Leptonic decays of charmed mesons with Wilson quarks

Fabian Joswig

G. S. Bali, S. Collins, J. Heitger, S. Kuberski, W. Söldner

29th July 2021

Topics of this talk

- Leptonic decay constants of the D and D_s mesons
  $\rightarrow$ Constraints on the CKM matrix elements V_{cd} & V_{cs}

$$i f_{D_q} p_\mu = \langle 0 | A_\mu^{qc} | D_q(p) \rangle$$

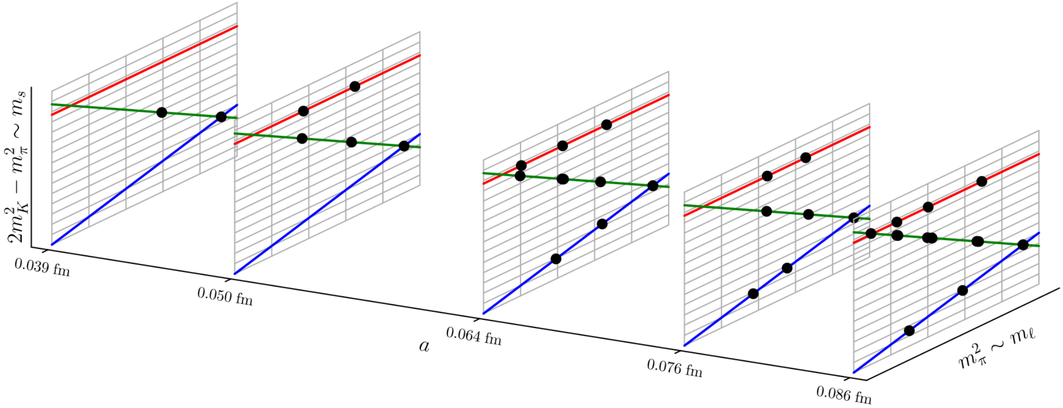
- Leptonic vector & tensor decay constants of the D^* and D_s^* mesons
  $\rightarrow$ Prediction of decay rates & comparison to QCD sum rule calculations

$$m_{D_s^*} f_{D_q^*} \epsilon_\mu^\lambda = \langle 0 | V_\mu^{qc} | D_q^*(p,\lambda) \rangle$$
$$i f_{D_q^*}^T (\epsilon_\mu^\lambda p_\nu - \epsilon_\nu^\lambda p_\mu) = \langle 0 | T_{\mu\nu}^{qc} | D_q^*(p,\lambda) \rangle$$

Coordinated Lattice Simulations $N_f = 2 + 1$ ensembles

Our analysis is based on 44 gauge field ensembles generated as part of the CLS initiative. Some features include:

- $N_f = 2 + 1$ $O(a)$ improved Wilson fermions
- Open or periodic boundary conditions in the temporal direction
- Five lattice spacings $a \approx 0.039 - 0.085$ fm
- Two physical point ensembles
- Three trajectories in the quark mass plane
- Algorithmic setup discussed in [1411.3982] & [1606.09039]


$2m_K^2 - m_\pi^2 \sim m_s$
0.039 fm
0.050 fm
a
0.064 fm
0.076 fm
0.086 fm
$m_\pi^2 \sim m_\ell$

[1903.12590]

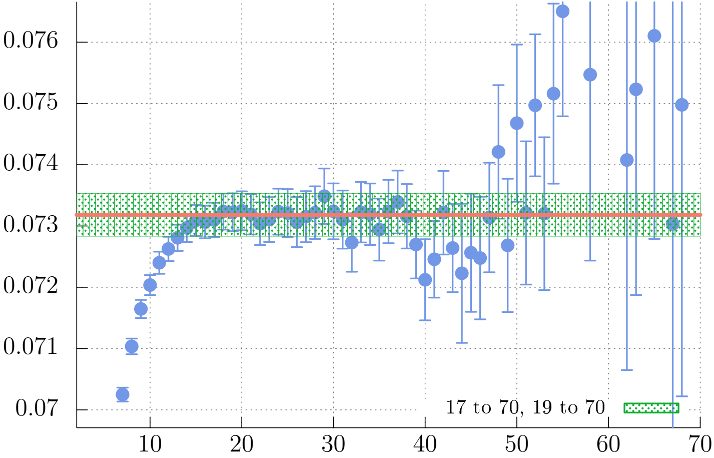
Bare decay constants obtained from spectral decomposition:

$$C^{qc,\mathrm{I}}_{\mathrm{A_0}\tilde{\mathrm{P}}}(x_0,y_0) \propto A^{qc}_{\mathrm{A_0}\tilde{\mathrm{P}}}\,\mathrm{e}^{-m_{\mathrm{D}_q}(x_0-y_0)}\,, \qquad f_{\mathrm{D}_{(q)}} = \frac{\sqrt{2}A^{qc}_{\mathrm{A_0}\tilde{\mathrm{P}}}}{\sqrt{A^{qc}_{\tilde{\mathrm{P}}\tilde{\mathrm{P}}}m_{\mathrm{D}_q}}}\,.$$

$$C^{qc}_{\tilde{\mathrm{P}}\tilde{\mathrm{P}}}(x_0,y_0) \propto A^{qc}_{\tilde{\mathrm{P}}\tilde{\mathrm{P}}}\,\mathrm{e}^{-m_{\mathrm{D}_q}(x_0-y_0)}\,,$$

- Point-to-all propagators with Gaussian smearing and APE-smoothed links
- Multiple source positions for each ensemble
- Region of groundstate dominance explored with two-state fits
- Matrix elements extracted with simultaneous one-state fits
- Two values for the valence charm quark mass on each ensemble

$$f^{\mathrm{eff}}_{\mathrm{D}_q}(x_0,y_0) = \frac{\sqrt{2}C^{qc,\mathrm{I}}_{\mathrm{A_0}\tilde{\mathrm{P}}}(x_0,y_0)}{\sqrt{C^{qc}_{\tilde{\mathrm{P}}\tilde{\mathrm{P}}}(x_0,y_0)m_{\mathrm{D}_q}\mathrm{e}^{-m_{\mathrm{D}_q}(x_0-y_0)}}}$$

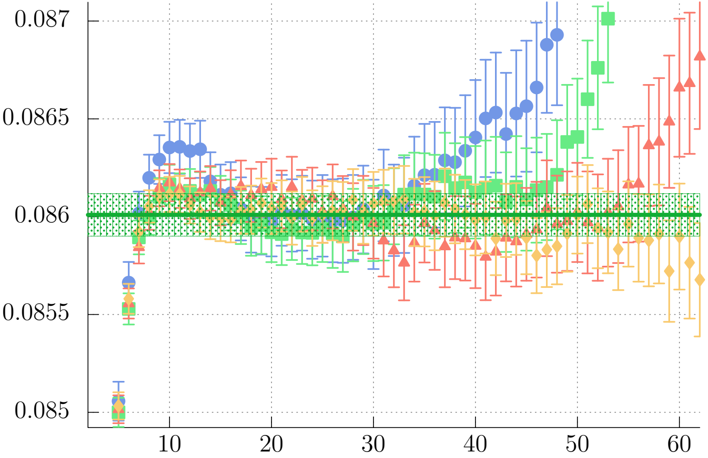


Effective decay constant f_D on ensemble E250

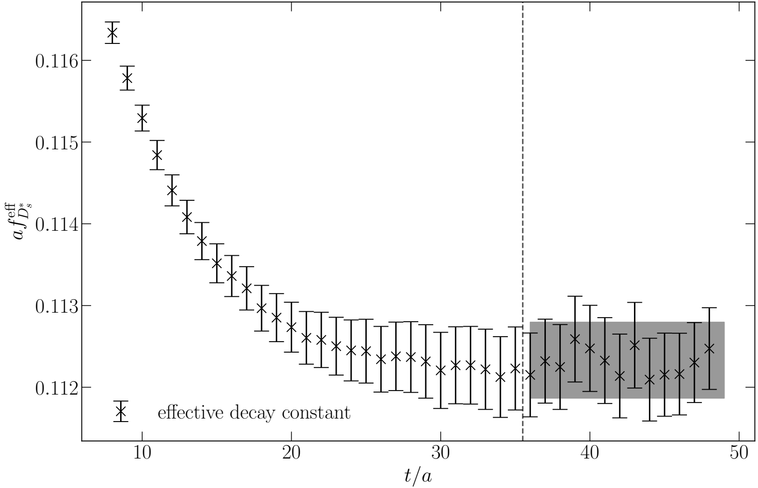
periodic boundary conditions

$T/a \times (L/a)^3 = 192 \times 96^3$

$m_\pi \approx 129$ MeV, $a \approx 0.064$ fm



Effective decay constant f_{D_s}
on ensemble D200
open boundary conditions
$T/a \times (L/a)^3 = 128 \times 64^3$
$m_\pi \approx 200\,\text{MeV}$, $a \approx 0.064\,\text{fm}$

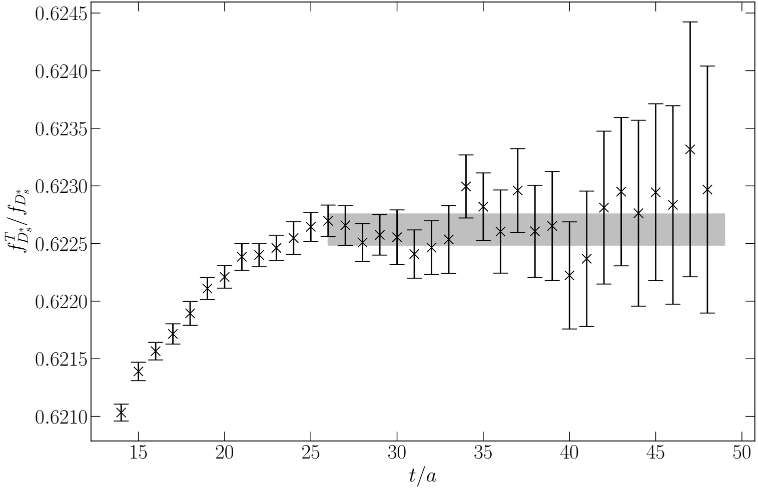


Extraction of $f_{D_s^*}$ on D200

open boundary conditions

$T/a \times (L/a)^3 = 128 \times 64^3$

$m_\pi \approx 200$ MeV, $a \approx 0.064$ fm



Extraction of $f^{\mathrm{T}}_{\mathrm{D}_s^*}/f_{\mathrm{D}_s^*}$ on D200

open boundary conditions

$T/a \times (L/a)^3 = 128 \times 64^3$

$m_\pi \approx 200$ MeV, $a \approx 0.064$ fm

Renormalization and improvement

Renormalization and improvement patterns as implied by Symanzik effective theory:

$$f^{\mathrm{R}}_{D_q} = Z_{\mathrm{A}}\left(1 + b_{\mathrm{A}} a m_{\mathrm{q},cq} + \bar{b}_{\mathrm{A}} a \mathrm{Tr}[M_{\mathrm{q}}]\right)\left(f^{(0)}_{D_q} + c_{\mathrm{A}} f^{(1)}_{D_q}\right) + \mathrm{O}(a^2)$$

$$f^{\mathrm{R}}_{D^*_q} = Z_{\mathrm{V}}\left(1 + b_{\mathrm{V}} a m_{\mathrm{q},cq} + \bar{b}_{\mathrm{V}} a \mathrm{Tr}[M_{\mathrm{q}}]\right)\left(f^{(0)}_{D^*_q} + c_{\mathrm{V}} f^{(1)}_{D^*_q}\right) + \mathrm{O}(a^2)$$

$$f^{\mathrm{T,R}}_{D^*_q} = Z_{\mathrm{T}}\left(1 + b_{\mathrm{T}} a m_{\mathrm{q},cq} + \bar{b}_{\mathrm{T}} a \mathrm{Tr}[M_{\mathrm{q}}]\right)\left(f^{\mathrm{T},(0)}_{D^*_q} + c_{\mathrm{T}} f^{\mathrm{T},(1)}_{D^*_q}\right) + \mathrm{O}(a^2)$$

Non-perturbative knowledge of

- Z_{A}, b_{A}, c_{A} – [1502.04999], [1604.05827], [1607.07090], [1808.09236]
- Z_{V}, b_{V}, c_{V} – [1805.07401], [1811.08209], [2010.09539]
- Z_{T}, c_{T} – [1910.06759], [2012.06284]

Chiral and continuum extrapolation

Position in the quark mass plane parametrized by:

$$\phi_\Delta \equiv 8t_0\left(m_K^2 - m_\pi^2\right), \quad \phi_4 \equiv 8t_0\left(m_K^2 + \tfrac{1}{2}m_\pi^2\right), \quad \phi_H \equiv \sqrt{8t_0}\,m_H$$

General ansatz inspired by $SU(3)$ chiral perturbation theory:

$$\sqrt{8t_0}f_{D_s}^R(m_\pi, m_K, 0) = f_0 + \bar{d}\,\phi_4 + \tfrac{2}{3}\delta d\,\phi_\Delta + b_{\log}\left(4\mu_K + \tfrac{4}{3}\mu_\eta\right) + d_H\,\phi_H + \ldots$$
$$\sqrt{8t_0}f_D^R(m_\pi, m_K, 0) = f_0 + \bar{d}\,\phi_4 - \tfrac{1}{3}\delta d\,\phi_\Delta + b_{\log}\left(3\mu_\pi + 2\mu_K + \tfrac{1}{3}\mu_\eta\right) + d_H\,\phi_H + \ldots$$

- with the chiral log defined as $\mu_X = m_X^2\log(m_X^2)$

Chiral and continuum extrapolation

We model the cutoff dependence via

$$\sqrt{8t_0} f^{R}_{D_{(s)}}(m_\pi^{\text{phys}}, m_K^{\text{phys}}, a) = \frac{a^2}{8t_0^*}\left(c + c_H\,\phi_H^2 + \bar{c}\,\phi_4 + c_{D_{(s)}}\,\phi_\Delta + \dots\right)$$

- f_D^R and $f_{D_s}^R$ are implicitly degenerate in the $SU(3)$ symmetric limit
- For m_H we either choose the flavor averaged D mass or the mass of the D_s meson

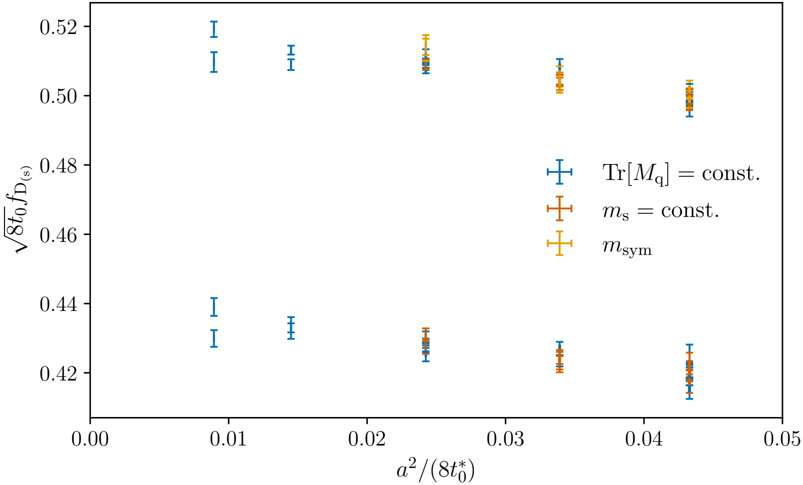
Chiral and continuum extrapolation

- Fully correlated fit with block diagonal covariance matrices (6×6 at the $SU(3)$ symmetric point, 9×9 away from it)

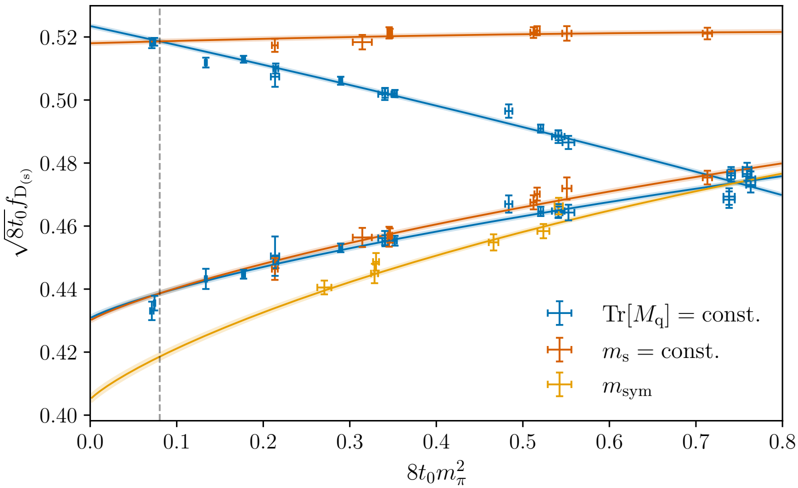
$$\chi^2 = \sum_{e=1}^{n_e} \delta f_e \, C_e^{-1} \, \delta f_e^T \,, \qquad \delta f_e = \left(f(p_i; \tilde{x}_e) - y_e, \tilde{x}_e - x_e \right) \,,$$

$$y_e = \left(f_{D_s}^{C1}, f_{D_s}^{C2}, f_{D}^{C1}, f_{D}^{C2} \right)_e \,, \qquad x_e = \left(m_\pi, m_K, m_{\bar{D}}^{C1}, m_{\bar{D}}^{C2}, 8t_0 \right)_e \,.$$

- Renormalization and improvement parameters as well as t_0^* taken into account via Gaussian priors



Lattice spacing dependence of a representative combined chiral and continuum extrapolation.



Light quark mass dependence of a representative combined chiral and continuum extrapolation.

- Chiral logarithm visible for f_D

Summary & Outlook

Final results for f_{D} & $f_{\mathrm{D_s}}$ soon
Expected error budget:

- Total statistical error of a few permille
- Scale setting error $\sim$ 0.8% (dominant)
  $\rightarrow$ Update see talk of W. Söldner
- Systematic error has to be quantified

Analysis of the vector and tensor decay constants at an earlier stage

- Bare vector decay constants can be obtained at $\sim$ 0.5% precision
- The bare ratios $f^{\mathrm{T}}_{\mathrm{D}^*_q}/f_{\mathrm{D}^*_q}$ can be extracted at sub permille precision.
  Dominant contribution from $Z_{\mathrm{T}}(\mu)$.