SU(3) symmetry breaking in f_B and f_{Bs}

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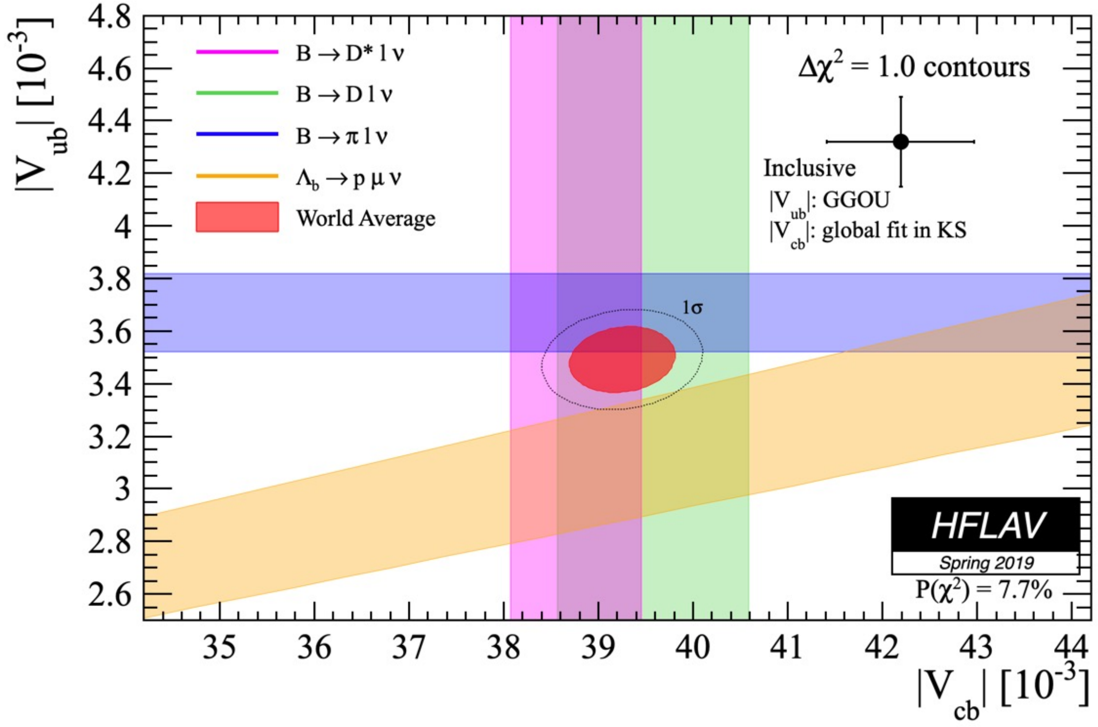
technische universität dortmund

THE UNIVERSITY of ADELAIDE

SPECIAL RESEARCH CENTRE FOR THE SUBATOMIC STRUCTURE OF MATTER

Decay constants and the Standard Model

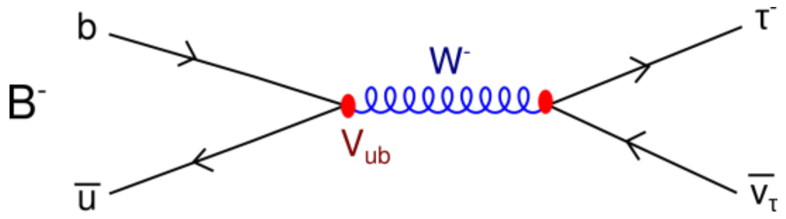


- Discrepancy between inclusive and exclusive measurements of $|V_{cb}|$ and $|V_{ub}|$
- Future exclusive measurements of $B^+ \to \tau^+ \nu$ may help
  - Decay constants instead of form factors
  - Independent exclusive measurement without charm background

$$\mathrm{Br}(B \to \tau\nu)_{\mathrm{SM}} = \frac{G_F^2 |V_{ub}|^2}{8\pi} \tau_B f_B^2 m_\tau^2 m_B \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2$$

$$\begin{bmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{bmatrix}$$

From leptonic and semileptonic B decays

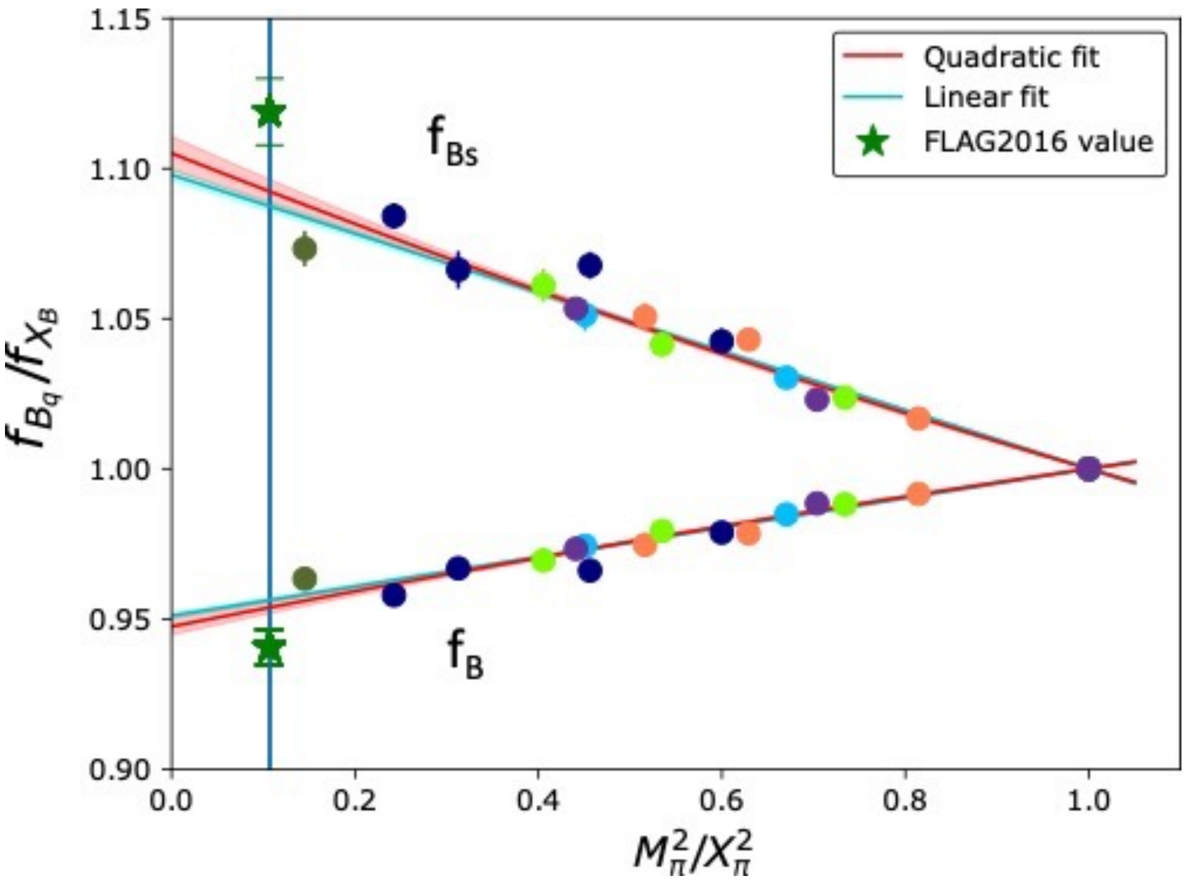


f_{Bs}/f_B current status and overview

- CSSM/UKQCD/QCDSF *fBs/fB*:
  - Four lattice spacings at $N_f = 2 + 1$
  - Ensembles with light and strange quark varying (SU(3) controlled)
  - Multiple *b* quarks used and best *b* is interpolated
- SYSTEMATICS:
  - Study of fit/extrapolation methods for final result
  - Largest contribution in systematics: choice of correlator fit windows
  - Currently working on improved methods!

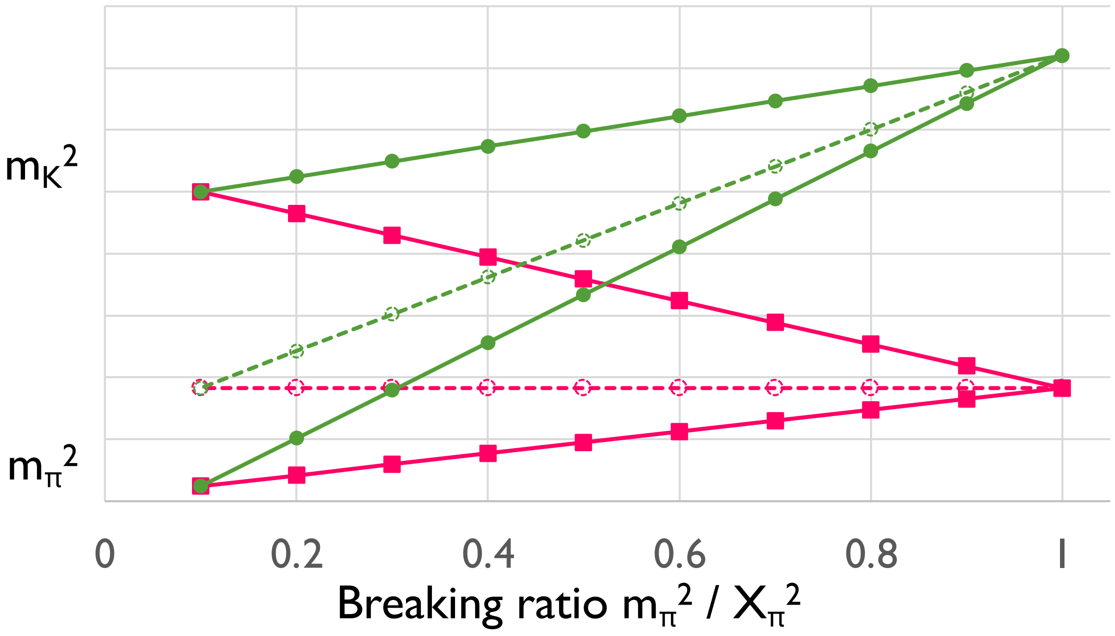
Central value: $m_\pi L > 4$ ensembles, single fit to all *a*:

$$\frac{f_{Bs}}{f_B} = 1.159 \pm 0.015\ (\mathrm{stat}) \pm 0.07\ (\mathrm{syst})$$



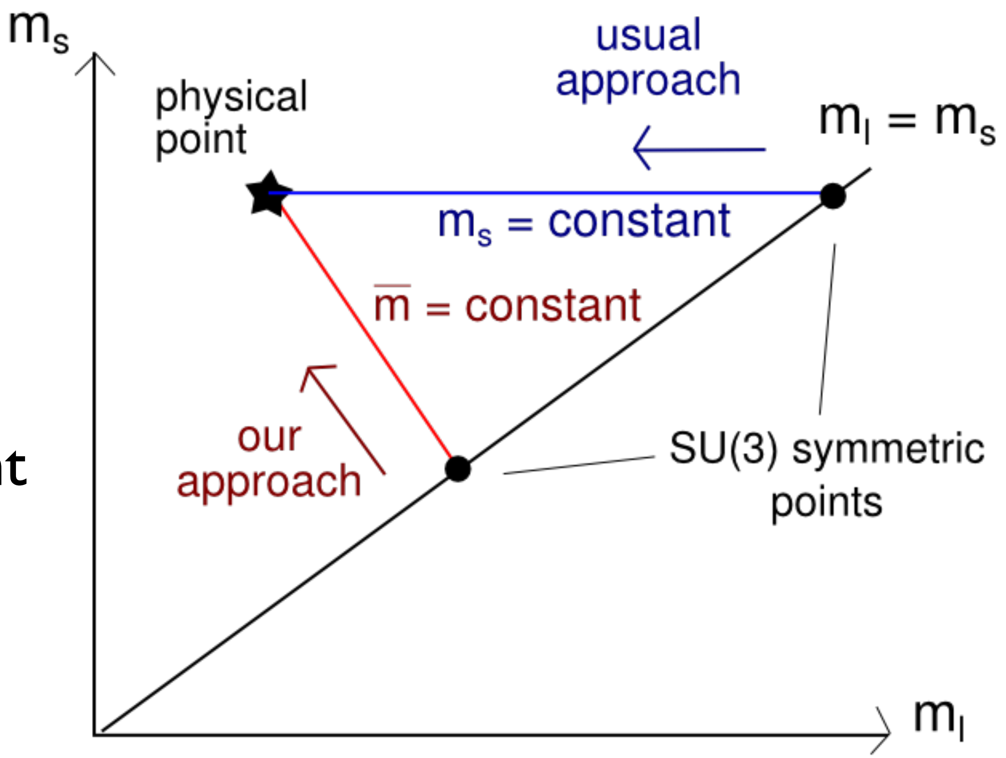
Lighter quarks and ensembles

- Order *a* improved Wilson clover action (sea and valence quarks)
- Overall principle: keep average mass of three lightest quarks constant across all ensembles



m_s = constant

$$\overline{m} = \tfrac{1}{3}\,(2m_l + m_s)$$



Flavour singlet quantities remain approx. constant ($O(\delta m)$ removed)

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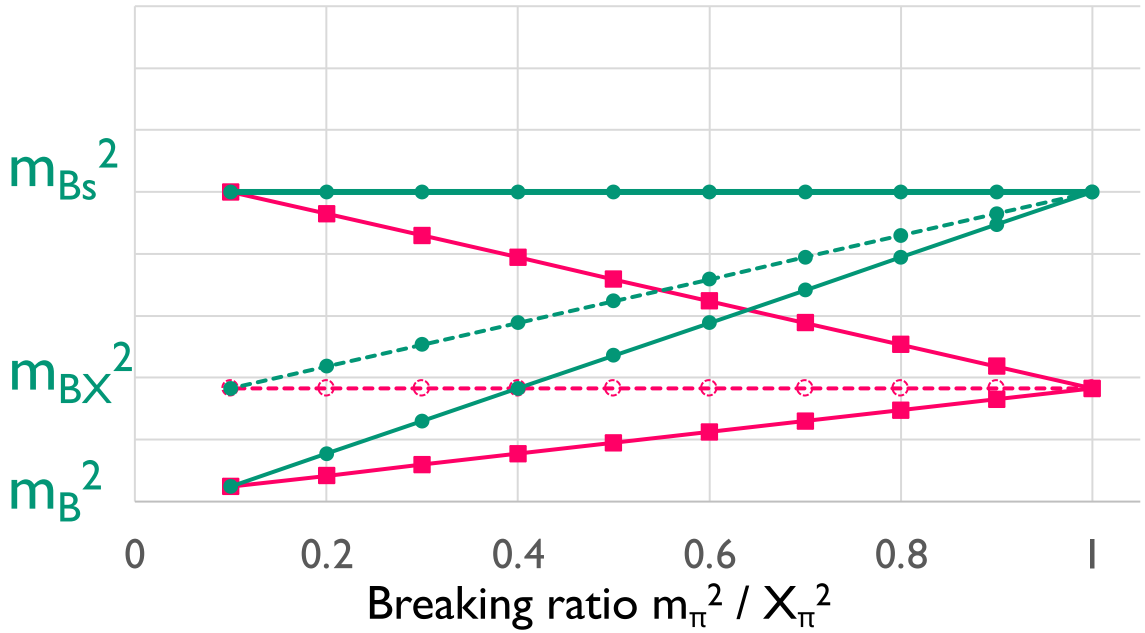

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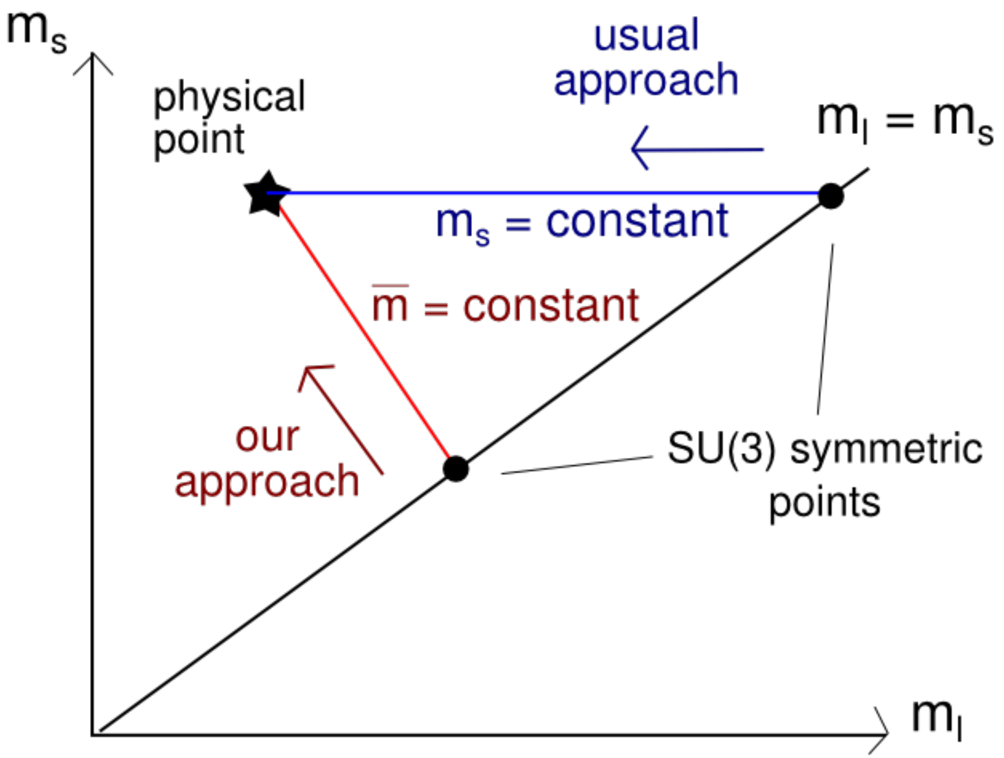


Flavour singlet quantities remain approx. constant ($O(\delta m)$ removed)

Generating *b*-quarks

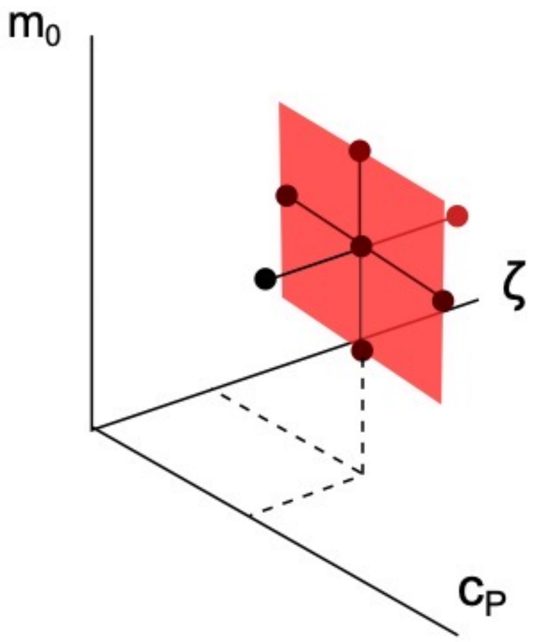
- Use an anisotropic, clover-improved action (Relativistic Heavy Quark Action), and then tune the free parameters to physical quantities for the B meson.

$$S_{lat} = a^4 \sum_{x,x'} \overline{\psi}(x') \left(m_0 + \gamma_0 D_0 + \zeta \vec{\gamma} \cdot \vec{D} - \frac{a}{2}(D^0)^2 - \frac{a}{2}\zeta(\vec{D})^2 + \sum_{\mu,\nu} \frac{ia}{4} c_P \sigma_{\mu\nu} F_{\mu\nu} \right)_{x,x'} \psi(x)$$

bare mass → spin-averaged meson mass

anisotropy → dispersion relation

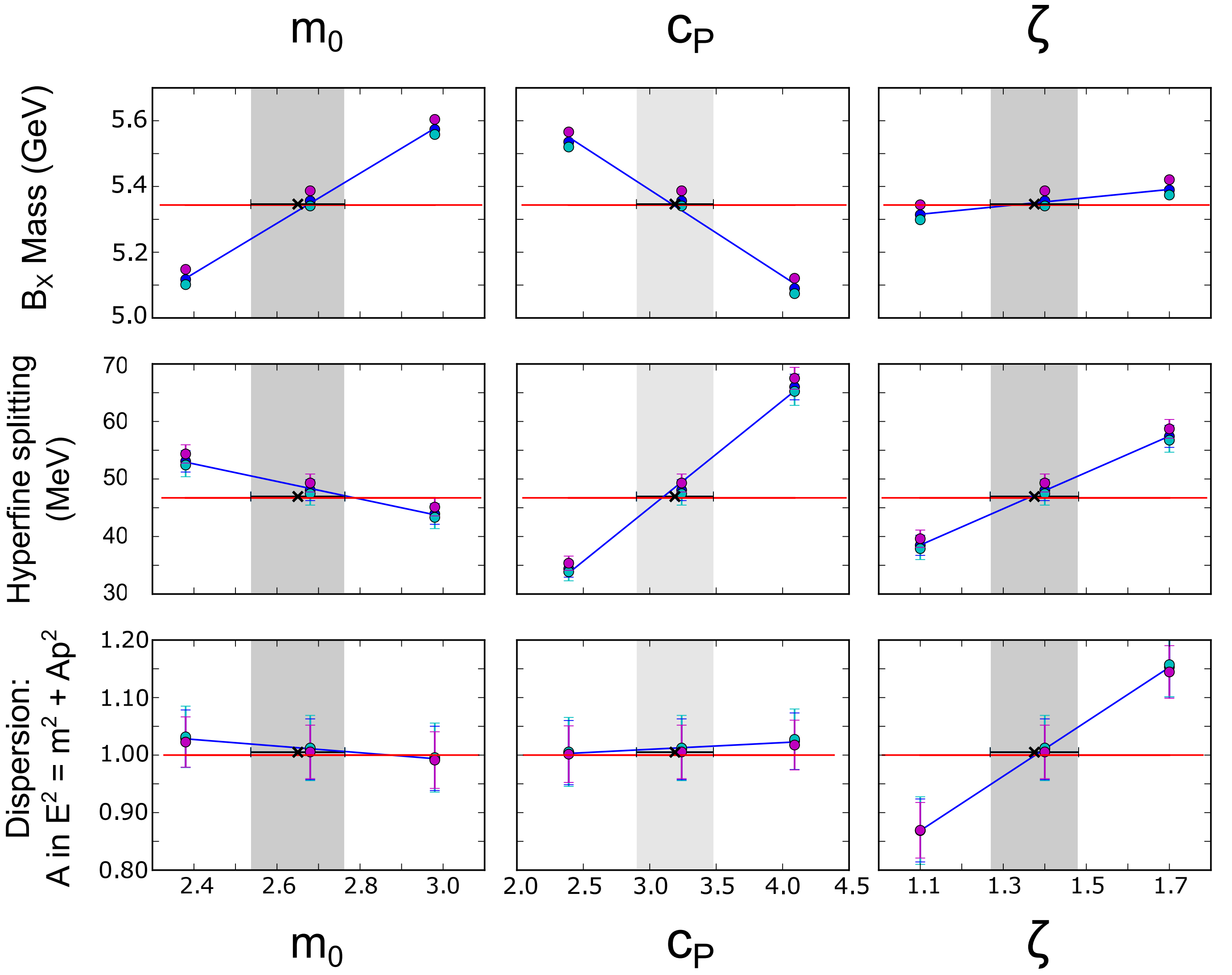
clover coefficient → hyperfine splitting between B* and B

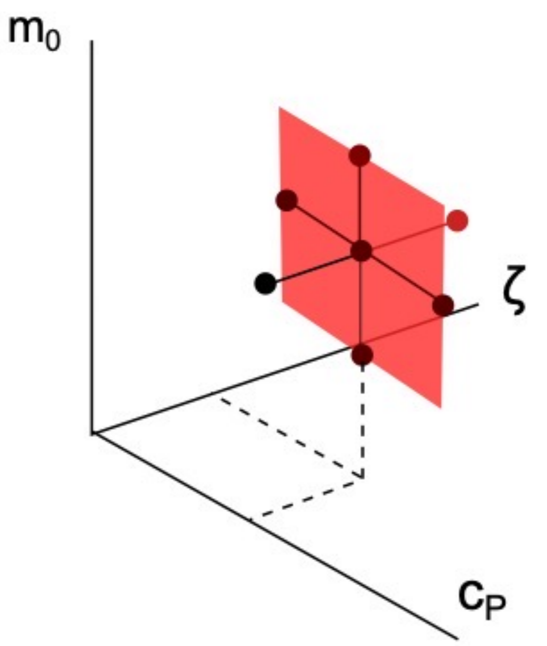


- Generate seven *b* quarks per ensemble in a "star" shape, and interpolate to the best *b*

Aoki, Y et al (2012). "Nonperturbative tuning of an improved relativistic heavy-quark action with application to bottom spectroscopy." *Physical Review D*, *86*(11), 116003. doi:10.1103/PhysRevD.86.116003

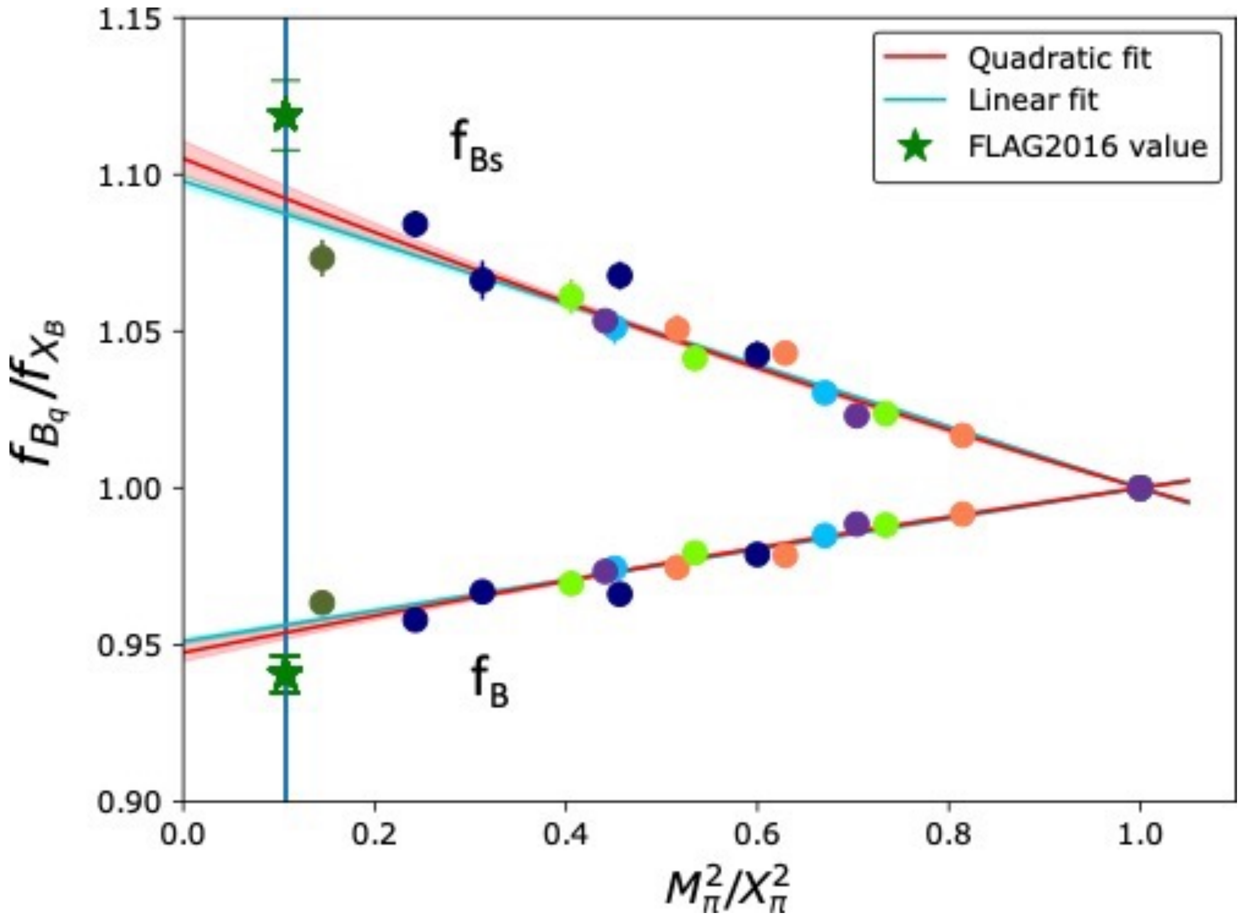
Tuning outcomes on one ensemble





Calculating f_B

- METHOD:
  - Calculate f_B or f_{Bs} for each *b* quark
  - Interpolate result at position of best *b* from tuning
  - Repeat for all ensembles
- In this figure:
  - Statistical error mostly from interpolation
  - Simple fit across all lattice spacings*
  - Good match with linear expectation of SU(3) expansion!

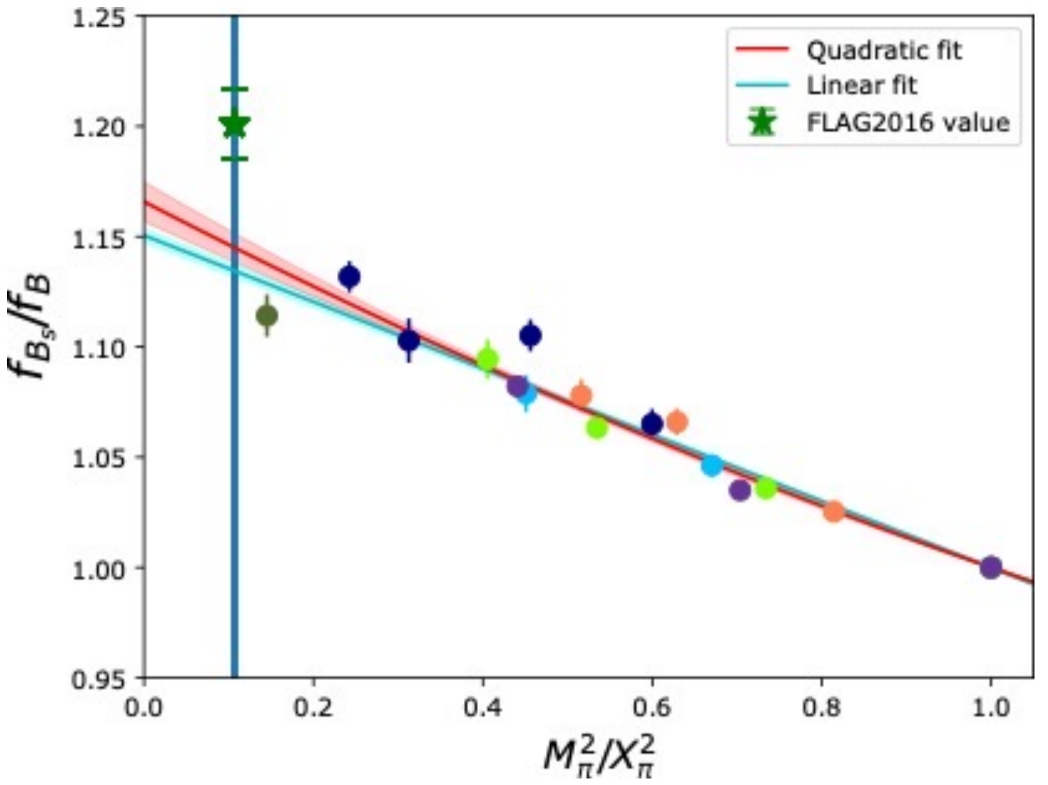


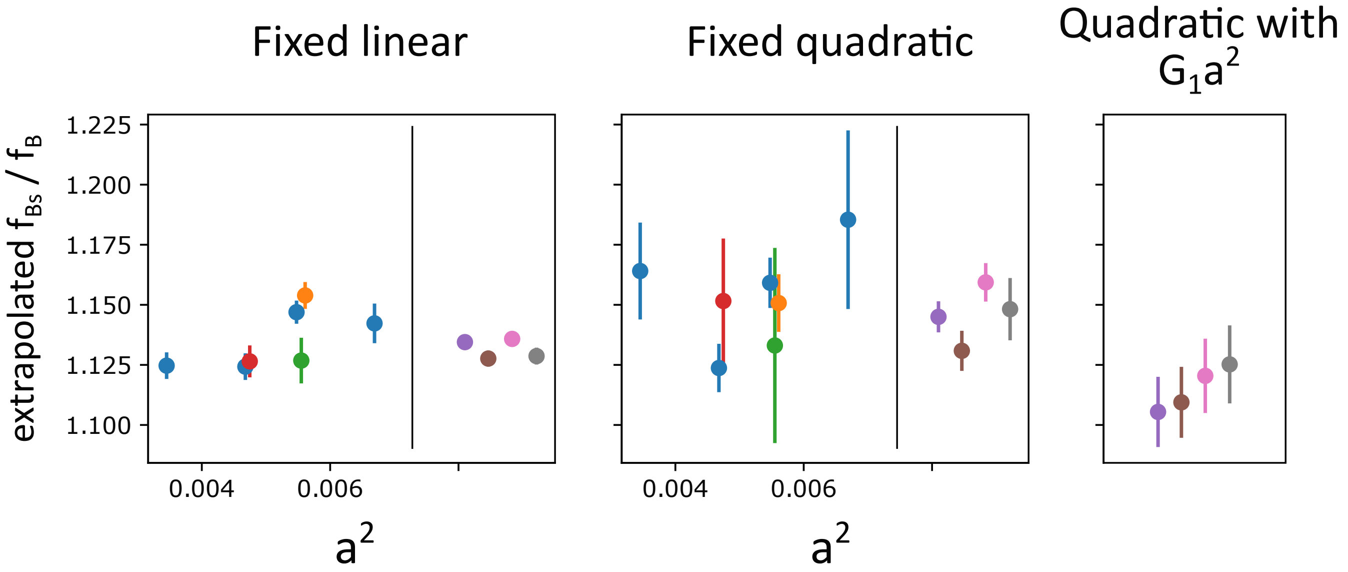
Ensemble information:
Size: 32^3 x 64 or larger
a (fm): 0.082, 0.074, 0.068, 0.059

*see backup slides for detailed ensemble information

Systematic studies in f_{Bs}/f_B: extrapolations

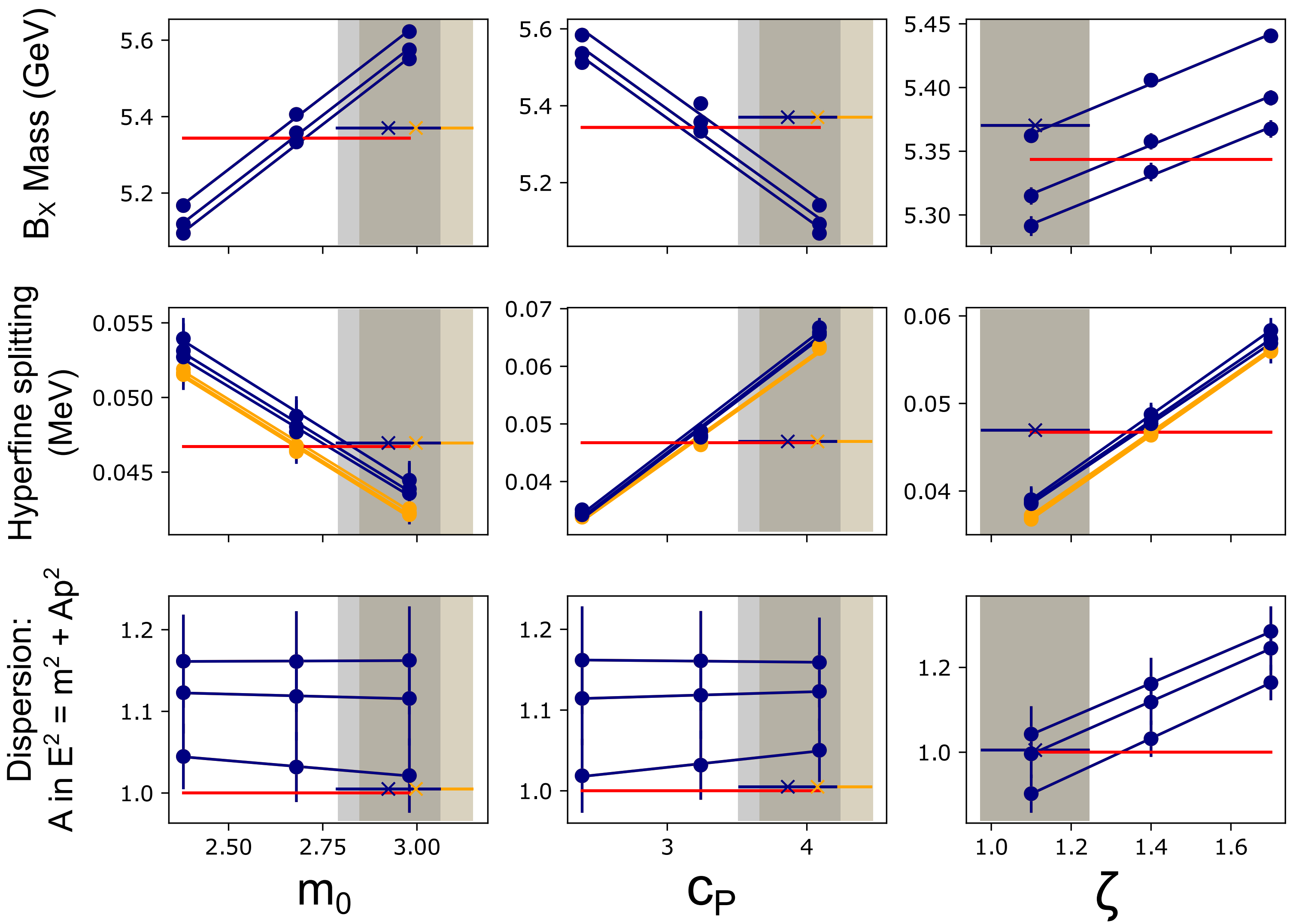
- Test of different extrapolation methods to physical pion and continuum
- Controlled SU(3) breaking of ensembles means f_{Bs}/f_B expected to be mostly linear





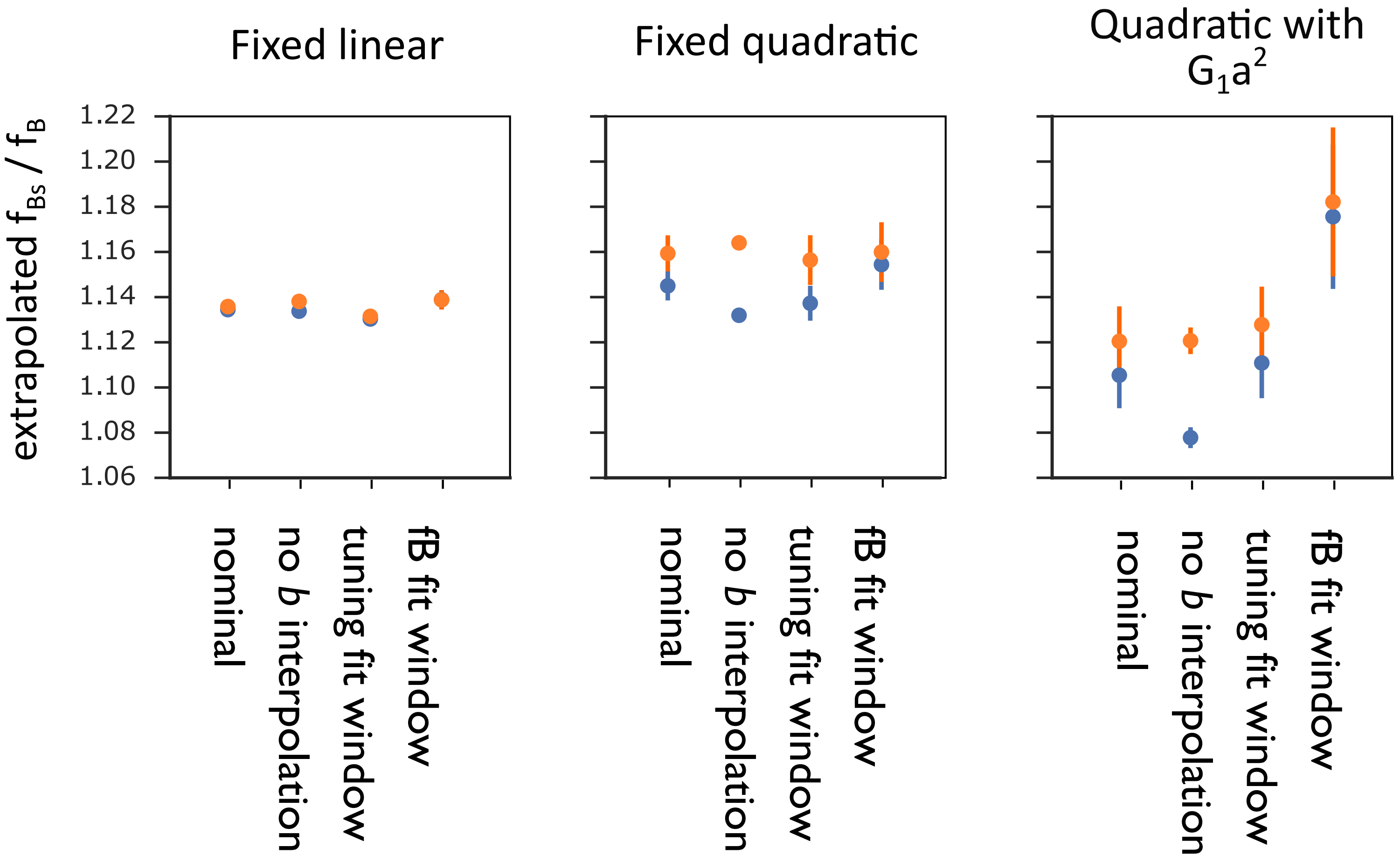
Data	Fit type	Value at physical	stat. error	χ^2/dof fit
All ensembles	Linear	1.134	0.003	1.8
	Quadratic	1.145	0.006	1.8
	Quadratic with a^2	1.105	0.015	1.3
$m_\pi L > 4$	Linear	1.136	0.003	1.8
	Quadratic	1.159	0.008	1.3
	Quadratic with a^2	1.120	0.015	0.9
FLAG value		1.201	0.016	

Systematic studies in f_{Bs}/f_B: correlators and tuning



- How much does the tuning affect the result?
  - Test nominal best tuning (dark blue) against refit of hyperfine splitting (pale orange)
  - Test effect of these interpolated tunings (cross) compared to the centre point in the tuning star

Systematic studies in f_{Bs}/f_B: correlators and tuning



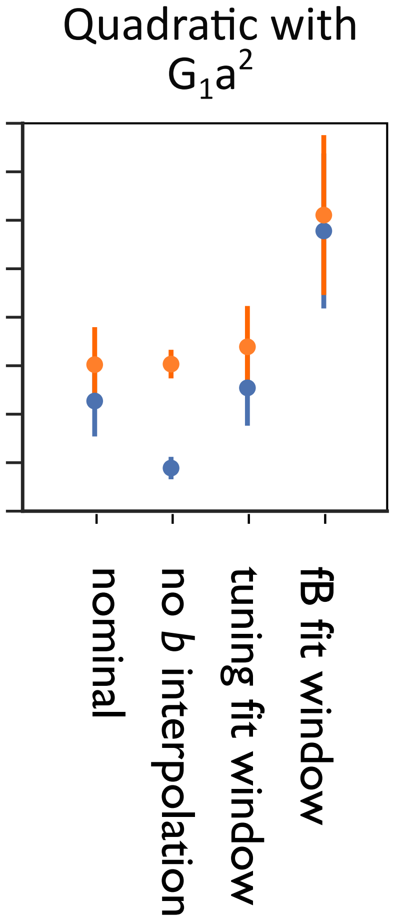
- Correlator fits for the tuning and the tuning itself affects the final f_{Bs}/f_B result
- Result is strongly affected by last result close to m_π
- The best f_B and f_{Bs} fit windows can be quite different for each *b* quark.

Fit to all ensembles

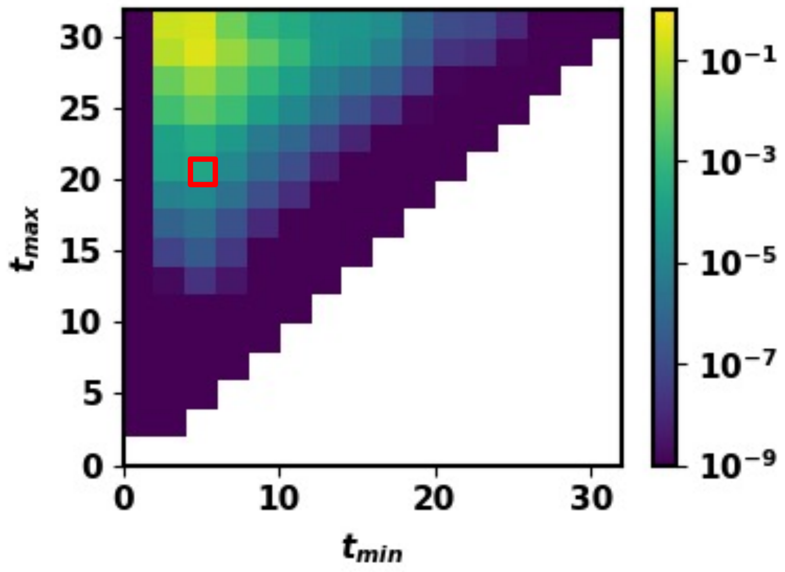
Fit to $m_\pi L > 4$ only

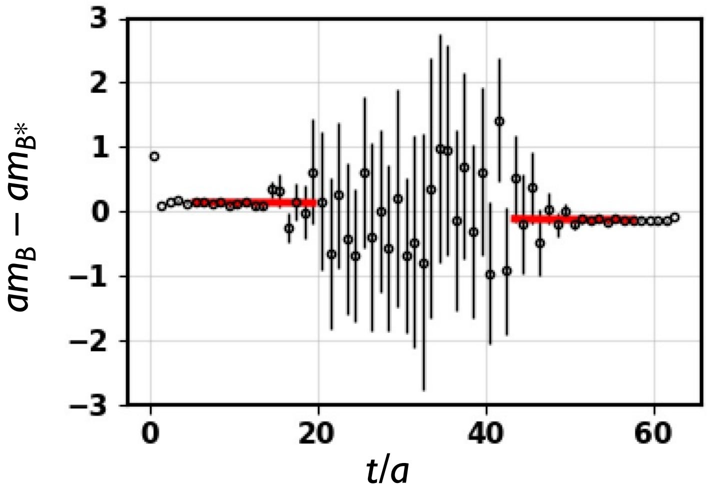
Improving fit reproduceability

- B meson fit procedure requires a large number of fits:
  - 7 *b* quarks * 2 lighter quarks * (6 correlators for tuning + 1 extra for simple f_B)
    = 98 fits per ensemble (after tuning is complete)
  - **Large number of fits: high chance of variation from analyst impacting result!**
- Updated approach: make use of weighted fitting methods
  - Calculate more fits but improve reliability
  - Better quantify the difference in quality between fits



Using a weighted fitting framework





- First tests of weighted fit in this B meson analysis
  - This example: fits to the hyperfine splitting
  - Example effective mass fit with plateau 5-20
- Testing different weighting systems:
  - p-value-like weights [1]

$$w^f = \frac{p_f (\delta E_0^f)^{-2}}{\sum_{f'=1}^{N_{\text{success}}} p_{f'} (\delta E_0^{f'})^{-2}} \qquad p_f = \frac{\Gamma(N_{\text{dof}}/2, \chi_f^2/2)}{\Gamma(N_{\text{dof}}/2)}$$

  - and Bayesian weights (example, left)[2]

$$w_i = \exp(-\frac{1}{2}\chi^2_{\nu,i} + N_{DOF,i})$$

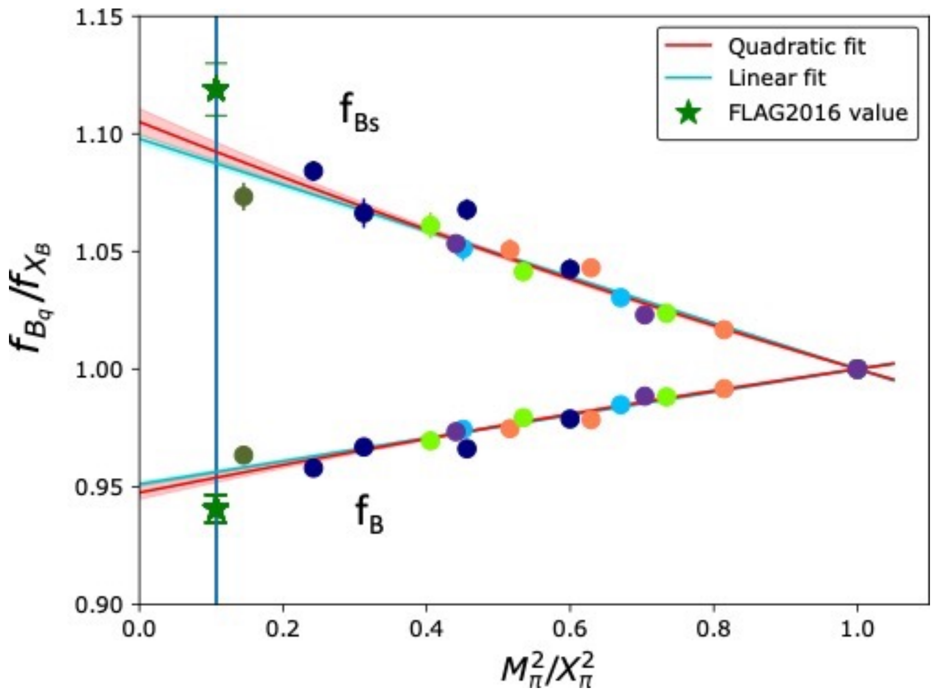
- See also: poster by Shanette De La Motte (location C6)

1 Beane, S. R. et al (2021). "Charged multi-hadron systems in lattice QCD+QED." *Physical Review D* https://doi.org/10.1103/PhysRevD.103.054504

2 William I. Jay and Ethan M. Neil (2021). "Bayesian model averaging for analysis of lattice field theory results" *Physical Review D* https://doi.org/10.1103/PhysRevD.103.114502

Summary

- We have developed a framework for calculating the decay constants f_{Bs} and f_B using the UKQCD/QCDSF ensembles
- Several different sources of systematic uncertainty affecting the result have been investigated
- Work is ongoing to improve our current estimate:
  - Adding new ensembles closer to the physical point
  - Using weighted fitting to make the process more consistent/improve f_B estimates



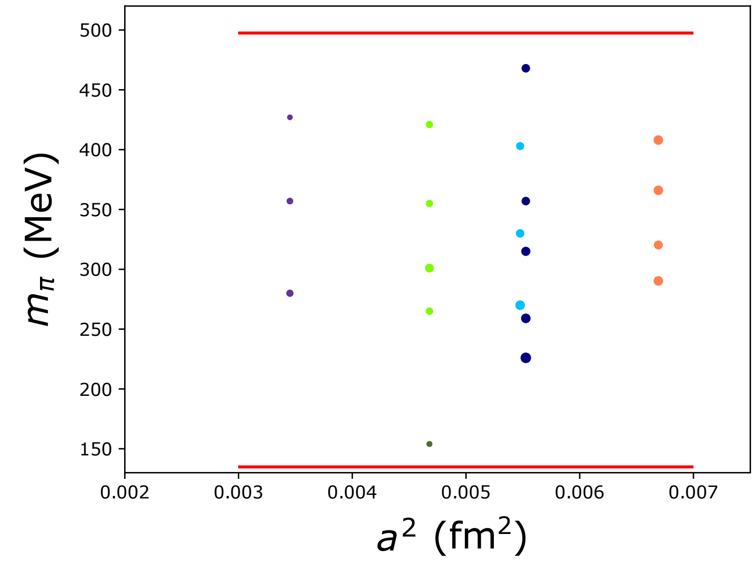
Source	-	+
Light vs strange 3pt corrections Z_V^{ss}/Z_V^{ll}	0	0.023
Changes to b tuning	0.007	0.007
Fitting to ensembles with light pion masses	0.015	0.015
Correlator fits used in the decay constant	0.07	0.07
SYSTEMATIC	**-0.071**	**+0.076**

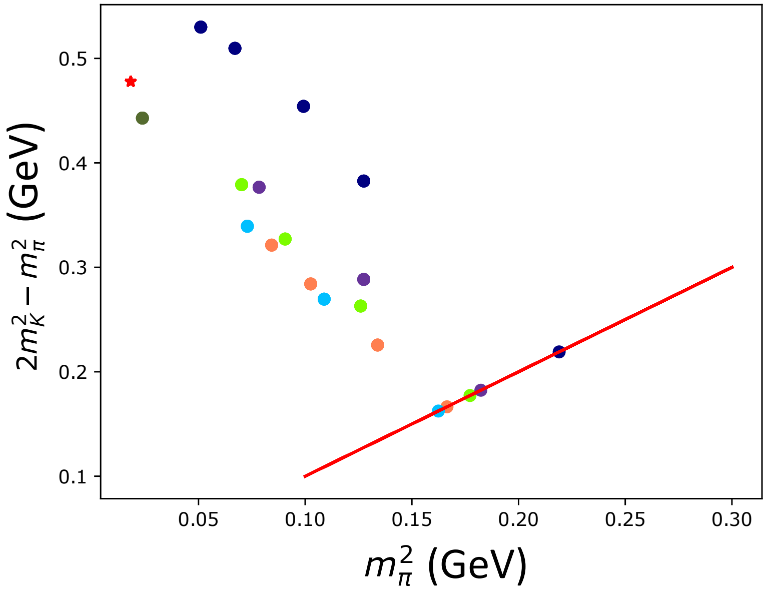
BACKUP SLIDES

Overview of ensembles used

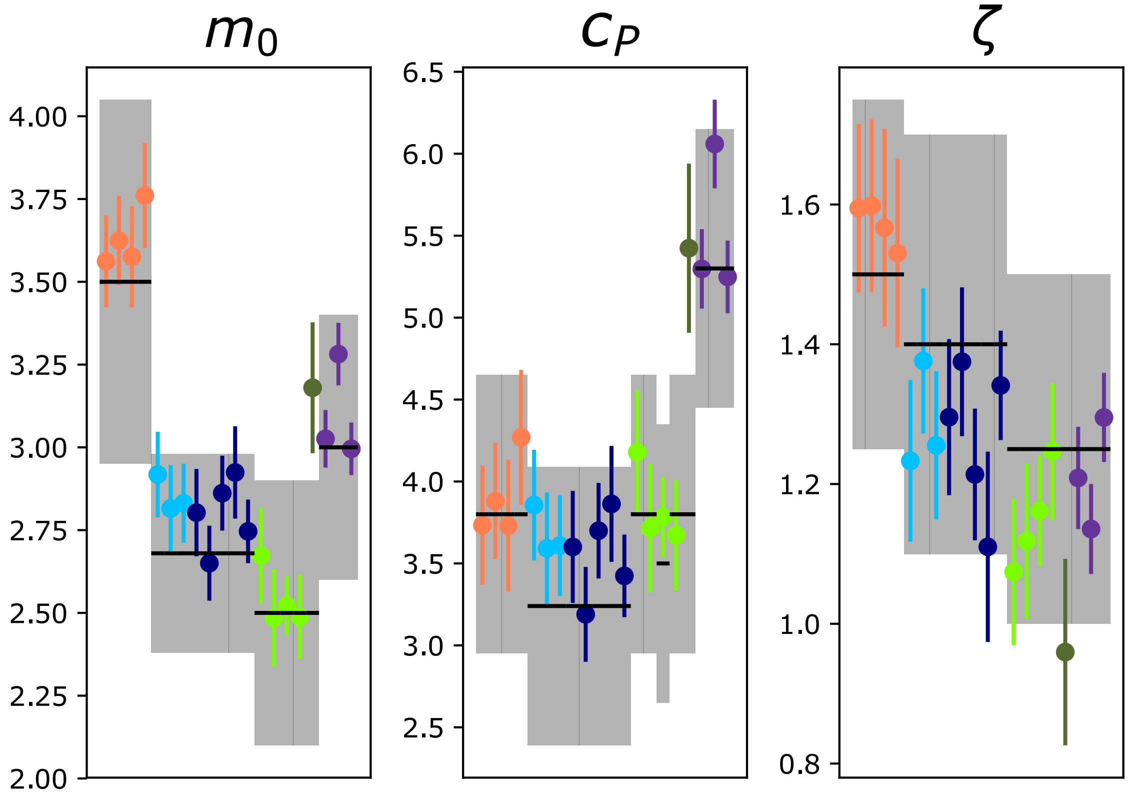
Table 1: Table of lattice ensembles used in this work. * indicates ensembles with a different value of $\overline{m}$, further from the physical $\overline{m}$. † indicates ensembles where multiple sources per configuration are used to produce additional samples. Marked ensembles use 2 randomised sources, except for the $64^3 \times 96$ sample with 4 randomised sources used.

β	a (fm)	Lattice volume	# Samples	$(\kappa_{\text{light}}, \kappa_{\text{strange}})$	m_π (MeV)	m_K (MeV)	
5.4	0.082	$32^3 \times 64$	1015	(0.11993 , 0.11993)	408	408	
			1004	(0.119989 , 0.119812)	366	424	
			877	(0.120048 , 0.119695)	320	440	
			1006	(0.120084 , 0.119623)	290	450	
5.5	0.074	$32^3 \times 64$	$677^\dagger$	(0.12095 , 0.12095)	403	403	
			786	(0.12104 , 0.12077)	331	435	
			1021	(0.121099 , 0.120653)	270	454	
		$32^3 \times 64$	778	(0.1209 , 0.1209)	468	468	*
			758	(0.12104 ,0.12062)	357	505	*
			$902^\dagger$	(0.121095 , 0.120512)	315	526	*
			1002	(0.121145 , 0.120413)	258	537	*
		$48^3 \times 96$	$1251^\dagger$	(0.121166 , 0.120371)	226	539	*
5.65	0.068	$48^3 \times 96$	500	(0.122005 , 0.122005)	412	412	
			500	(0.122078 ,0.121859)	355	441	
			$845^\dagger$	(0.12213 ,0.121756)	302	457	
			576	(0.122167 , 0.121682)	265	474	
		$64^3 \times 96$	$320^\dagger$	(0.122227 , 0.121563)	155	480	
5.8	0.059	$48^3 \times 96$	298	(0.12281 , 0.12281)	427	427	
			415	(0.12288 ,0.12267)	357	456	
			525	(0.12294 , 0.122551)	280	477	





Tuned b parameters on each ensemble



Grey band: size of tuning star

Coloured spot: tuned position for that ensemble

Table 1: The calculated 'best' tuning parameters and error margins for each of the ensembles used. * denotes ensembles with a different value of $\overline{m}$, further from the physical $\overline{m}$, represented in dark blue in all Figures. † denotes the near-physical 64^3x96 ensemble which has extrapolated parameters

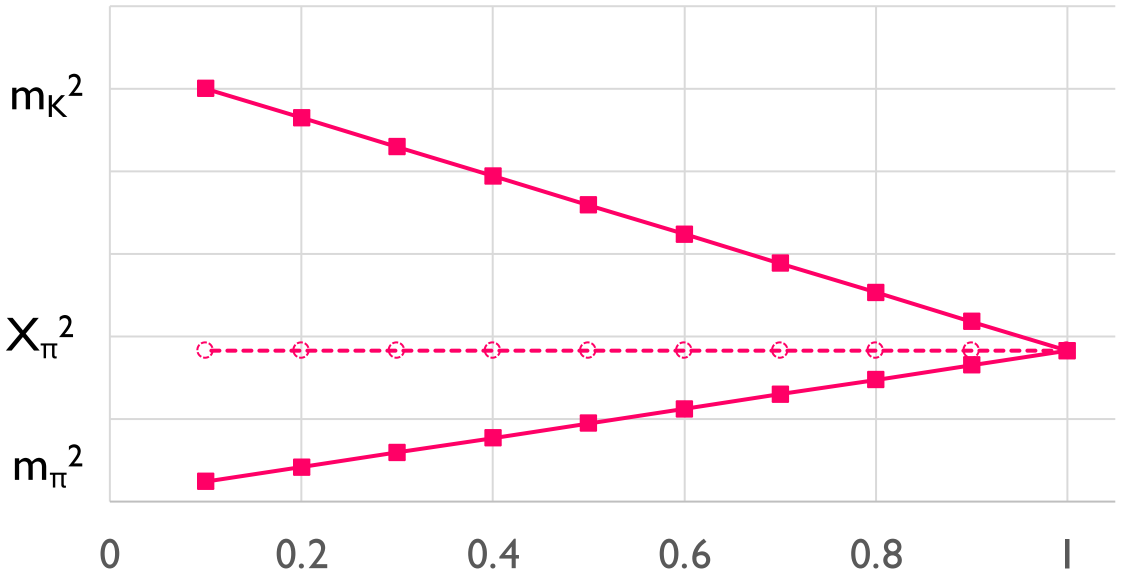
β	κ_l	m_0	c_P	ζ
5.4	0.11993	3.56 ± 0.14	3.73 ± 0.36	1.59 ± 0.12
	0.119989	3.62 ± 0.13	3.88 ± 0.35	1.60 ± 0.12
	0.120048	3.58 ± 0.15	3.73 ± 0.40	1.57 ± 0.14
	0.120084	3.76 ± 0.16	4.27 ± 0.41	1.53 ± 0.14
5.5	0.12095	2.92 ± 0.13	3.86 ± 0.34	1.23 ± 0.12
	0.12104	2.82 ± 0.13	3.59 ± 0.34	1.38 ± 0.10
	0.121099	2.83 ± 0.12	3.61 ± 0.31	1.26 ± 0.11
5.5*	0.1209	2.80 ± 0.13	3.60 ± 0.34	1.30 ± 0.11
	0.12104	2.65 ± 0.11	3.19 ± 0.29	1.37 ± 0.11
	0.121095	2.86 ± 0.11	3.70 ± 0.29	1.21 ± 0.09
	0.121145	2.92 ± 0.14	3.86 ± 0.35	1.11 ± 0.14
	0.121166	2.75 ± 0.10	3.42 ± 0.25	1.34 ± 0.08
5.65	0.122005	2.67 ± 0.14	4.18 ± 0.38	1.07 ± 0.10
	0.122078	2.48 ± 0.15	3.72 ± 0.39	1.12 ± 0.11
	0.12213	2.52 ± 0.09	3.78 ± 0.24	1.16 ± 0.08
	0.122167†	2.49 ± 0.13	3.67 ± 0.34	1.25 ± 0.10
5.8	0.122227	3.18 ± 0.20	5.42 ± 0.52	0.96 ± 0.13
	0.12281	3.03 ± 0.09	5.30 ± 0.24	1.21 ± 0.07
	0.12288	3.28 ± 0.09	6.06 ± 0.27	1.14 ± 0.06
	0.12294	3.00 ± 0.08	5.25 ± 0.22	1.30 ± 0.06

Choosing light and strange quarks

Light flavour singlets on QCDSF configurations, including:

$$X_\pi^2 = \frac{1}{6}\left(M_{K^+}^2 + M_{K^0}^2 + M_{\pi^+}^2 + M_{\pi^-}^2 + M_{\overline{K}^0}^2 + M_{K^-}^2\right),$$

Expected behaviour



Measured behaviour

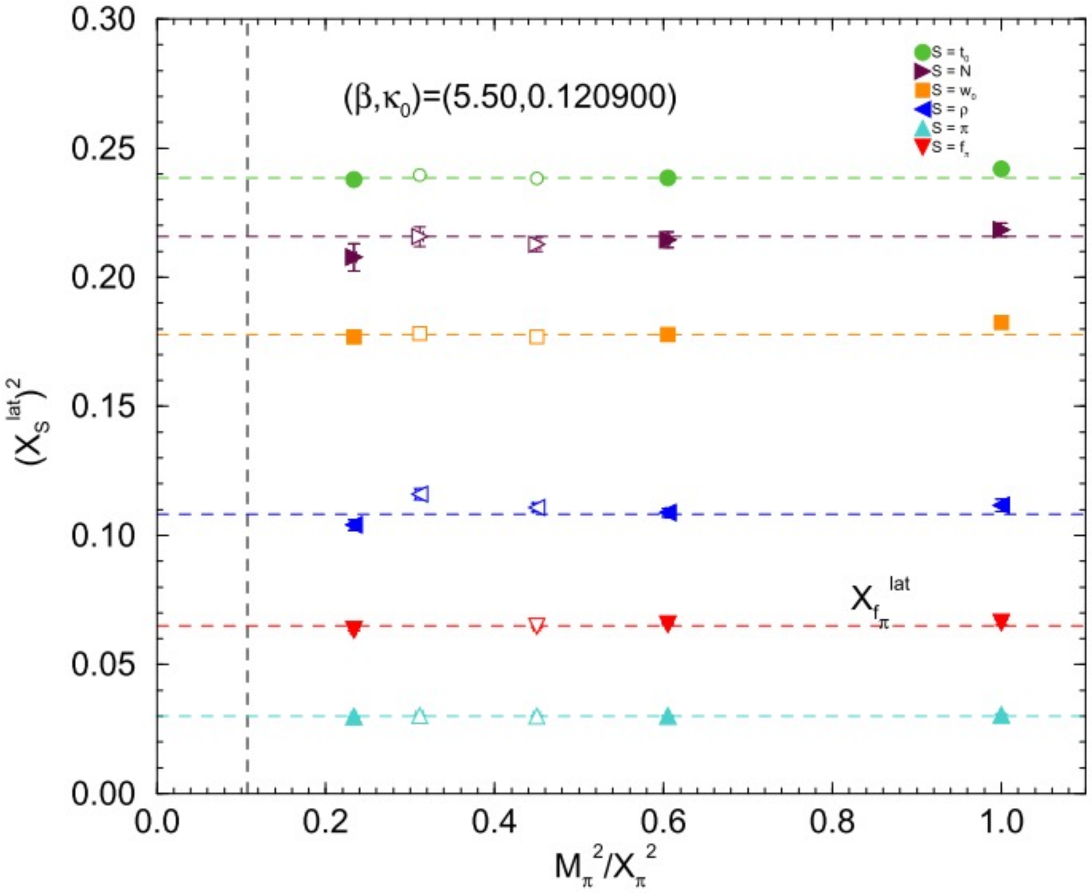
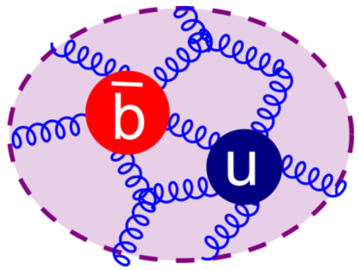


Figure seen in: https://arxiv.org/pdf/1711.02485.pdf

Expanding for f_B and fit functions

$$\frac{f_B(q\bar{b})}{f_{X_B}} = 1 + G(\delta\mu_q) + (H_1 + H_2)\delta\mu_q^2$$
$$- (\tfrac{2}{3}H_1 + H_2)(\delta m_u^2 + \delta m_d^2 + \delta m_s^2)$$
$$+ \ldots$$

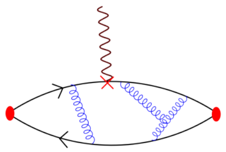


Type of fit	Functional form
Linear	$G_0\left(M_\pi^2/X_\pi^2 - 1\right) + 1$
Quadratic	$H\left(M_\pi^2/X_\pi^2 - 1\right)^2 + G_0\left(M_\pi^2/X_\pi^2 - 1\right) + 1$
Quadratic with a^2	$H\left(M_\pi^2/X_\pi^2 - 1\right)^2 + (G_0 + G_1 a^2)\left(M_\pi^2/X_\pi^2 - 1\right) + 1$

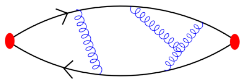
Calculating the decay constant f_{Bq}

$$f_B = \frac{\hbar c}{a} Z_\Phi \left[\Phi_B^0 + c_A \Phi_B^1 \right]$$

Renormalisation factor:
Ratio of 2 point and 3 point functions with constant coefficient $\rho=1$



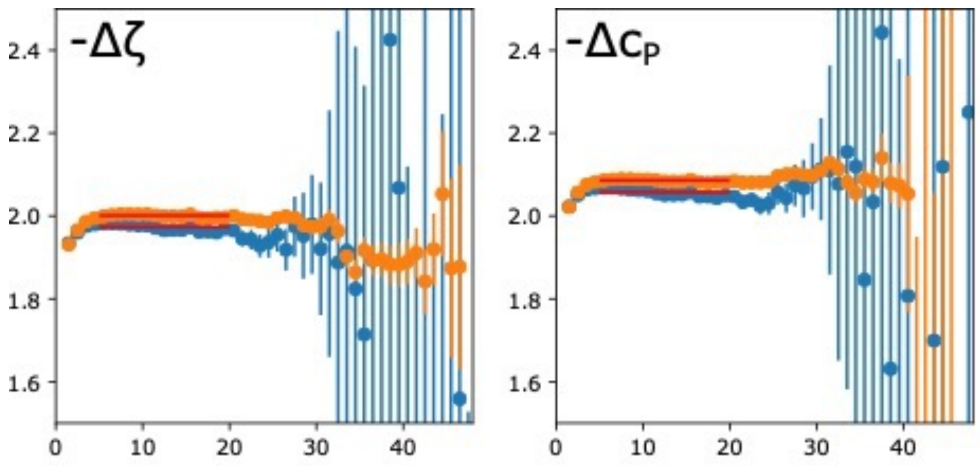
Lattice decay constant:
2 point functions with different operators in the quark propagators, and mass of B



Improvement term:
2 point correlators & coefficient c_A

Currently take $c_A=0$, Exact value can be calculated using perturbative QCD

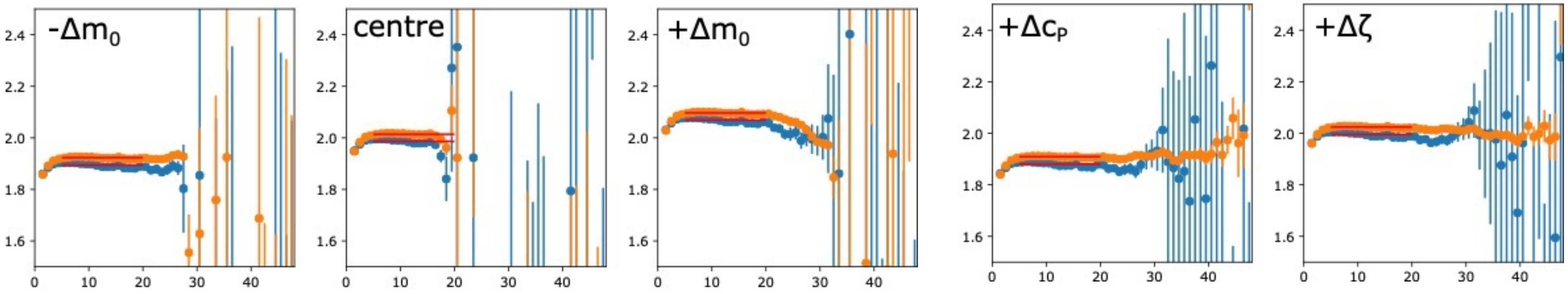
A closer look at the decay constant fits [nominal]



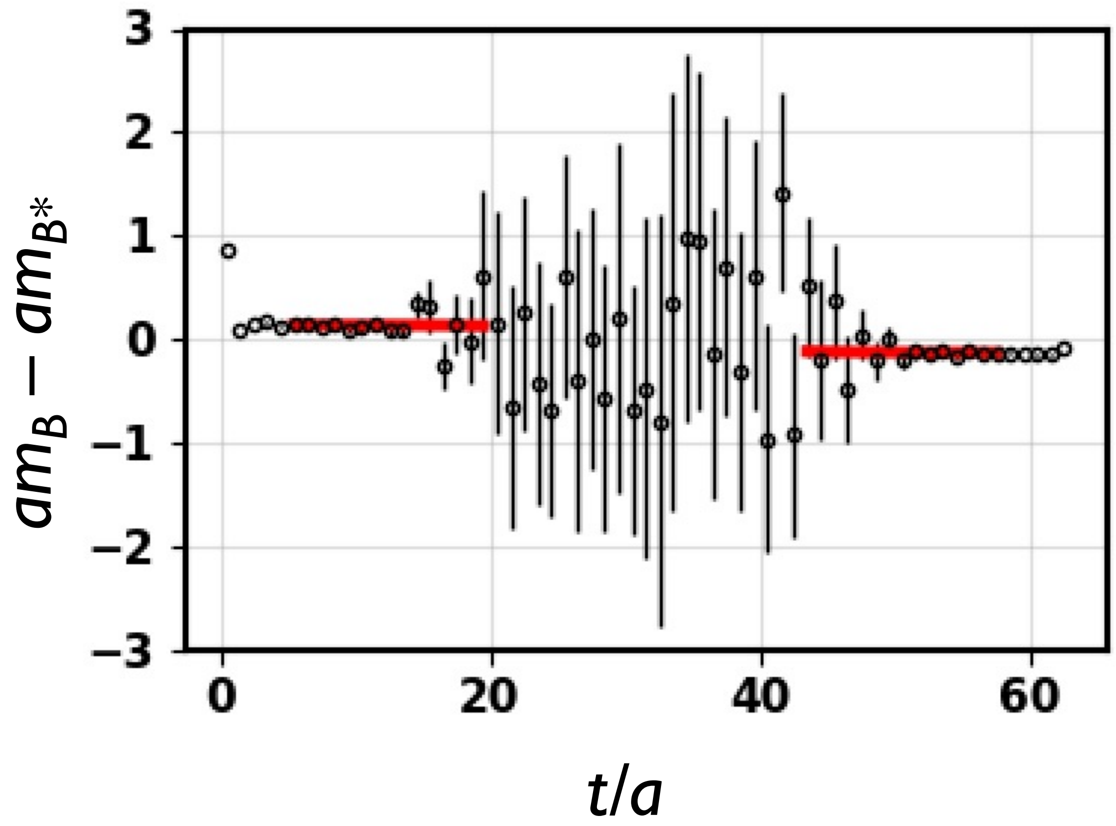
- Fits to $$\mathcal{C}_{AP} = \frac{\langle \Omega | [\chi_A]_0 | B_q \rangle \, \langle B_q | \chi_P | \Omega \rangle}{2M_B},$$
- Observe: plateau on central *b* is much shorter than for other *b*

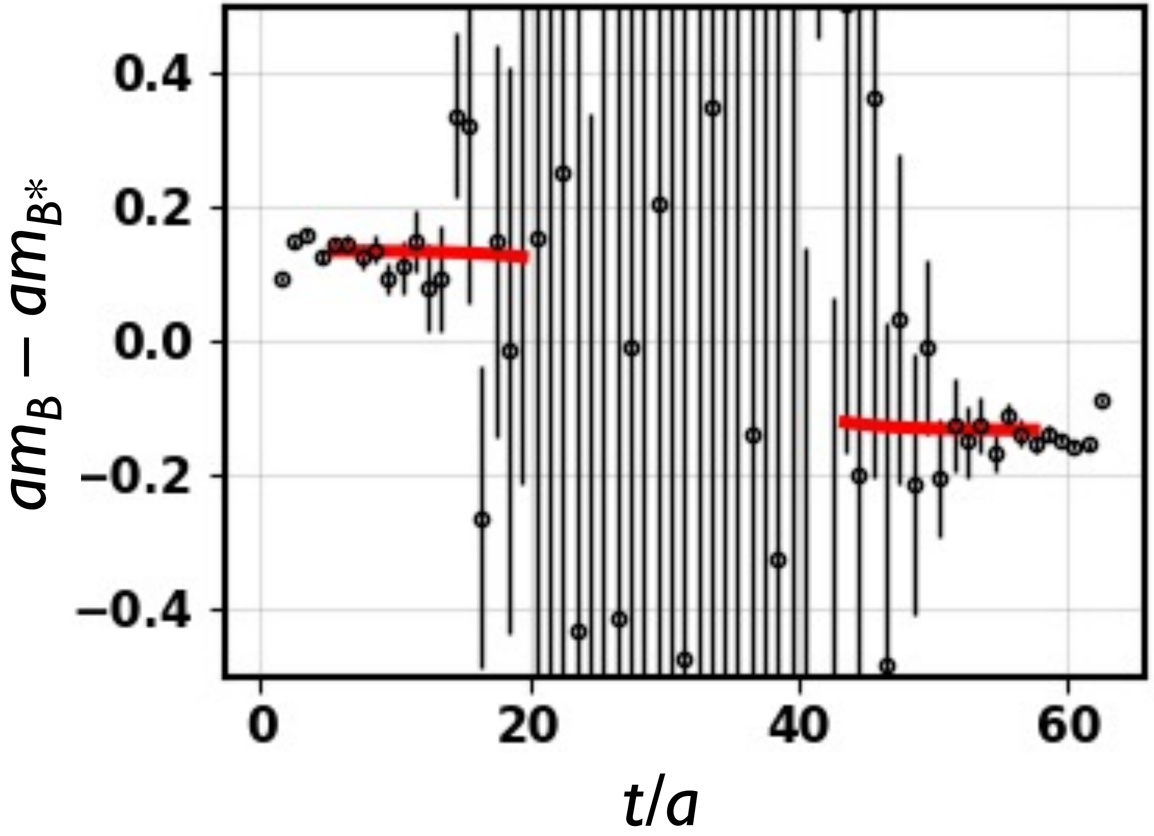
Correlator for f_B

Correlator for f_{Bs}



A closer look at the hyperfine splitting [weighted]





Function: $A\left(e^{-mt} + e^{-m(T-t)}\right)$
Window: 5-20 + 43-58
Bayesian weight: 6.0×10^{-6}