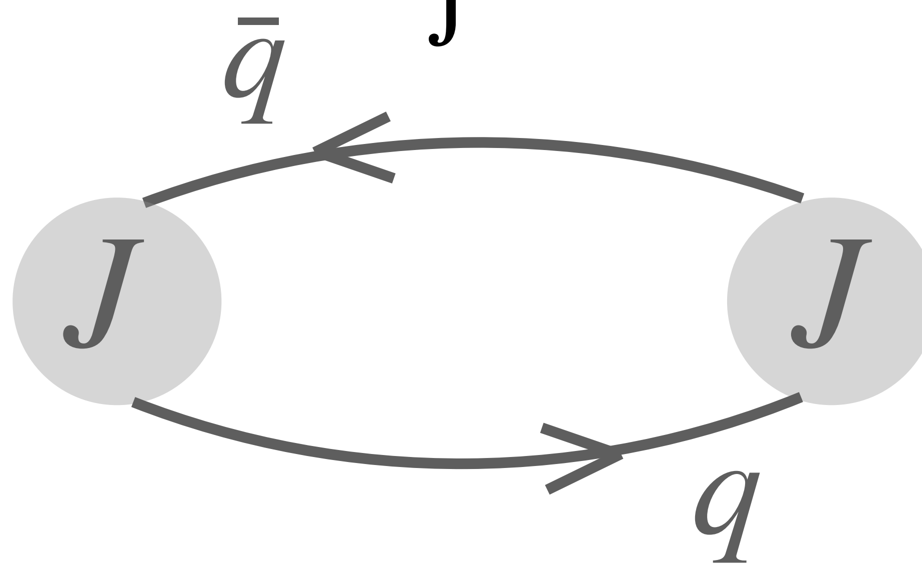
requirements:

- typical energy scale is large enough to apply the perturbation theory
- discretization error under control

current-current correlator

vacuum polarization function

$$\Pi(Q^2) := i\int d^4x e^{iQ\cdot x}\langle J(x)J(0)\rangle$$ (- subtraction)



- typical scale: Q^2
- perturbation $O(\alpha_s^3, \alpha_s^4)$ 😀
- lattice calculation 😀
- OPE 😐

$\Lambda_{\rm QCD}^2 \ll Q^2 \ll 1/a^2$

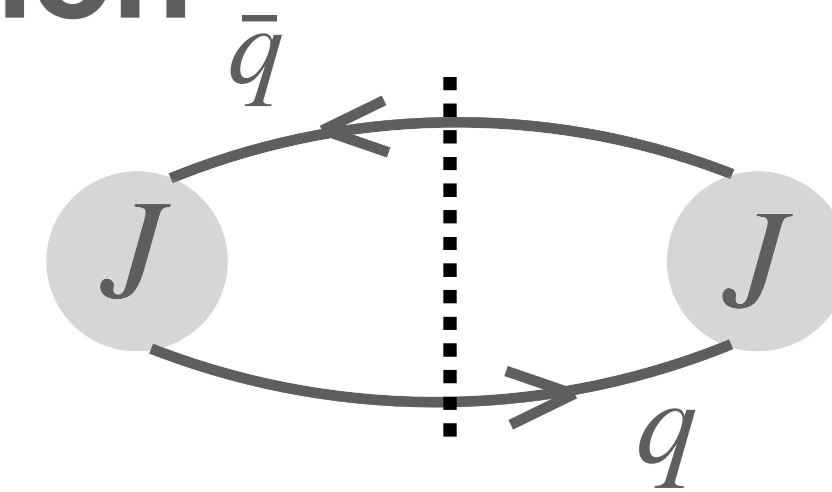
$Q \gtrsim 1.8$ GeV for convergence of OPE

severe window problem

[Hudspith, Lewis, Maltman, Shintani, 18]

[Shifman, Vainshtein, Zakharov, 79]

Borel transformation



We compute the weighted spectrum employed in the QCD sum rule

Borel transformed HVP

$$\tilde{\Pi}(M^2) := \mathscr{B}_M[\Pi(Q^2)] = \frac{1}{M^2}\int ds\; e^{-s/M^2}\rho(s)$$

$$\rho(s) = \frac{1}{\pi}\mathrm{Im}\,\Pi(s+i\epsilon) \sim \sum_n |\langle n|J|0\rangle|^2\delta(s-E_n^2)$$

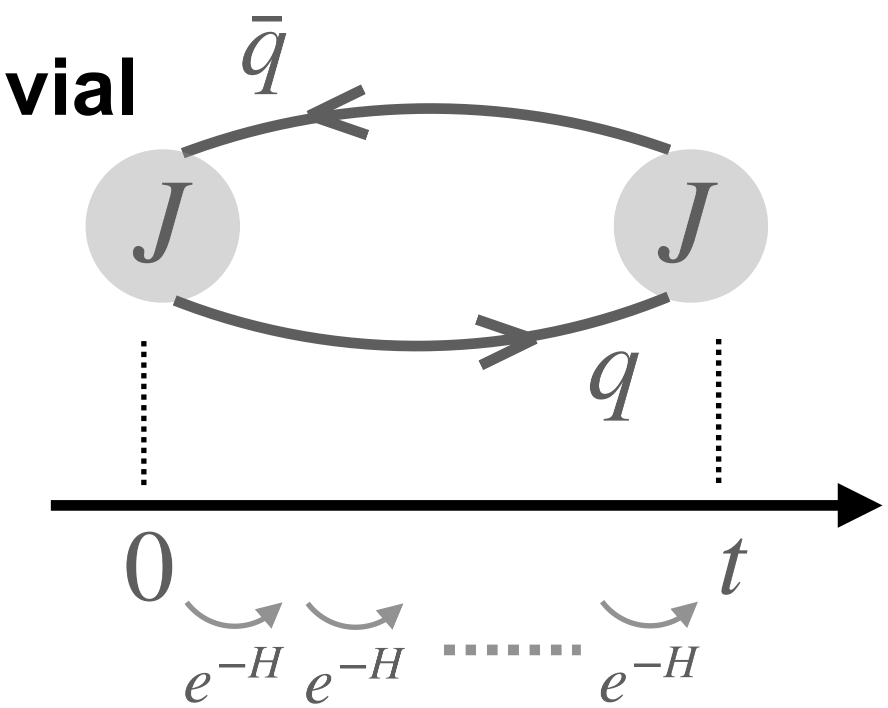
Matching condition

$$\tilde{\Pi}^{\overline{\mathrm{MS}}}(\mu;M^2) = \left(Z^{\overline{\mathrm{MS}}/\mathrm{lat}}(\mu,a)\right)^2\tilde{\Pi}^{\mathrm{lat}}(a;M^2)$$

- typical scale: Borel mass M
- perturbation 😀 ← $O(\alpha_s^3, \alpha_s^4)$
- OPE 😀 ← $\mathscr{B}_M\left[\frac{1}{Q^{2n}}\right] = \frac{1}{(n-1)!}\frac{1}{M^{2n}}$
- lattice calculation → for $s\bar{s}$ state [TI, S. Hashimoto, arXiv:2103.06539]

Transfer matrix expansion (1)

lattice computation of $\tilde{\Pi}(M^2)$ is nontrivial



◆ current-current correlator

$$C(t) = \langle J(t)J(0)\rangle = \int d\omega\, e^{-\omega t}\omega^2\rho(\omega^2)$$

ill-posed problem $\qquad (\omega^2 = s)$

$\rho(\omega^2)$

$C(t) \longrightarrow \tilde{\Pi}(M^2)$

[G. Bailas, S. Hashimoto, TI, PTEP2020 043B07]

compute the spectral sum by the transfer matrix e^{-H}

$$C(t=n) = \langle J|(e^{-H})^n|J\rangle,\ \rho(\omega^2) = \langle J|\delta(\omega - H)|J\rangle$$

$$\tilde{\Pi}(M^2) = \frac{1}{M^2}\int ds\, e^{-s/M^2}\rho(s) = \sum_n a_n(M^2)\langle J|(e^{-H})^n|J\rangle$$

expanded in $e^{-\omega}$

Transfer matrix expansion (2)

Chebyshev expansion ←

- orthogonal polynomial
- $|T_j^*(x)| \leq 1$ for $0 \leq x \leq 1$

$$\tilde{\Pi}(M^2) \simeq \frac{c_0^*(M^2)}{2} C(0) + \sum_{j=1}^{N} c_j^*(M^2) \langle T_j^*(e^{-H}) \rangle$$

$c_j^*(M^2)$ is determined to reproduce the integral $\int ds\, e^{-s/M^2} \rho(s)$

$$= 2 \int d\omega\, \omega e^{-\omega^2/M^2} \rho(\omega^2)$$

(shifted) Chebyshev polynomial

$$T_1^*(x) = 2x - 1,\ T_2^*(x) = 8x^2 - 8x + 1, \cdots$$

$$\langle T_1^*(e^{-H}) \rangle = 2C(1) - C(0),\ \langle T_2^*(e^{-H}) \rangle = 8C(2) - 8C(1) + C(0), \cdots$$

$x^n \longrightarrow C(n)$ correlators from lattice simulations

Setup

- JLQCD ensemble

  Nf = 2+1 Möbius domain-wall fermion

β	a^{-1}[GeV]	$L^3 \times T(\times L_5)$	#meas	am_{ud}	am_s
4.17	2.453(4)	$32^3 \times 64\ (\times 12)$	800	0.007	0.04
4.35	3.610(9)	$48^3 \times 96\ (\times 8)$	600	0.0042	0.025
4.47	4.496(9)	$64^3 \times 96\ (\times 8)$	400	0.0030	0.015

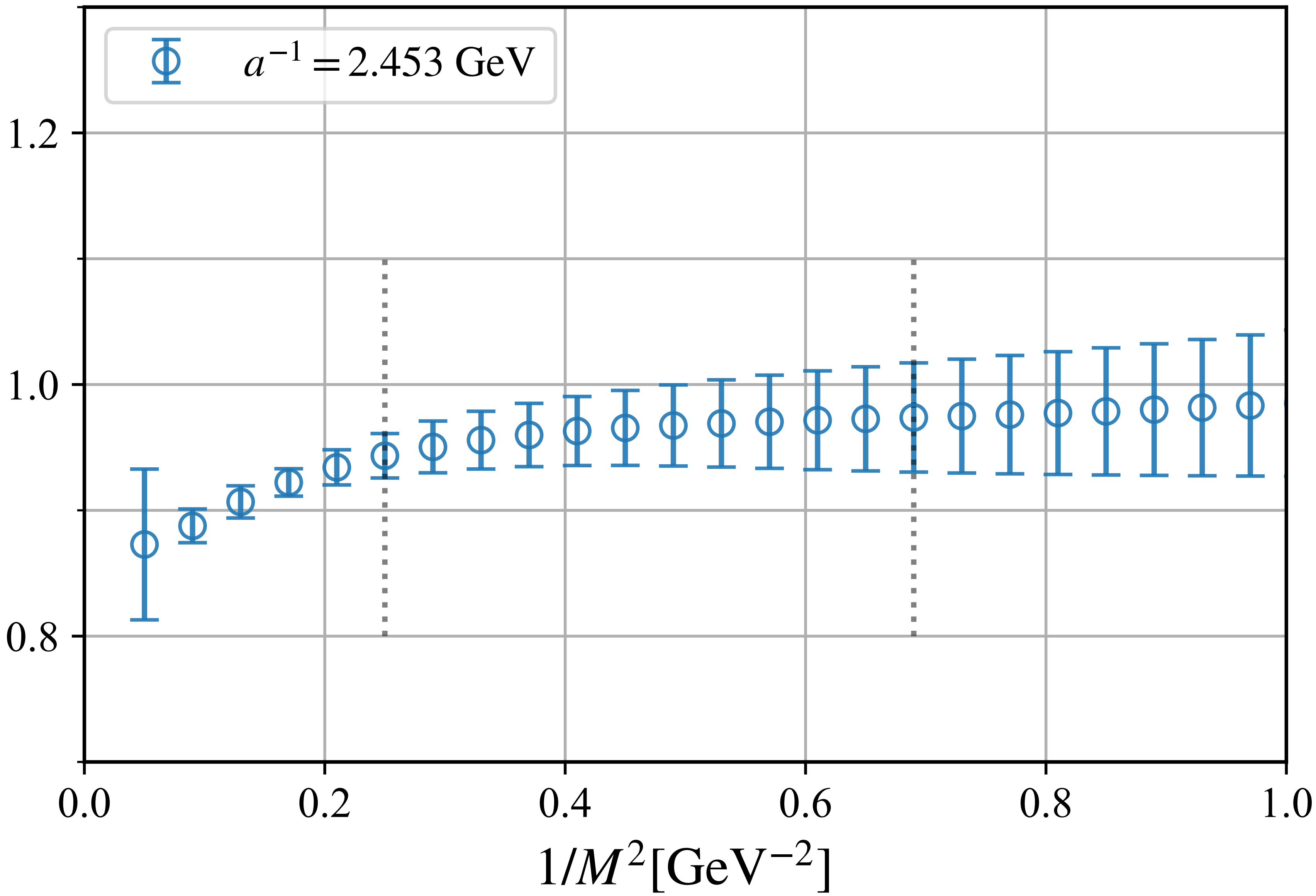
We compute the renormalization constant for the vector current

- $\tilde{\Pi}(M^2)$ known to $O(\alpha_s^4)$
- $J_i = \bar{q}\gamma_i q$ ⟶ **no anomalous dimension**

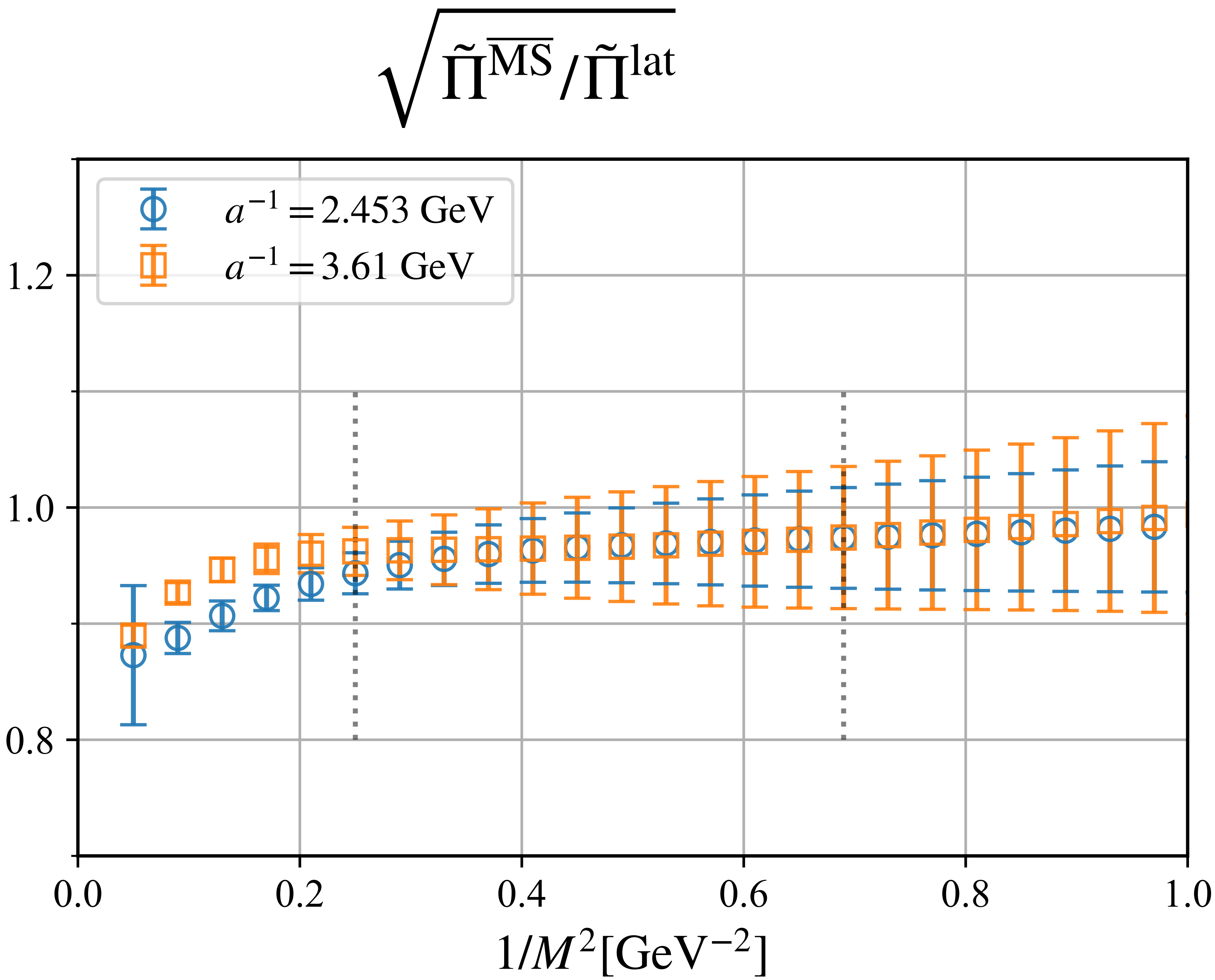
$$\tilde{\Pi}^{\overline{\mathrm{MS}}}(\mu; M^2) = \left(Z^{\overline{\mathrm{MS}}/\mathrm{lat}}(\mu, a)\right)^2 \tilde{\Pi}^{\mathrm{lat}}(a; M^2)$$

Result

$$\sqrt{\tilde{\Pi}^{\overline{\mathrm{MS}}}/\tilde{\Pi}^{\mathrm{lat}}}$$

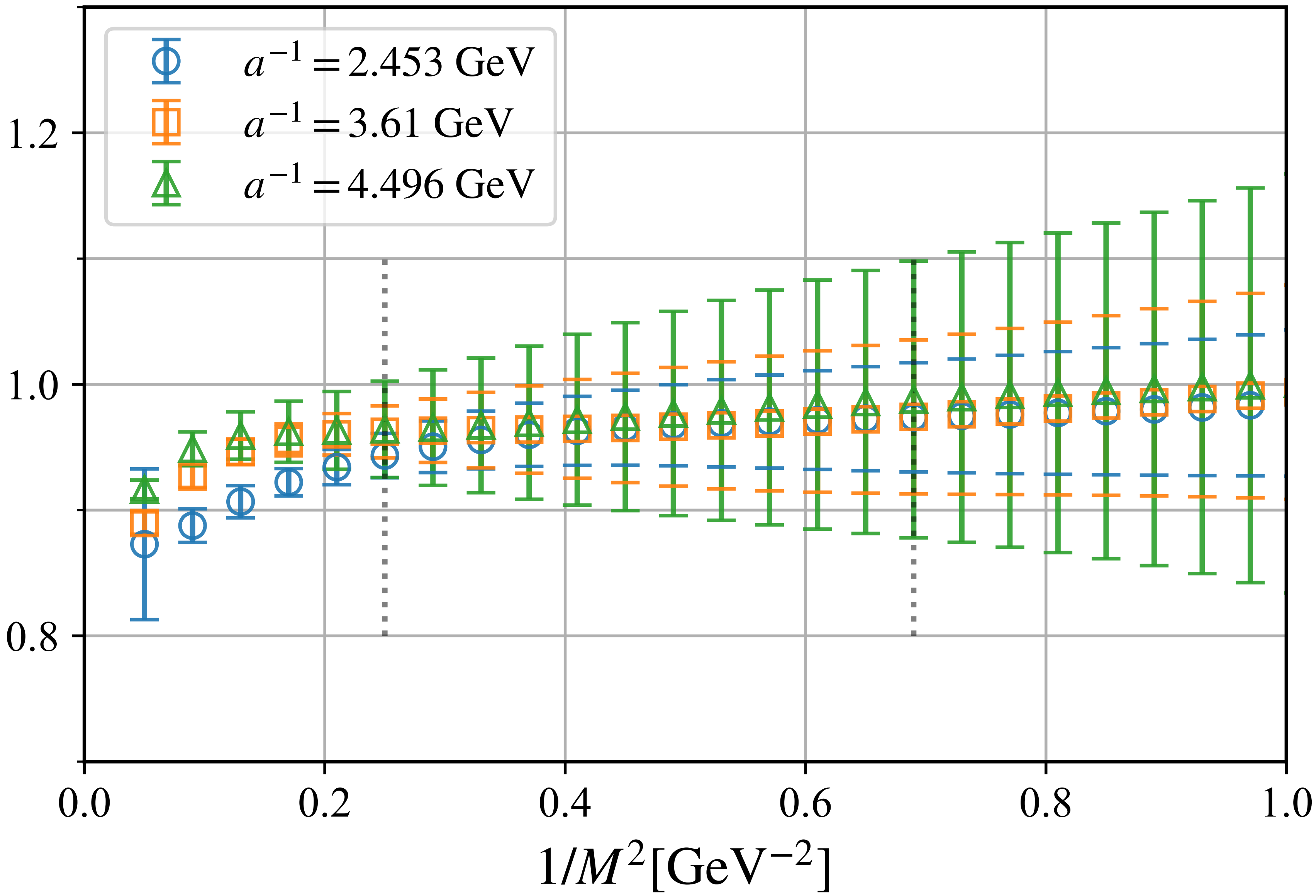


Result


$\sqrt{\tilde{\Pi}^{\overline{\rm MS}}/\tilde{\Pi}^{\rm lat}}$
$a^{-1} = 2.453$ GeV
$a^{-1} = 3.61$ GeV
1.2
1.0
0.8
0.0
0.2
0.4
0.6
0.8
1.0
$1/M^2[\mathrm{GeV}^{-2}]$

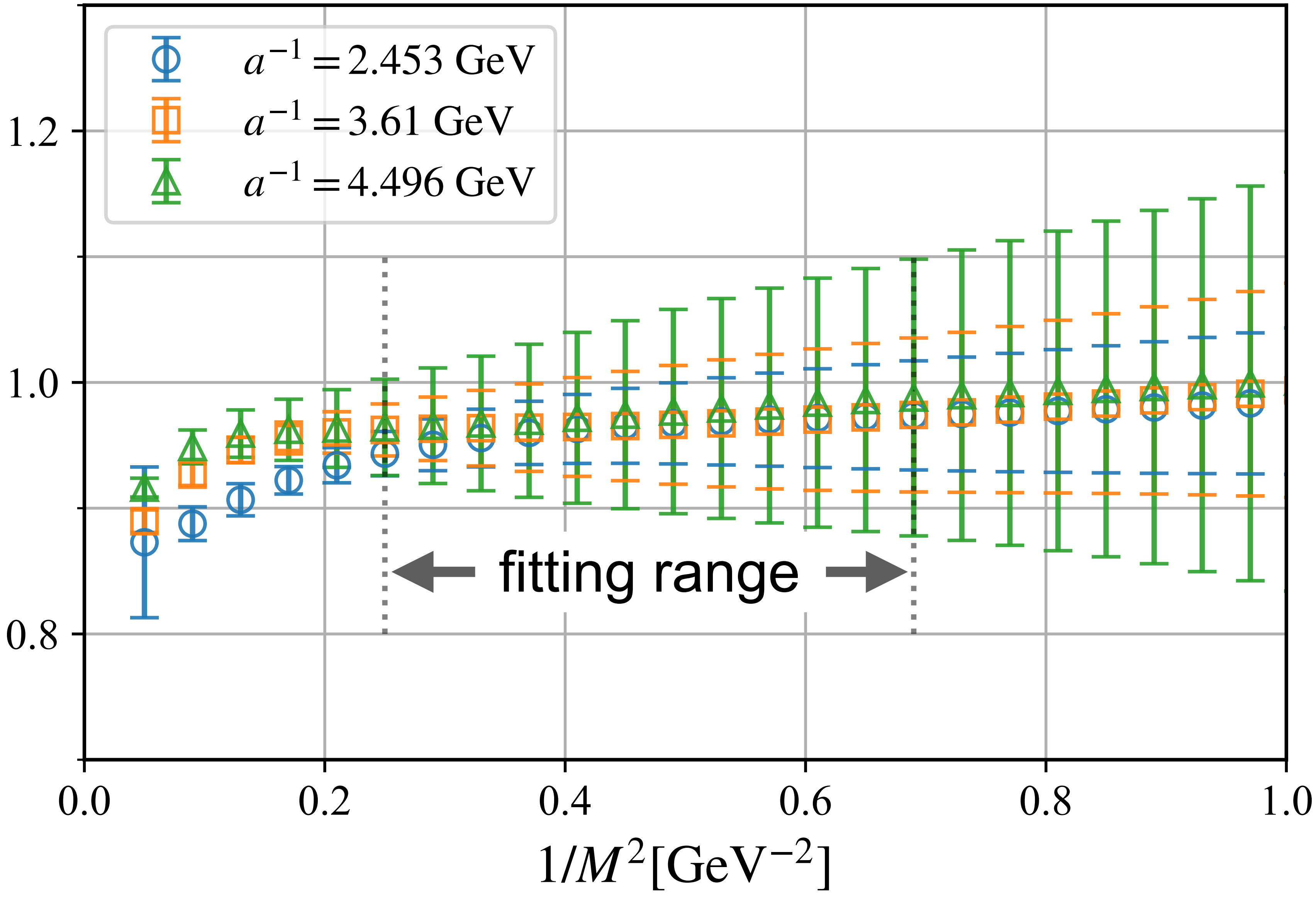
Result

$$\sqrt{\tilde{\Pi}^{\overline{\rm MS}}/\tilde{\Pi}^{\rm lat}}$$



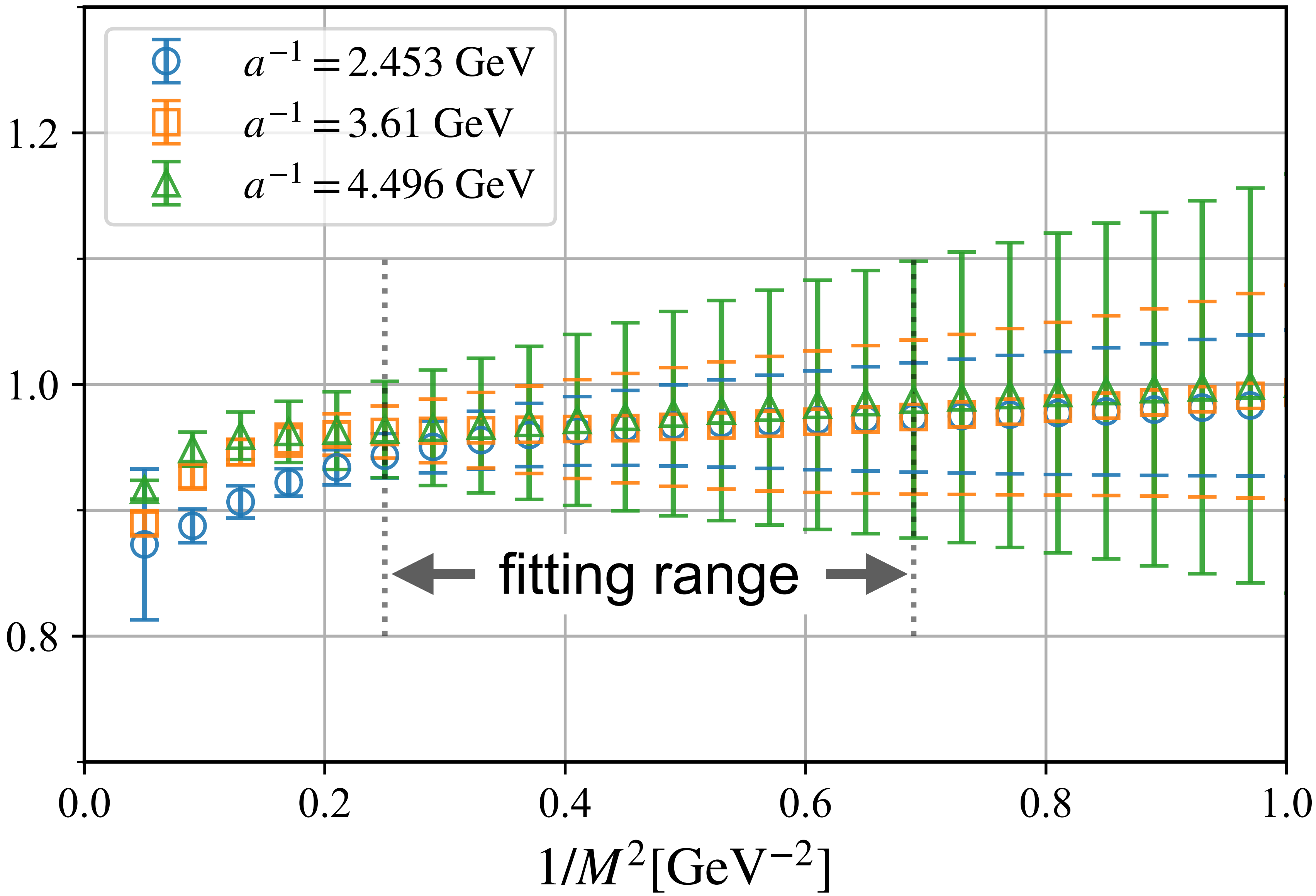
Result

global fit $\sqrt{\tilde{\Pi}^{\overline{\rm MS}}/\tilde{\Pi}^{\rm lat}} = Z_V^{\overline{\rm MS}/{\rm lat}}(a) + c_0 M^2 a^2 + c_1 \Lambda^4/M^4$



Result

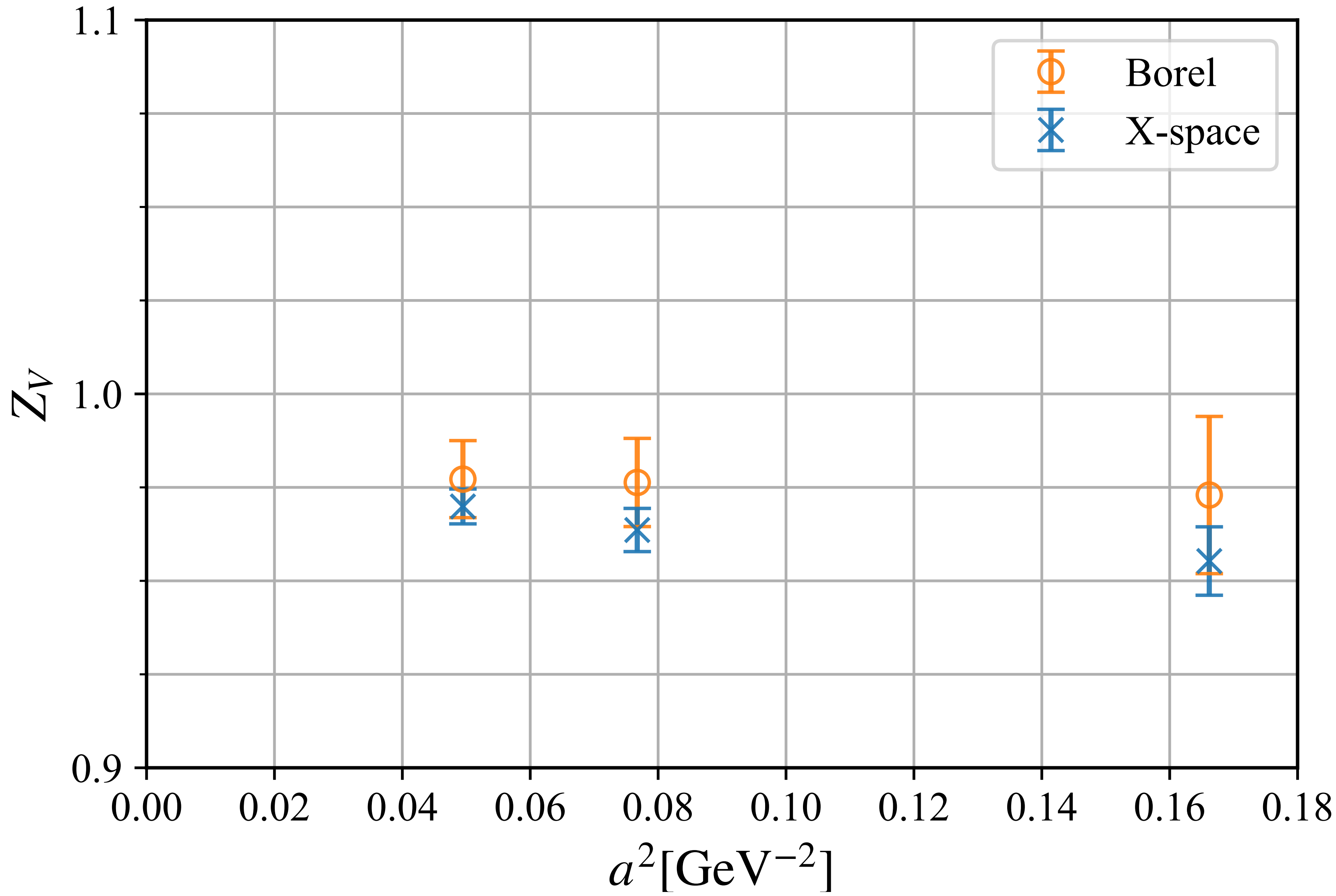
global fit $\sqrt{\tilde{\Pi}^{\overline{\mathrm{MS}}}/\tilde{\Pi}^{\mathrm{lat}}} = Z_V^{\overline{\mathrm{MS}}/\mathrm{lat}}(a) + c_0 M^2 a^2 + c_1 \Lambda^4/M^4$



a^{-1} [GeV]	our work
2.453	0.973(21)
3.610	0.976(12)
4.496	0.977(10)

comparison with X-space method

all results are consistent within statistical error



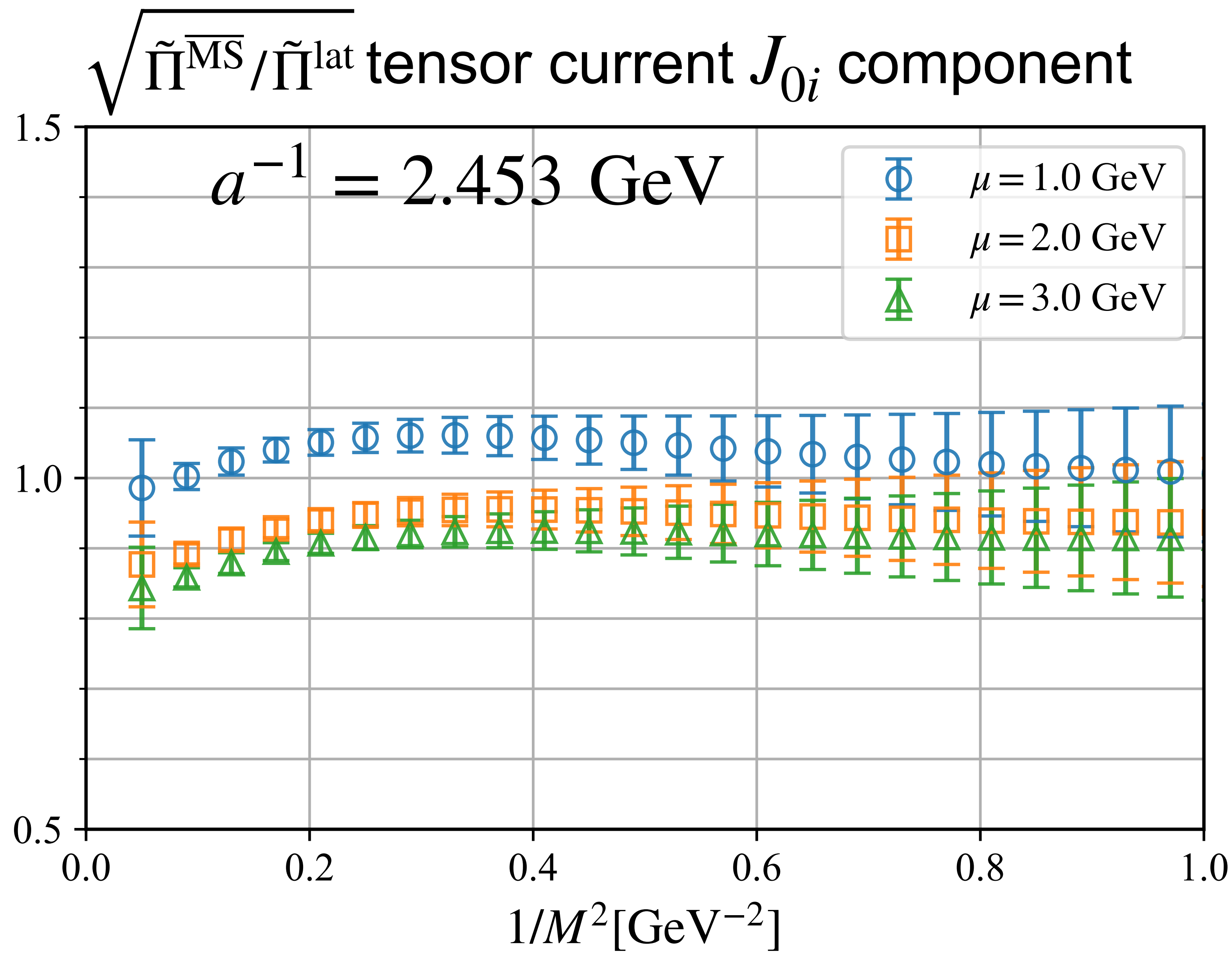
- taking the renormalization constant from the X-space correlator as a reference

  [M. Tomii et al. PRD 94, 054504]

- Error is larger, but obtained with limited ensembles.

For other operators

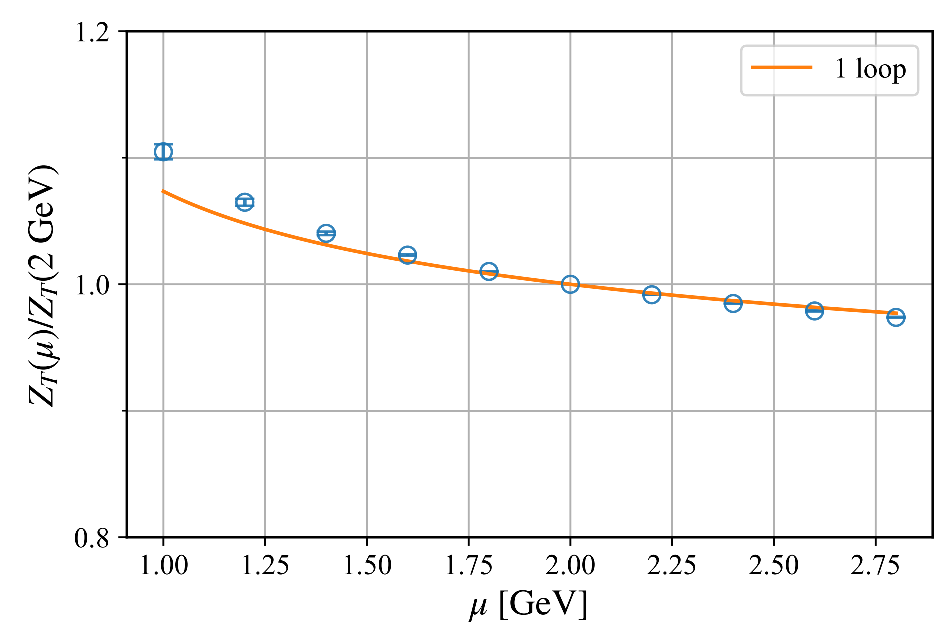
we extend our method to the other operators



- $J_S = \bar{q}q$ and $J_P = \bar{q}\gamma_5 q$
- $J_{\mu\nu} = \bar{q}\sigma_{\mu\nu}q$

non-zero anomalous dim.

→ Z_S and Z_T depend on μ unlike $Z_V(a)$



Summary

We propose a renormalization method

- Based on the Borel transform following SVZ.
- High-order perturbative expansion of $\tilde{\Pi}(M^2)$ is available.
- The scale parameter M^2 is continuous and easily adjustable through Chebyshev expansion.
- The result for the vector current agrees with another renormalization method.
- Computation of the renormalization constant for other operators underway