Comparison of lattice QCD results for inclusive semi–leptonic decays of B mesons with the OPE

Sandro Mächler, Paolo Gambino (U. of Turin & INFN Turin)
Shoji Hashimoto, Takashi Kaneko (KEK, SOKENDAI)

Lattice 2021, Jul 26-30 2021

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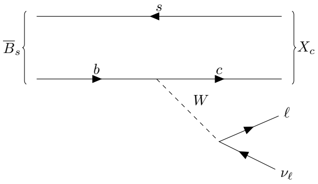
Inclusive B decays



- Inclusive decays are independent of the final state. Through an OPE short and long distance QCD effects are separated.
- Short distance effects are encoded in perturbative Wilson coefficients and long distance effects are parametrized by matrix elements of local operators $\rightarrow$ double expansion in α_s and $\Lambda_{\mathrm{QCD}}/m_b$
- Inclusive B decays allow for an extraction of V_{cb}. There is a long standing discrepancy between inclusive and exclusive determinations of V_{cb}.

In general inclusive observables are smeared differential distributions, given by double series in Λ_{QCD}/m_b and α_s:

$$M_i = M_i^{(0,0)} + \frac{\alpha_s}{\pi} M_i^{(1)} + \frac{\mu_\pi^2}{m_b^2} \left(M_i^{(\pi,0)} + \frac{\alpha_s}{\pi} M_i^{(\pi,1)} \right) + \frac{\mu_G^2}{m_b^2} \left(M_i^{(G,0)} + \frac{\alpha_s}{\pi} M_i^{(G,1)} \right) + \frac{\rho_D^3}{m_b^3} M_i^{(D,0)} + \frac{\rho_{LS}^3}{m_b^3} M_i^{(LS,0)} + \cdots ,$$

where for instance

$$\mu_\pi^2 = \frac{1}{M_B} \left\langle B \left| \bar{b}_v \left(i \vec{D} \right)^2 b_v \right| B \right\rangle , \qquad \mu_G^2 = \frac{1}{M_B} \left\langle B \left| \bar{b}_v \frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b_v \right| B \right\rangle .$$

- The non–perturbative parameters $\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3$ are extracted from fits to experimental data.
- Furthermore the quark masses m_b and m_c are crucial input parameters.
- Is there an independent method for determining the parameters?
- Yes! There is now a method for computing inclusive observables on the lattice. See Shoji Hashimoto "Composition of the inclusive semi–leptonic decay of B meson", Wednesday 9:45pm.
- **Idea:** Treat lattice simulations as a virtual laboratory to probe the non–perturbative dynamics.

Inclusive decay rate

We consider the decay

$$B\left(m_B,\mathbf{0}\right)\rightarrow X_c\left(\omega,-\boldsymbol{q}\right)\ell\left(E_\ell,\boldsymbol{p}_\ell\right)\overline{\nu}_\ell\left(q_0-E_\ell,\boldsymbol{q}-\boldsymbol{p}_\ell\right).$$

The triple differential decay rate can be written as

$$\frac{\mathrm{d}^3\Gamma}{\mathrm{d}\boldsymbol{q}^2\mathrm{d}\omega\mathrm{d}E_\ell}=\frac{G_\mathrm{F}^2\left|V_{cb}\right|^2}{8\pi^3}L_{\mu\nu}W^{\mu\nu}$$

with the hadronic tensor

$$W^{\mu\nu}\left(p,q\right)=\sum_{X_c}\frac{\left(2\pi\right)^3}{2E_B}\delta^{(4)}\left(p-q-p_X\right)\left\langle\overline{B}\left|J_q^{\mu\dagger}\right|X_c\right\rangle\left\langle X_c\left|J_q^{\nu}\right|\overline{B}\right\rangle,$$
$$J_q^\mu=\bar{c}\gamma^\mu\left(1-\gamma_5\right)b=V^\mu-A^\mu.$$

In the following we keep the VV and AA contributions separate, and distinguish polarizations parallel and perpendicular to the W–boson's three–momentum $\boldsymbol{q}$.

The inclusive total decay rate is given by

$$\begin{aligned}\Gamma &\propto \int_0^{\boldsymbol{q}^2_{\max}} \mathrm{d}\boldsymbol{q}^2 \int_{\sqrt{m_D^2+\boldsymbol{q^2}}}^{m_B-\sqrt{\boldsymbol{q^2}}} \mathrm{d}\omega \int_{\frac{q_0^2-\sqrt{\boldsymbol{q^2}}}{2}}^{\frac{q_0^2+\sqrt{\boldsymbol{q^2}}}{2}} \mathrm{d}E_\ell L_{\mu\nu} W^{\mu\nu} \\ &\equiv \int_0^{\boldsymbol{q}^2_{\max}} \mathrm{d}\boldsymbol{q}^2 \frac{\sqrt{\boldsymbol{q^2}}}{3} \overline{X}\left(\boldsymbol{q}^2\right).\end{aligned}$$

- The functions $\overline{X}^{\parallel}_{VV}(\boldsymbol{q}^2), \overline{X}^{\perp}_{VV}(\boldsymbol{q}^2), \overline{X}^{\parallel}_{AA}(\boldsymbol{q}^2), \overline{X}^{\perp}_{AA}(\boldsymbol{q}^2)$ were considered in a pilot numerical study in arXiv:2005.13730 at $\mathcal{O}(1/m_b^2)$.
- We expanded this analysis including $\mathcal{O}(1/m_b^3)$ corrections and the leading $\mathcal{O}(\alpha_s)$ perturbative corrections, again for an unphysically light $m_b^{\rm kin}(1{\rm GeV}) = 2.7$ GeV, and $\overline{m}_c(2{\rm GeV}) = 1.093$ GeV.
- For the non–perturbative parameters we employ the results of the recent fit of Bordone, Capdevila and Gambino, arXiv:2107.00604, in the kinetic scheme and $\alpha_s = 0.32$.
- Note that the semileptonic fit was performed at the physical m_b for B_d mesons, while here the b quark is unphysically light!

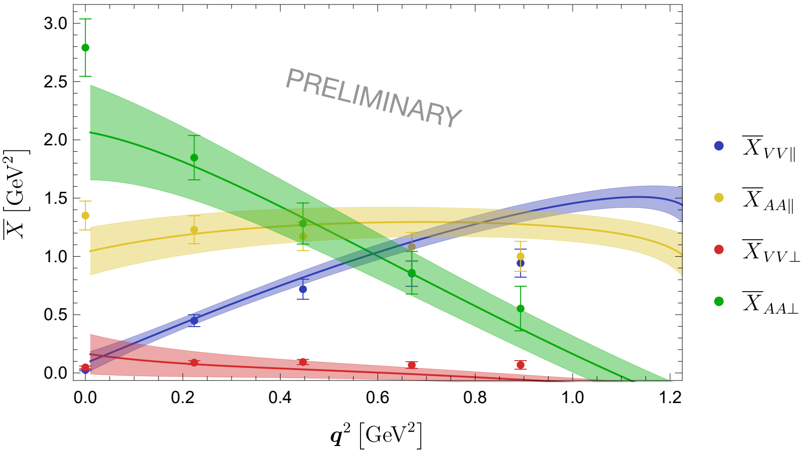


Figure: Decay rate at fixed $\boldsymbol{q}^2$. The dots and solid lines correspond to lattice data and OPE prediction at $\mathcal{O}\left(1/m_b^3, \alpha_s\right)$ respectively. The shaded bands include the experimental uncertainty on the non–perturbative parameters and theoretical uncertainty due to the truncation of the double series.

Lepton energy moments

In order to reduce the uncertainties one can consider normalized quantities like the moments of the lepton energy E_ℓ,

$$\langle E_\ell^n \rangle = \frac{\int \mathrm{d}\boldsymbol{q}^2 \mathrm{d}\omega \mathrm{d}E_\ell E_\ell^n \frac{\mathrm{d}^3\Gamma}{\mathrm{d}\boldsymbol{q}^2 \mathrm{d}\omega \mathrm{d}E_\ell}}{\int \mathrm{d}\boldsymbol{q}^2 \mathrm{d}\omega \mathrm{d}E_\ell \frac{\mathrm{d}^3\Gamma}{\mathrm{d}\boldsymbol{q}^2 \mathrm{d}\omega \mathrm{d}E_\ell}}$$

Again we study them at fixed $\boldsymbol{q}^2$ and define

$$\langle E_\ell^n \rangle \left(\boldsymbol{q}^2\right) = \frac{1}{\overline{X}\left(\boldsymbol{q}^2\right)} \int \mathrm{d}\omega \mathrm{d}E_\ell \frac{3}{\sqrt{\boldsymbol{q}^2}} E_\ell^n L_{\mu\nu} W^{\mu\nu}.$$

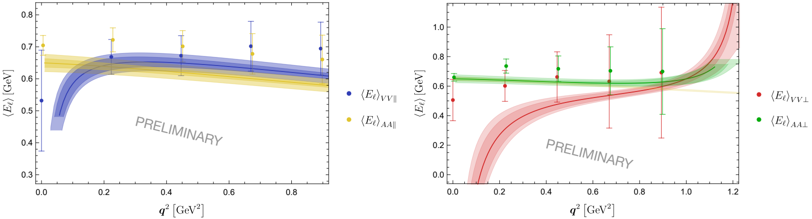


Figure: First lepton energy moment at fixed $\boldsymbol{q}^2$. The dots and solid lines correspond to lattice data and OPE prediction at $\mathcal{O}\left(1/m_b^3, \alpha_s\right)$ respectively. The shaded bands include the experimental uncertainty on the non–perturbative parameters and theoretical uncertainty due to the truncation of the double series.

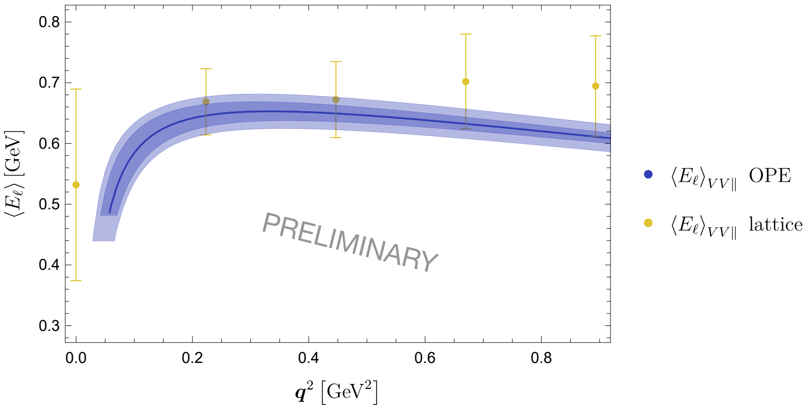


Figure: First lepton energy moment at fixed $\boldsymbol{q}^2$. The dots and solid lines correspond to lattice data and OPE prediction at $\mathcal{O}\left(1/m_b^3, \alpha_s\right)$ respectively. The shaded bands include the experimental uncertainty on the non–perturbative parameters and theoretical uncertainty due to the truncation of the double series.

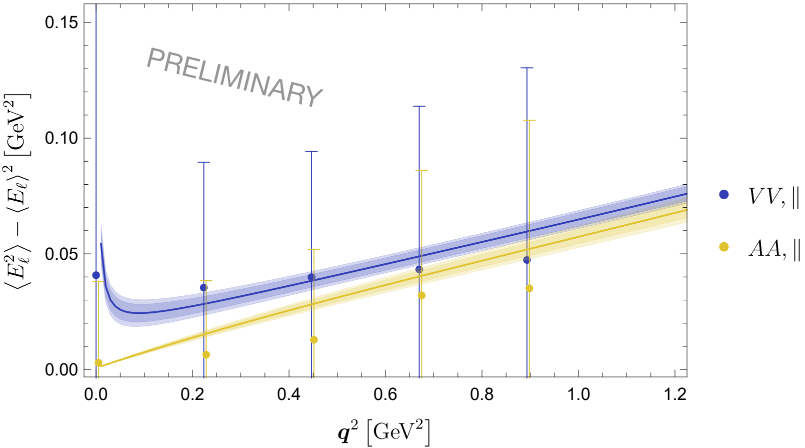
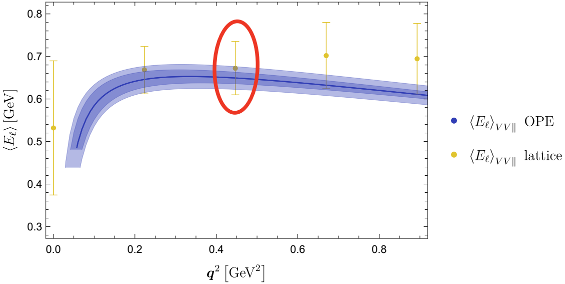


Figure: Variance of the charged lepton energy at fixed $\boldsymbol{q}^2$. The dots and solid lines correspond to lattice data and OPE prediction at $\mathcal{O}\left(1/m_b^3, \alpha_s\right)$ respectively. The shaded bands include the experimental uncertainty on the non–perturbative parameters and theoretical uncertainty due to the truncation of the double series.

- We now have the possibility to fit the OPE results to lattice data and determine the parameters $\mu_\pi^2, \mu_G^2, \rho_D^3, \rho_{LS}^3$ in an independent way.
- E.g. we have

$$\langle E_\ell \rangle_{VV,\parallel}(0.45\text{GeV}^2) = -0.024\text{GeV} + 0.439 m_b - 0.419 m_c + 0.054\text{GeV}^{-1}\mu_G^2 - 0.175\text{GeV}^{-1}\mu_\pi^2 - 0.006\text{GeV}^{-2}\rho_D^3 - 0.099\text{GeV}^{-2}\rho_{LS}^3$$



Summary & Outlook

- The framework to compute inclusive observables on the lattice is in place.
- Lattice QCD and OPE predictions agree remarkably well.
- The next steps include:
  - Considering additional observables, such as hadronic energy moments.
  - SV sum rules.
  - Studying a smooth integration kernel.
  - Lattice simulations closer to the physical m_b.