

Light meson physics and scale setting from a mixed action with Wilson twisted mass valence quarks

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>cls The ALPHA Collaboration logo features the word 'ALPHA' in a bold, red, serif font with a blue horizontal line above the 'A'. Below it, the word 'Collaboration' is written in a smaller, black, sans-serif font.

Lattice 2021, July 30, 2021

mixed action

- ▶ **setup**: mixed action with Wilson twisted mass (Wtm) valence quarks on CLS $N_f = 2 + 1$ ensembles

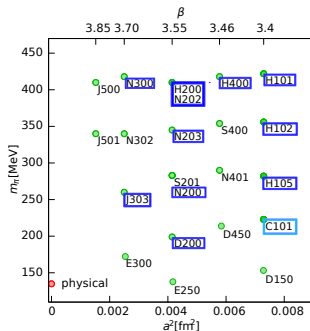
motivation:

- ▶ alternative/complementary way to control lattice artefacts
 \rightsquigarrow universality
- ▶ **target**: leptonic and semi-leptonic decays in the heavy-quark sector
 see talks by Alessandro Conigli [Thu.,5:45] & by Julien Frison [Thu.,14:15]
- ▶ **this work**: light-quark sector
 \rightsquigarrow sea/valence matching, scale setting, quark masses

sea sector: $N_f = 2 + 1$ CLS [1411.3982, 1608.08900, 1712.04884]

- ▶ lattice action:
 - ▶ gauge action: Lüscher-Weisz gauge action (tISym)
 - ▶ fermion action: $N_f = 2 + 1$ Wilson fermions with non-perturbative c_{sw}
- ▶ open boundary conditions in time
 - ▶ connected topological sectors \rightsquigarrow smooth flow of topological charge [Lüscher, 1009.5877; Lüscher & Schaefer, 1105.4749]
 - ▶ relevant for heavy-quark physics
- ▶ lattice spacings: $a \approx 0.087, 0.077, 0.065, 0.050, 0.037$ fm

$$M_\pi L \geq 3.9$$



based on [1712.04884]

see talks by Wolfgang Soeldner [Tue.,13:00] & by Ben Strassberger [Fri.,7:00]

chiral trajectory: $N_f = 2 + 1$

$$M_q = \text{diag} (m_{q_u}, m_{q_d}, m_{q_s})$$

$$\text{tr} M_q = m_{q_u} + m_{q_d} + m_{q_s} = \text{const.}$$

- ▶ constant $O(a \text{tr} M_q)$ cutoff effects

renormalised chiral trajectory

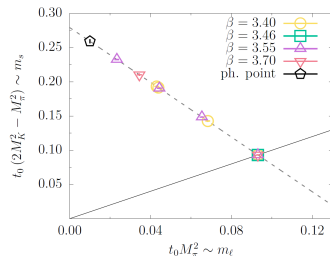
[Bruno, Korzec, Schaefer, 1608.08900]

$$\phi_4 \equiv 8f_0 \left(\frac{1}{2} M_\pi^2 + M_K^2 \right) = \frac{1}{2} \phi_2 + \phi_K = \text{const} = \phi_4^{\text{phys}}$$

- ▶ ϕ_4^{phys} is not known a priori
- ▶ small mass corrections of chiral trajectory through a Taylor expansion
- ▶ $O = f(\{A_i\})$ $\bar{A}_i = \langle A_i \rangle$

$$f(m') = f(m) + (m' - m) \frac{df}{dm}$$

$$\frac{df}{dm} = \sum_i \frac{\partial f}{\partial \bar{A}_i} \left[\left\langle \frac{\partial A_i}{\partial m} \right\rangle - \left\langle (A_i - \bar{A}_i) \left(\frac{\partial S}{\partial m} - \overline{\frac{\partial S}{\partial m}} \right) \right\rangle \right]$$



valence quarks: Wilson twisted mass

[ALPHA, hep-lat/0101001; Frezzotti and Rossi hep-lat/0306014, Pena et al., hep-lat/0405028]

- ▶ valence action

$$D_{\text{Wtm}} = D_{\text{W}}^{\text{SW}} + m \pm i\gamma_5 \mu$$

- ▶ maximal twist $\omega = \frac{\pi}{2}$:

$$\begin{aligned} m &= m_{\text{Cr}} && \rightsquigarrow m_{12}^{\text{val}} = 0 \\ \mu &= \{\mu_{ud}, \mu_s, \mu_c\} \end{aligned}$$

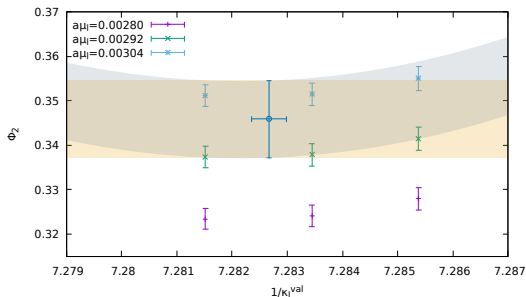
- ▶ properties:

- ▶ absence of $\mathcal{O}(a\mu)$ lattice artefacts at maximal twist
- ▶ μ acts as an infrared cutoff
- ▶ valence flavour breaking cutoff effects \rightsquigarrow SW term

- ▶ **mixed action**: match sea & valence quark masses (at maximal twist)

performed using M_π and M_K after mass-shifts to renormalized chiral trajectory
as a check, we use direct quark mass matching

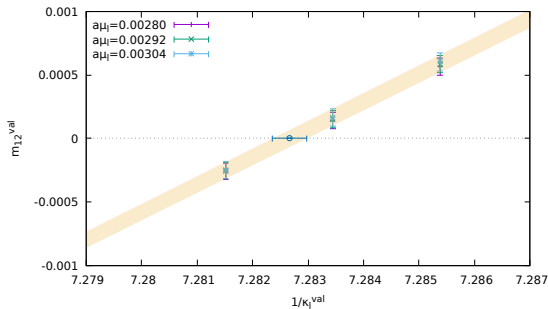
matching of mixed action



$$\phi_4 \equiv 8t_0 \left(\frac{1}{2} M_\pi^2 + M_k^2 \right) = 1.117(12) [1.1\%]$$

$$\phi_2 \equiv 8t_0 M_\pi^2$$

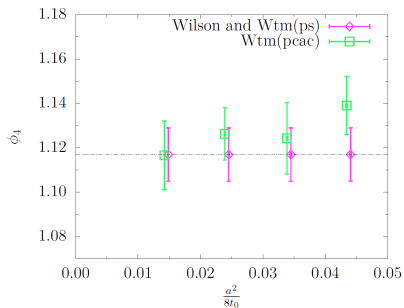
$$M_\pi^2 \propto \sqrt{\left(m_{12}^2|_v \right)^2 + \left(\mu_1^p \right)^2}$$



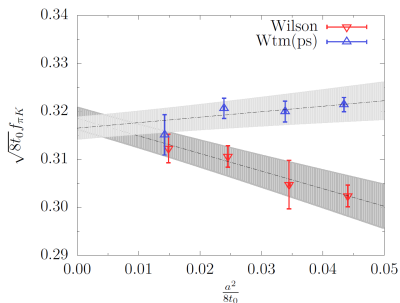
$$\begin{aligned} \sigma &= 0.086 \text{ fm} \\ M_\pi &= 280 \text{ MeV} \\ M_\pi L &= 3.9 \end{aligned}$$

continuum-limit scaling

$$\phi_A \equiv 8t_0 \left(\frac{1}{2} M_\pi^2 + M_K^2 \right)$$



$$f_{\pi K} \equiv \frac{2}{3} \left(\frac{1}{2} f_\pi + f_K \right)$$

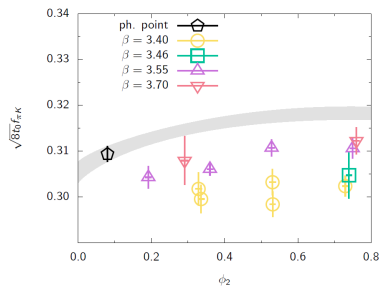


symmetric point: $m_\ell = m_s$, $M_\pi = M_K = 420$ MeV

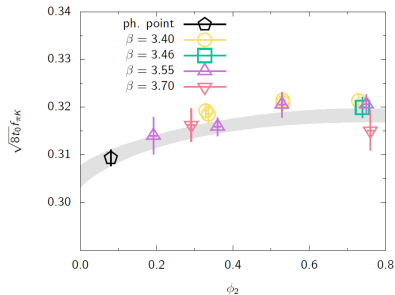
continuum-chiral extrapolation of $f_{\pi K}$

$$f_{\pi K} \equiv \frac{2}{3} \left(\frac{1}{2} f_{\pi} + f_K \right)$$

Wilson



Wilson twisted mass

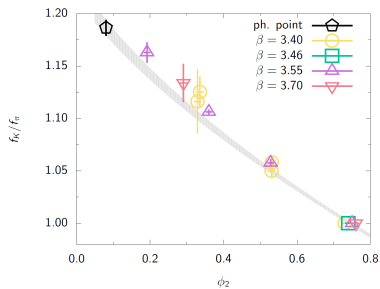


Systematic effects [ongoing]:

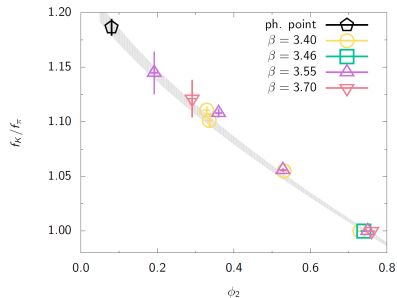
- ▶ SU(3) NLO χ PT + Taylor series of f_{π} , f_K , f_K/f_{π} , $f_{\pi K}$
- ▶ cuts in M_{π}
- ▶ discretization effects: $O(a^2)$, $O(\phi_2 a^2)$
- ▶ excited states contamination

continuum-chiral extrapolation of f_K/f_π

Wilson



Wilson twisted mass

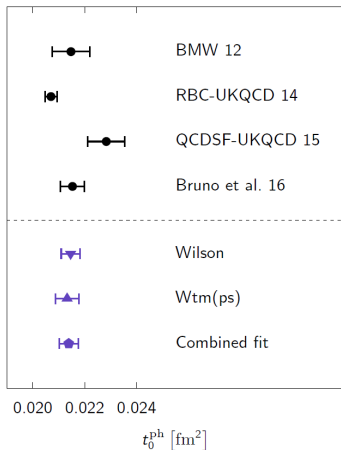


Systematic effects [ongoing]:

- ▶ SU(3) NLO χ PT + Taylor series of f_π , f_K , f_K/f_π , $f_{\pi K}$
- ▶ cuts in M_π
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- ▶ excited states contamination

scale setting: $t_0 \quad N_f = 2 + 1$

preliminary



$\sqrt{t_0}$ uncertainty: $\sim 1\%$

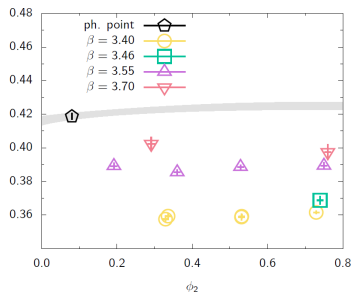
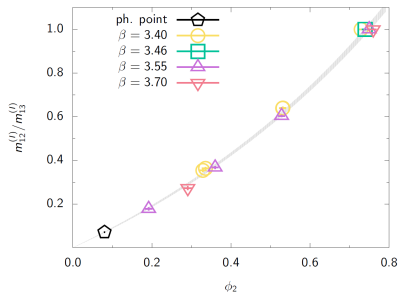
physical input: M_π, M_K, f_π, f_K [FLAG & PDG]

light-quark masses

non-perturbative renormalization and running [Schrödinger Functional] [\[ALPHA, 1802.05243\]](#)

$$\frac{m_{12}}{m_{13}} \propto \frac{2m_{ud}}{m_{ud}+m_s}$$

$$\frac{1}{\sqrt{8f_0}} \left(\frac{m_{12}^p}{M_\pi^2} + 2 \frac{m_{13}^p}{M_K^2} \right)$$



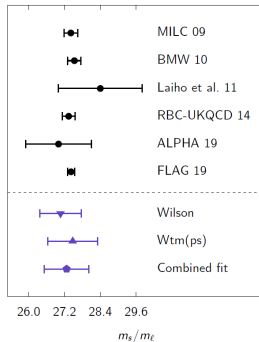
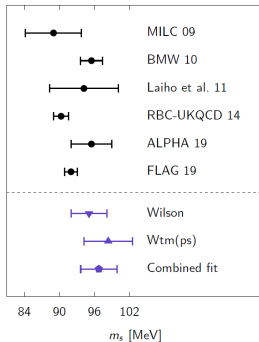
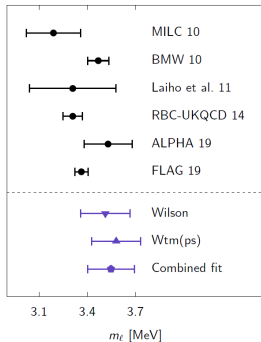
Wilson twisted mass [similar behaviour for Wilson]

Systematic effects [ongoing]:

- ▶ SU(3) NLO χ PT + Taylor series of m_{12} , m_{13} , m_{12}/m_{13} , $m_{12}/M_\pi^2 + 2m_{13}/M_K^2$
- ▶ cuts in M_π
- ▶ discretization effects: $O(\alpha^2)$, $O(\phi_2 \alpha^2)$
- ▶ excited states contamination

light-quark masses $N_f = 2 + 1$

preliminary



m_ℓ & m_s in $\overline{\text{MS}}$ at 2 GeV

conclusions

- ▶ mixed action: Wilson twisted mass on Wilson fermions
- ▶ scale setting: t_0 combining Wilson & Wtm
- ▶ determination of quark masses : m_{ud} , m_s
- ▶ ongoing analysis of systematic effects
- ▶ extend analysis to lighter ensembles and finer lattice spacing
- ▶ heavy-quark physics

see talks by Alessandro Conigli [Thu.,5:45] & by Julien Frison [Thu.,14:15]