University of Colorado Boulder

NSF GRADUATE RESEARCH FELLOWSHIP PROGRAM

Determination of the continuous beta function of SU(3) Yang-Mills theory

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In collaboration with Anna Hasenfratz, Jake van Sickle and Oliver Witzel

Renormalization group (RG) beta function

- RG beta functions describe the dependence of renormalized couplings $g(\mu)$ on the energy scale μ of some physical process

$$\beta(g^2) \equiv \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2} g^2(\mu)$$

- Gradient flow (GF) defines a renormalized coupling that runs with $\mu^2 \propto (8t)^{-1}$

$$g^2_{\mathrm{GF}}(t; L, g_0^2) = \frac{128\pi^2}{3(N^2-1)} \frac{1}{1+\delta(t/L^2)} \langle t^2 E(t) \rangle$$

- By setting $\mu^2 \propto (cL)^{-1}$, one can define the discrete step-scaling beta function
  - Permits only a single scale (set by L)
    - Consequently, step-scaling works well in the deconfined phase
- In the infinite-volume limit one can define a beta function that runs continuously with the GF flow time
  - Explored by a number of groups throughout the years
    - Holland, K. et al.
    - Hasenfratz, A. and Witzel, O.
    - A number of talks at this conference

[Fodor, Z., Holland, K., Kuti, J. et al. *JHEP* 11 (2012) 007]

[Fodor, Z., Holland, K., Kuti, J., Nogradi, C., Wong, K. H. *EPJ WoC* **175**, 08027 (2018)]

[Hasenfratz, A., Witzel, O. *PoS*, LATTICE2019 (**2019**) 094]

[Hasenfratz, A., Witzel, O. *PRD* **101**, 034514 (2019)]

[Monahan, C., Wed. at 9:15 PM]

[Holland, K., after this talk]

[Kuti, J., Thurs. at 9:00 PM]

[Hasenfratz, A., Thurs. at 9:15 PM]

The continuous beta function method

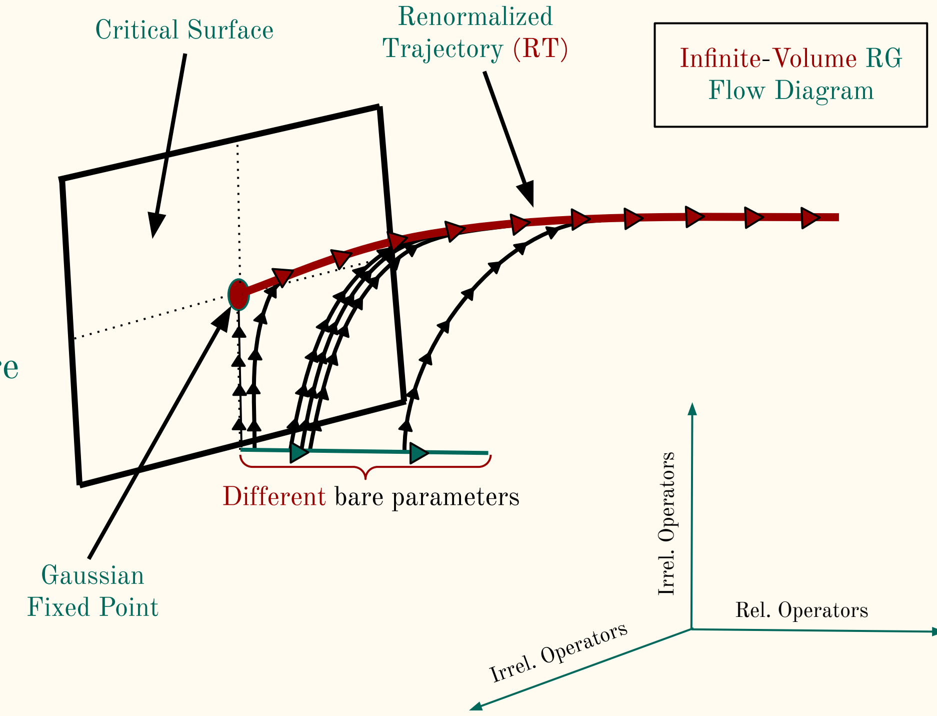
[Carosso, A., Hasenfratz, A., Neil, E. *PRL* **121**, 201601 (2018)]

- Gradient flow describes an RG transformation when combined with a rescaling step in the calculation of expectation values

$$\beta_{\mathrm{GF}}\left(g^2_{\mathrm{GF}}\right) \equiv \mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\, g^2_{\mathrm{GF}}(\mu) \overset{\mu^2 \propto (8t)^{-1}}{\overbrace{=}} -t\frac{\mathrm{d}}{\mathrm{d}t}\, g^2_{\mathrm{GF}}(t)$$

$$\frac{g^2_{\mathrm{GF}}(t)}{4\pi} \equiv \alpha_{\mathrm{GF}}(t)$$

- First step in calculation is an extrapolation to the infinite-volume limit
  - ➢ Allows for the presence of multiple scales
  - ➢ The RG trajectories of actions with different bare parameters overlap sufficiently close to the RT
- Continuum limit is an extrapolation of $\beta_{\mathrm{GF}}\left(t; g_0^2\right)$ to $a^2/t \to 0$ at fixed α_{GF}
  - ➢ Equivalent to tuning $g_0^2(a)$ to zero at fixed dimensionful flow time t



Gradient flow beta function for SU(3) pure gauge Yang-Mills theory

[Dalla Brida, M., Ramos, A. *EPJC* **79**, 720 (2019)]

The gradient flow coupling at high-energy and the scale of SU(3) Yang–Mills theory

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[2] School of Mathematics and Hamilton Mathematics Institute, Trinity College Dublin, Dublin 2, Ireland

- We aim to demonstrate that the continuous beta function method can be applied in the confined phase

- We focus on the SU(3) pure gauge Yang-Mills system
  - No complications arise from introducing fermions

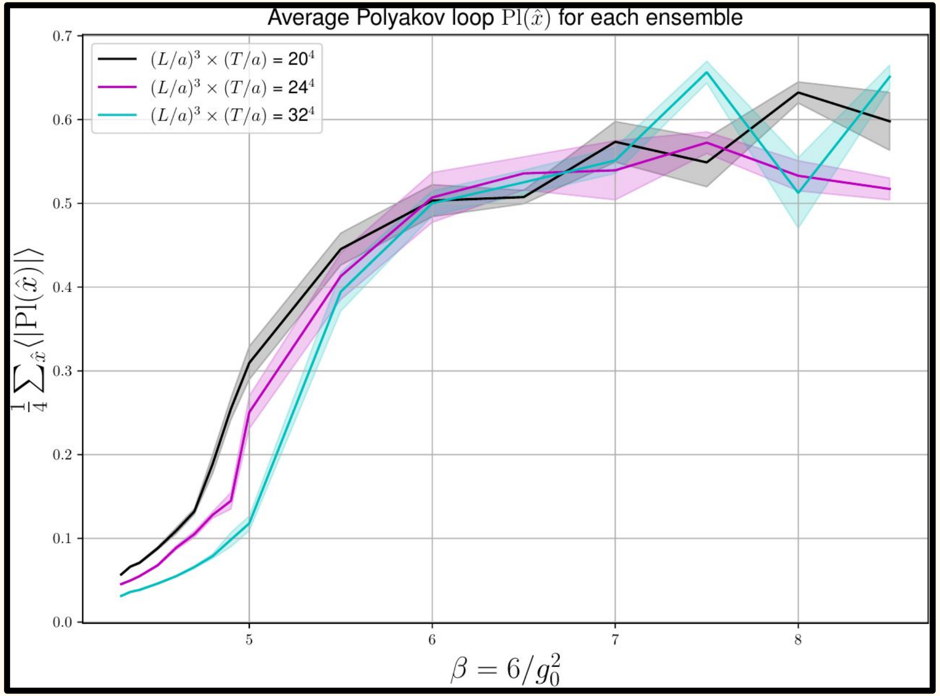
- Most recent study of the RG beta function from GF for SU(3) Yang-Mills uses step-scaling
  - Goes out to $\alpha_{\mathrm{GF}} \sim 1$

Simulation and Analysis

[Ramos, A., Sint, S. *EPJC* **76**, 720 (2019)]

- Simulations are performed with a pure gauge Symanzik action using GRID
  - Bare gauge couplings in the range $4.3 \leq \beta \leq 8.5$
  - Volumes used $(L/a)^3 \times (T/a) = 20^4$, 24^4 and 32^4
  - Ensembles generated using hybrid Monte Carlo

- Gradient flow performed with Wilson flow and Zeuthen flow using QLUA
  - Flow (F) and operator (O) combinations to be abbreviated
  - ZS (Zeuthen flow + Symanzik operator) fully $\mathcal{O}(a^2)$ improved

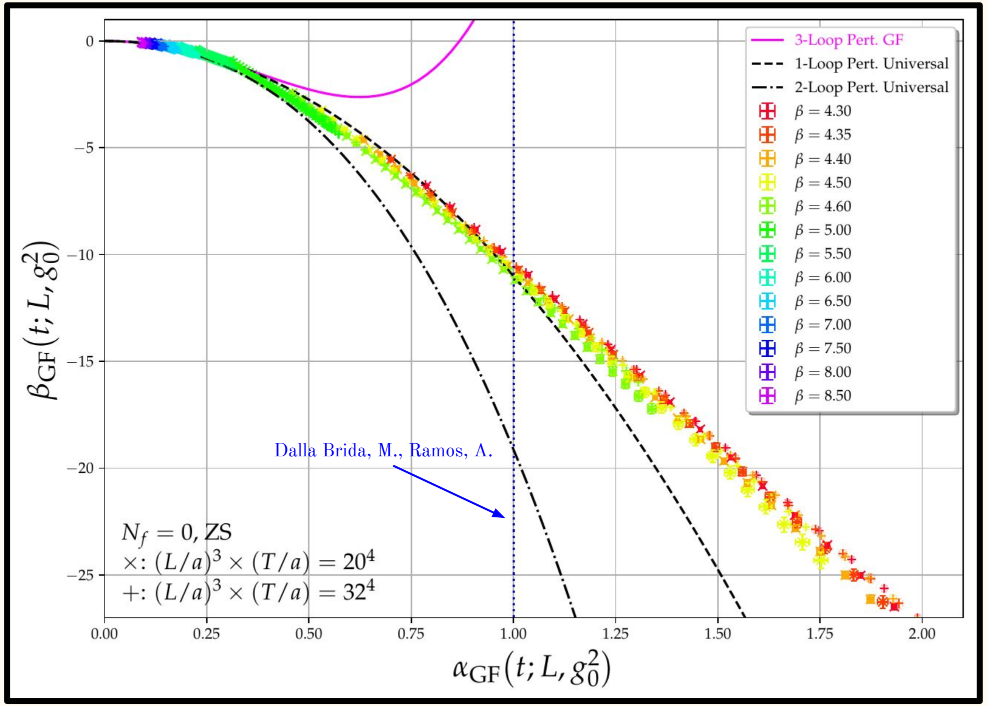


Step 0: Looking at the raw data

[Dalla Brida, M., Ramos, A. *EPJC* **79**, 720 (2019)]

[Harlander, R., Neumann, T. *JHEP* 06 (2016) 161]

- The raw data exhibits a number of attractive features
  - Considerable overlap between different bare gauge couplings
  - Small finite-volume effects
  - Overlap with perturbation theory at weak coupling



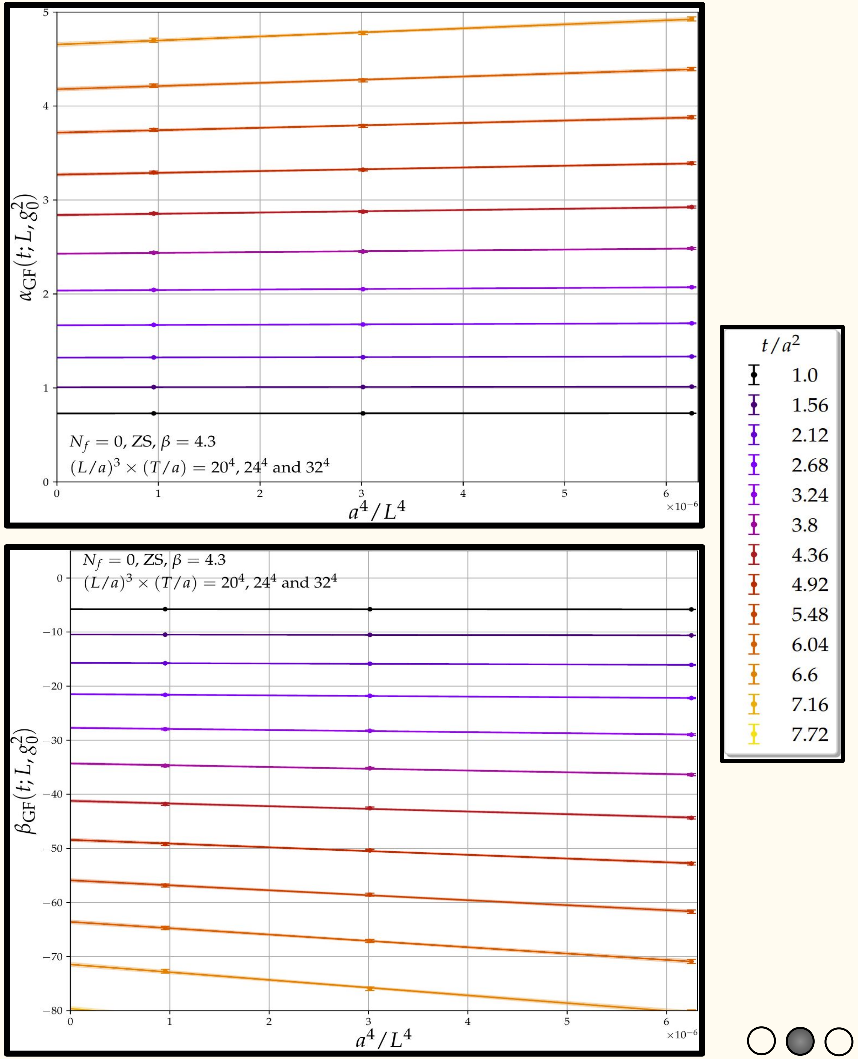
Step 1: Infinite-volume extrapolation

- Extrapolation done at fixed t/a^2 and bare gauge coupling for each operator

- Extrapolating function is linear in a^4/L^4
  - Motivated by scaling with L/a in deconfined phase
  - Investigations of other possible scalings with L/a are underway

- $\alpha_{\mathrm{GF}}(t; L, g_0)$ and $\beta_{\mathrm{GF}}(t; L, g_0)$ exhibit weak dependence on L/a

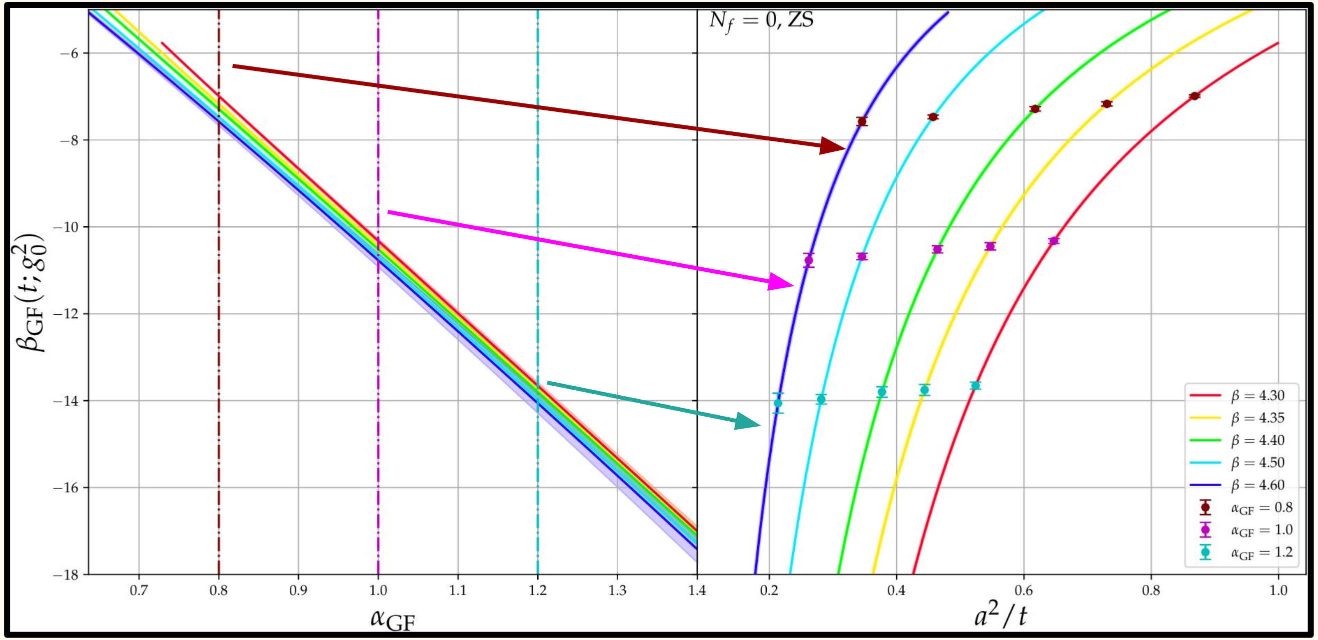
[Hasenfratz, A., Witzel, O., *PRD* **101**, 034514 (2019)]



Step 2: Continuum extrapolation of $\beta_{\mathrm{GF}}(t; g_0^2)$

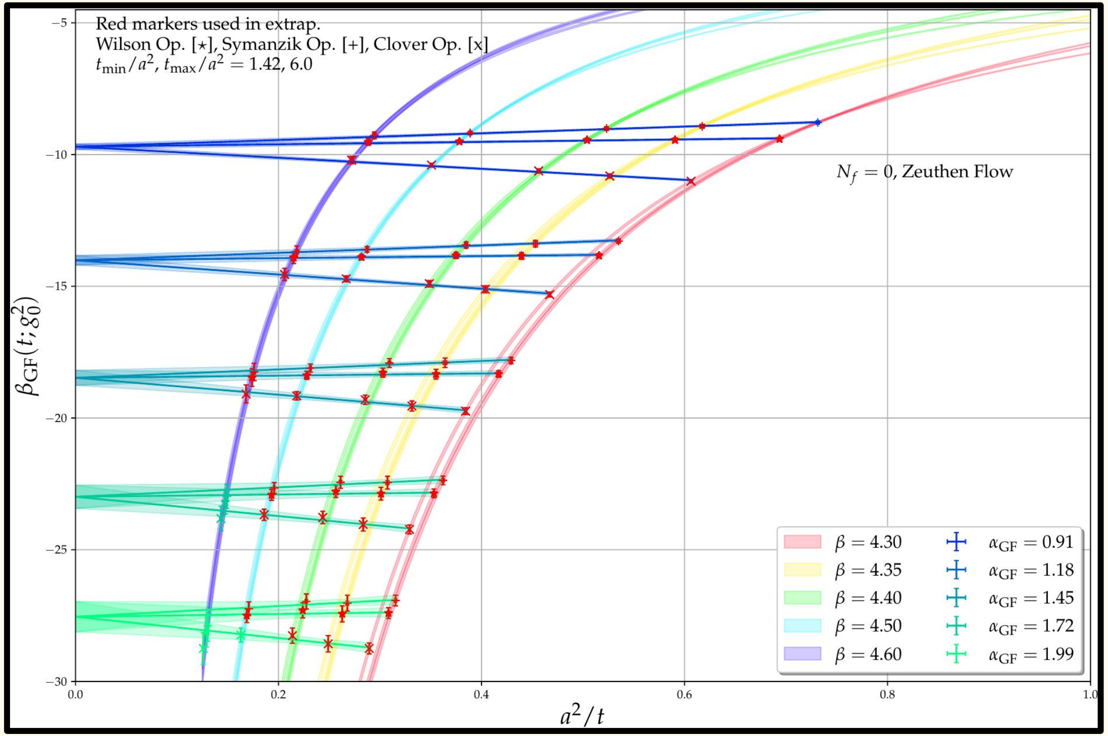
- Continuum limit taken by fixing α_{GF} and extrapolating $\beta_{\mathrm{GF}}(t; g_0^2)$ to $a^2/t \to 0$

- Interpolation of $\beta_{\mathrm{GF}}(t; g_0^2)$ and t/a^2 done in $\alpha_{\mathrm{GF}}(t; g_0^2)$



Step 2: Continuum extrapolation of $\beta_{\mathrm{GF}}(t; g_0^2)$

- We do a simultaneous fit using three operators
  - Extrapolating function is linear in a^2/t
  - Correlations between operators accounted for using SVD cuts
    - Investigations of better ways to deal with the correlations are ongoing

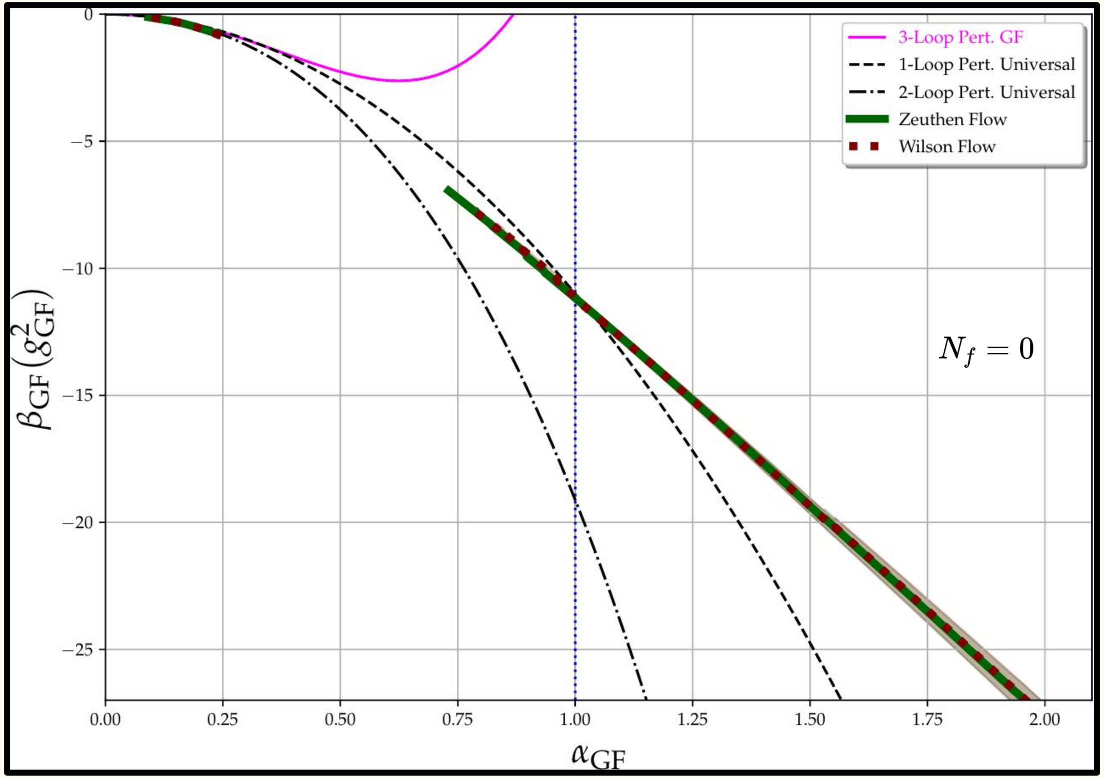


Final Results and Conclusions

[Dalla Brida, M., Ramos, A. *EPJC* **79**, 720 (2019)]

[Harlander, R., Neumann, T. *JHEP* 06 (2016) 161]

- Analysis in the confined phase
  - Agreement within error between Zeuthen flow and Wilson flow
- Analysis in the deconfined phase
  - Requires an extra interpolation between bare gauge couplings
  - Matches perturbation theory
  - Agreement within error between Zeuthen flow and Wilson flow
- Analysis in the deconfinement transition
  - Requires a more careful investigation of scaling with L/a
  - Missing from this iteration of the analysis



Final Results and Conclusions

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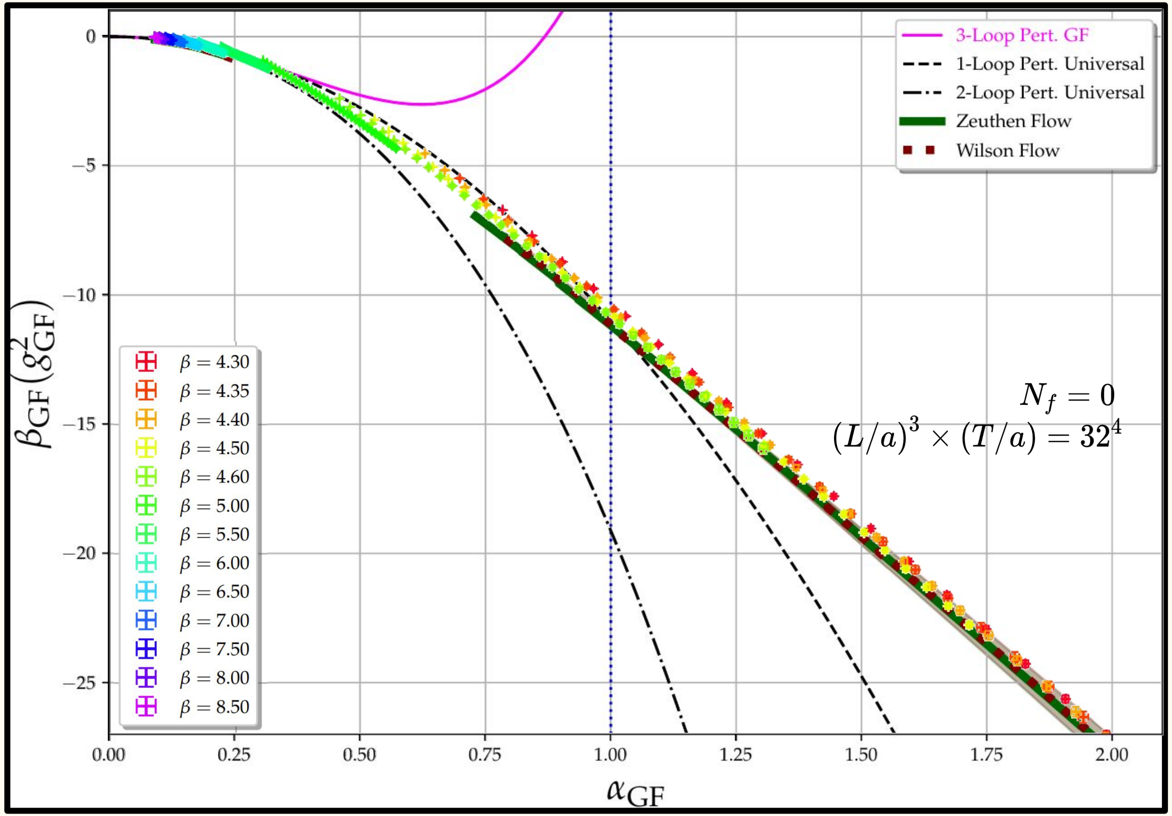


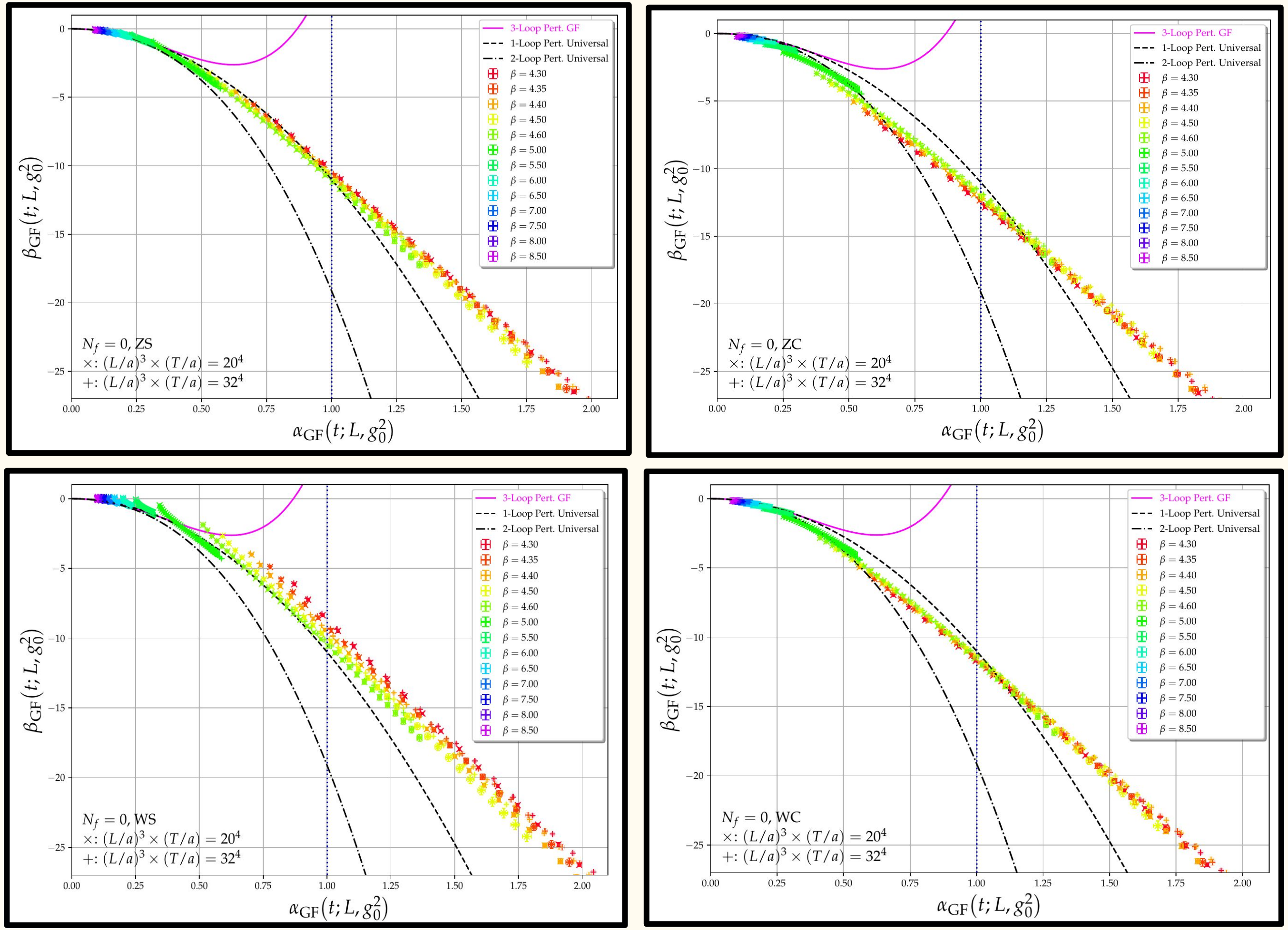
Acknowledgements

U. Colorado: RMACC Summit

USQCD: BNL SDCC

NSF GRFP

Supplemental Slides



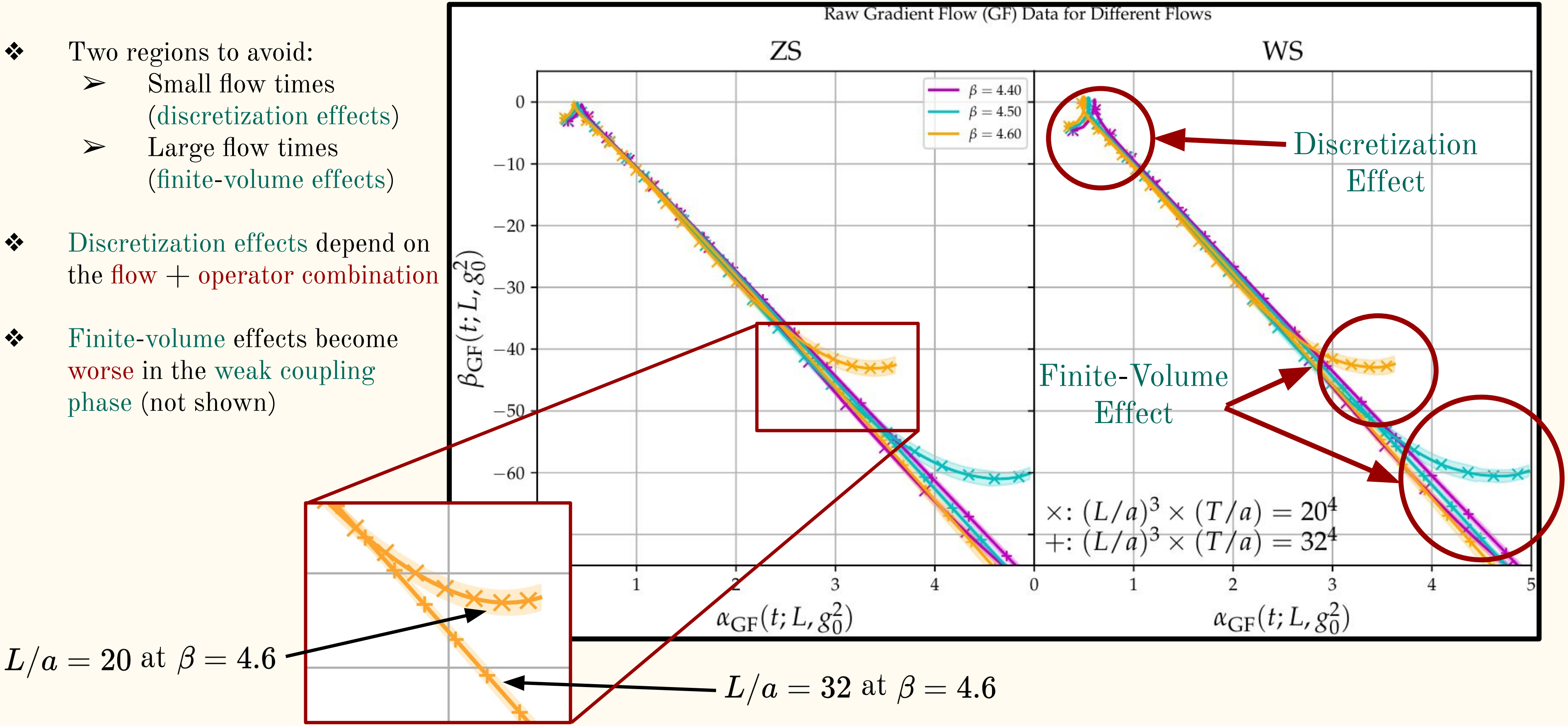
Raw data with different flow+operator combinations

[Dalla Brida, M., Ramos, A. *EPJC* **79**, 720 (2019)]

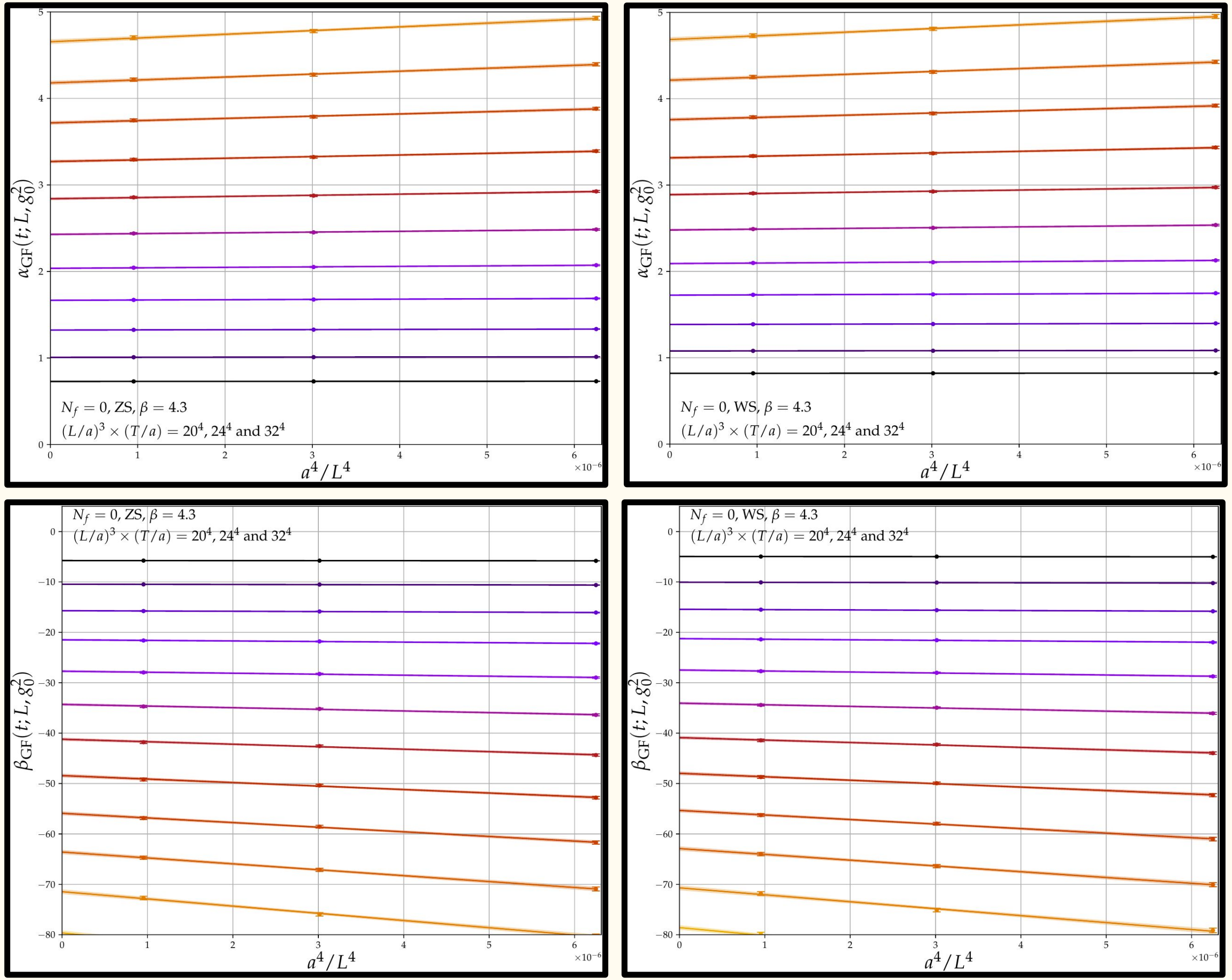
[Harlander, R., Neumann, T. *JHEP* 06 (2016) 161]

Various effects that appear in raw data

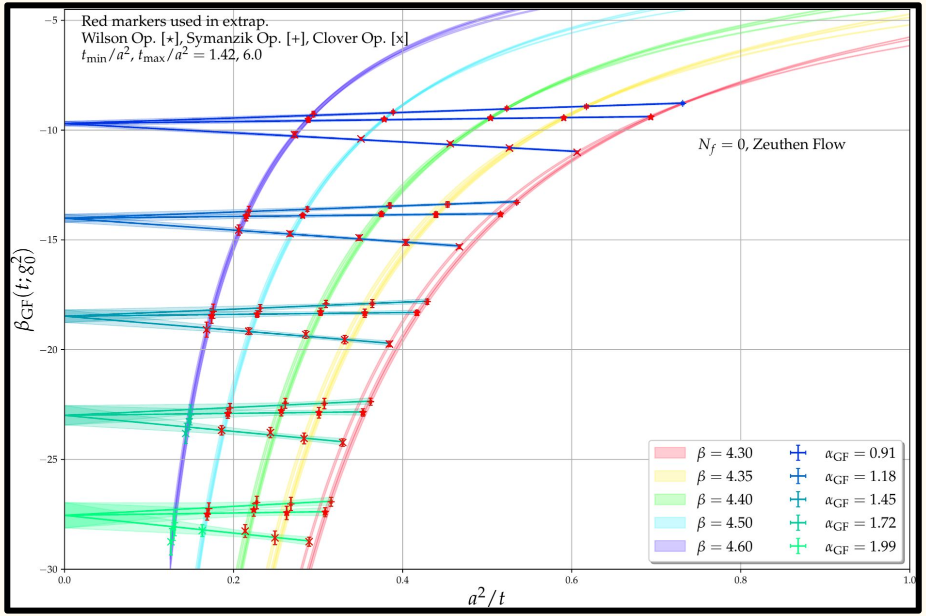
- Two regions to avoid:
  - Small flow times (discretization effects)
  - Large flow times (finite-volume effects)
- Discretization effects depend on the flow + operator combination
- Finite-volume effects become worse in the weak coupling phase (not shown)

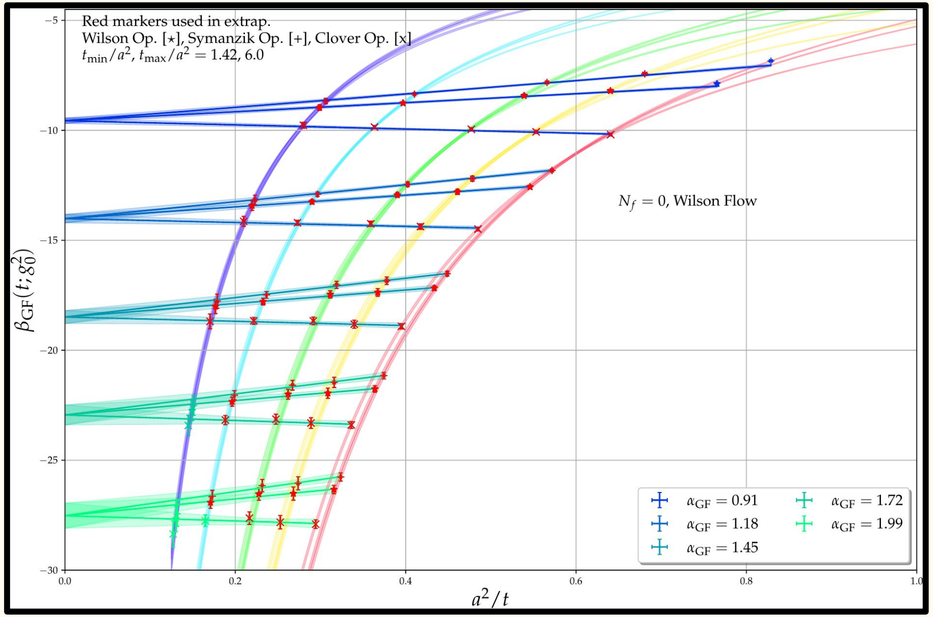


Infinite-volume extrapolation for Symanzik operator with different flows



Continuum extrapolation for different flows





Final results for different flows and different ranges of bare gauge couplings