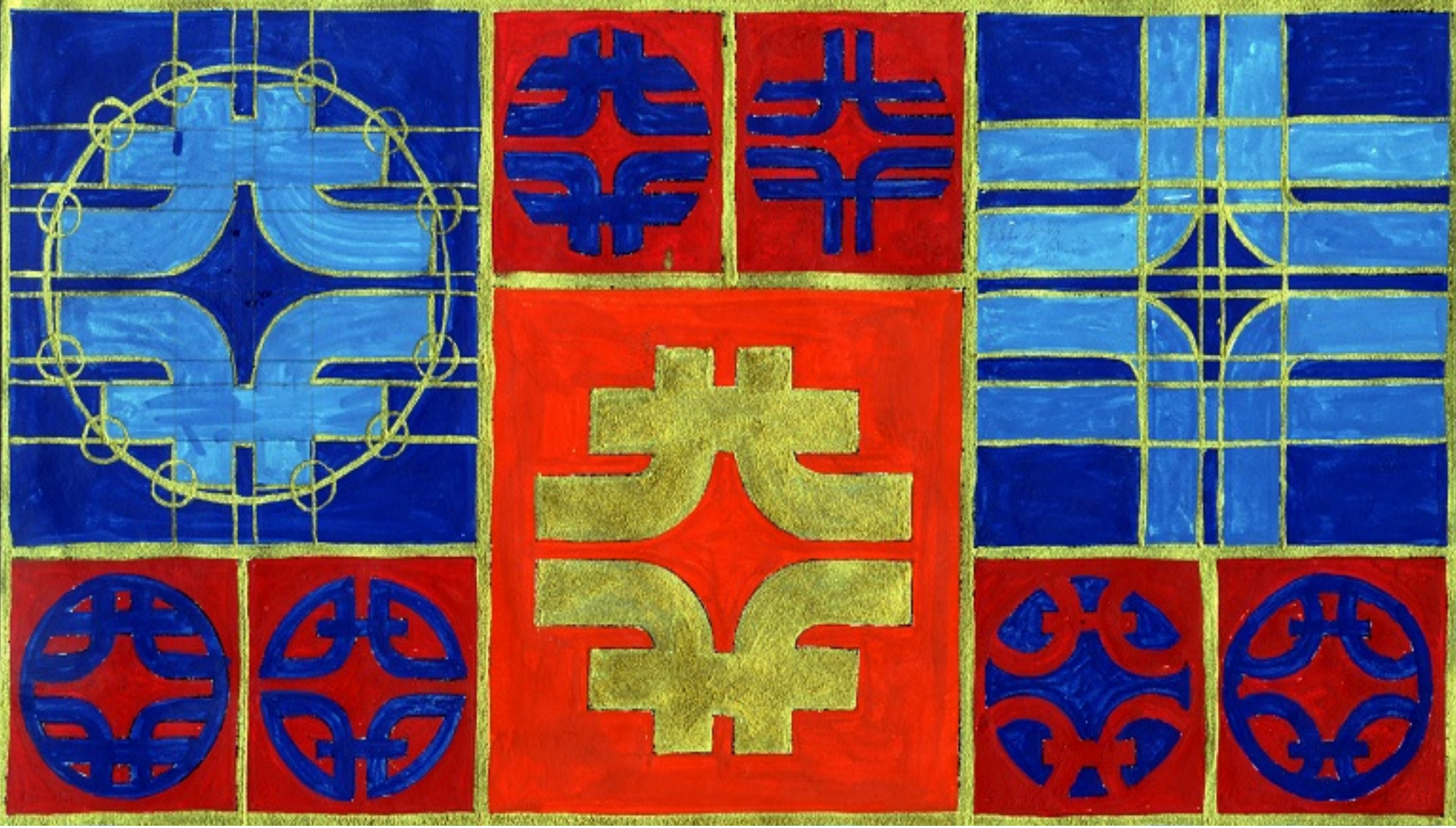


D→P Semileptonic Decays with Highly Improved Staggered Quarks

William I. Jay (Fermilab) for FNAL Lattice & MILC Collaborations

Lattice 2021 — 29 July 2021

Outline

- Scope of the talk
- The gauge ensembles
- Setup of calculation
- Preliminary results
- Summary and next steps

Special thanks to my colleagues in the all-HISQ subgroup:

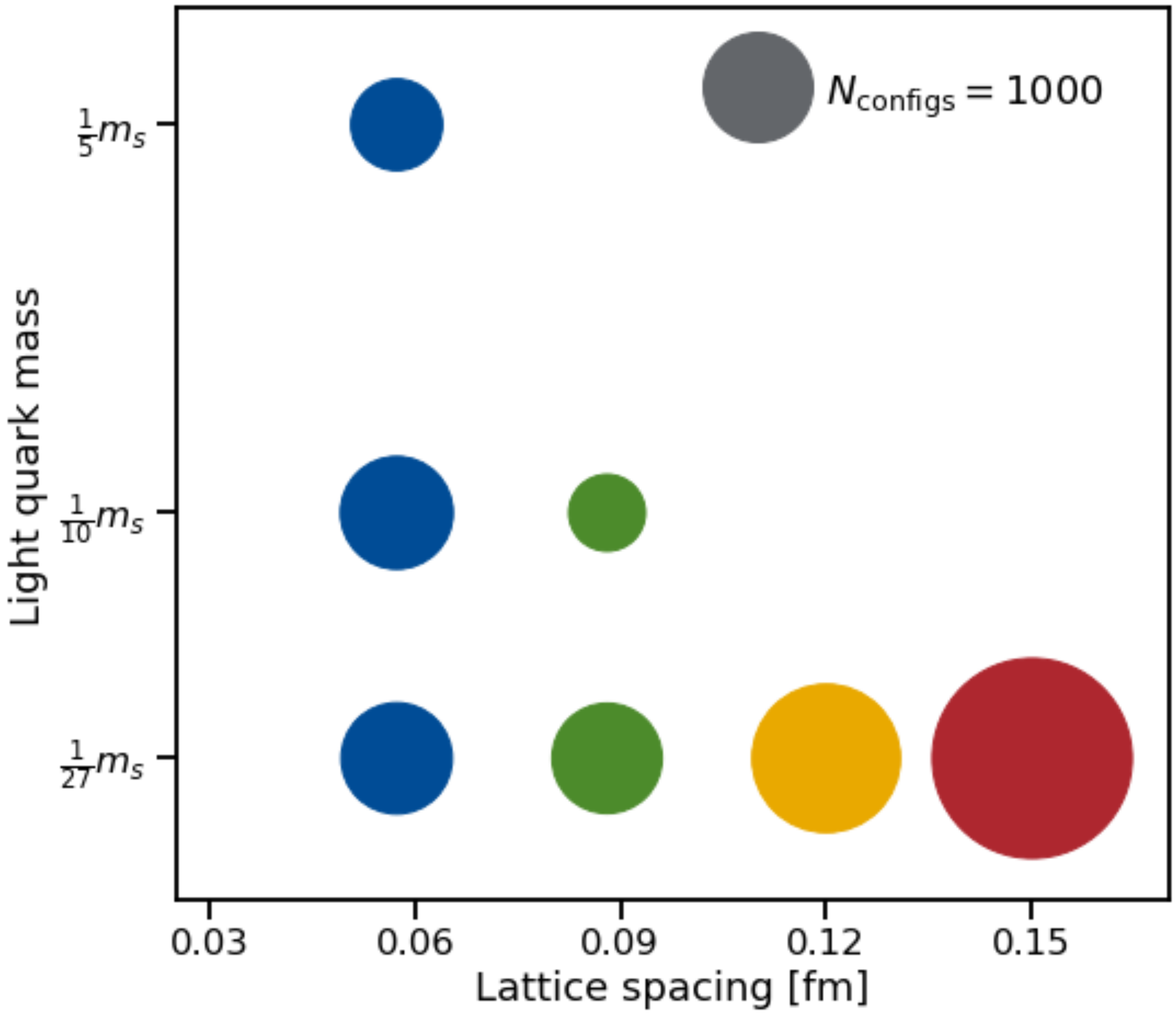
Carleton DeTar, Aida El-Khadra, Elvira Gámiz, Zech Gelzer,

Steve Gottlieb, Andreas Kronfeld, Andrew Lytle, Jim Simone

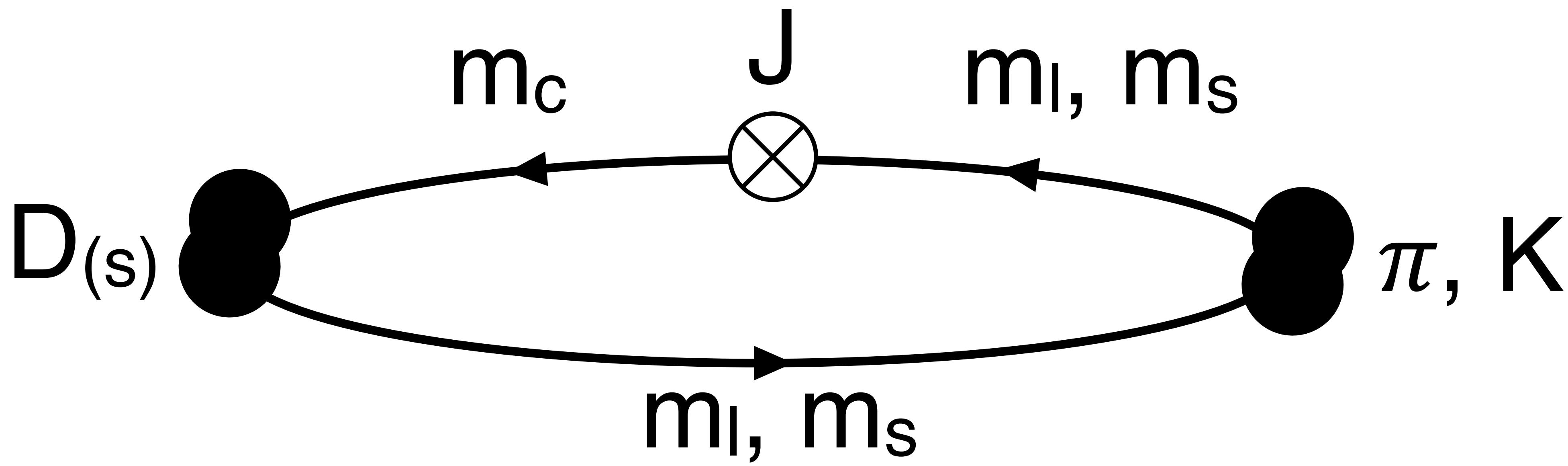
Scope: the all-HISQ campaign

- HISQ ensembles with N_f =(2+1+1) dynamical sea quarks generated by the MILC collaboration
- Valence quarks:
  - Light and strange quarks match the sea
  - Heavy quarks: range from 0.9 m_c up to cutoff (ma~1)
- **Campaign Goal**: form factors for decays of B, D mesons to pseudoscalars
  - D mesons: $D_{(s)} \rightarrow \pi$, K
  - B mesons: $B_{(s)} \rightarrow D_{(s)}$, π, K
- Eventual target: lattice spacings from 0.15 fm—0.03 fm
- **This talk**: preliminary results for
  - 4 lattice spacings: 0.15, 0.12, 0.09, 0.06 fm
  - Decays of D mesons only
- For B decays: see Andrew Lytle's talk in this session
- All 3pt functions are fully blinded

The gauge ensembles



3pt correlators



- We compute scalar, vector, and tensor currents
- Kinematic setup:
  - Work in rest frame of mother hadron $D_{(s)}$
  - Vary the energy of the recoiling daughter hadron π, K

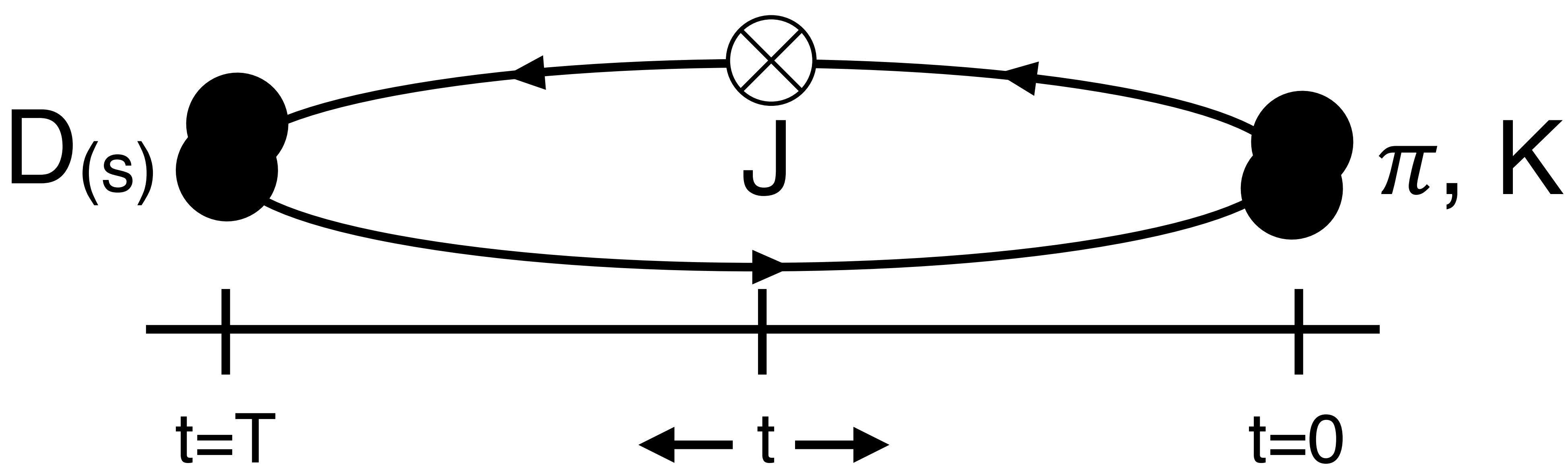
3pt correlators

- As usual, the spectral decomposition reads:

$$C_3(t;T) = \langle \mathcal{O}_D(T) J(t) \mathcal{O}_\pi(0) \rangle$$

$$\sim \langle 0 | \mathcal{O}_D | D \rangle \underbrace{\langle D | J | \pi \rangle} \langle \pi | \mathcal{O}_\pi | 0 \rangle \, e^{-m_D(T-t)} e^{-m_\pi t}$$

(bare) transition matrix element ~ desired form factor



Form factors from 2pt + 3pt

- Recall the ratio R(T, t)

$$R(T,t) = (\text{factors}) \times \frac{C_3(T,t)}{\sqrt{C_2^L(t) C_2^H(T-t)}} e^{+E_L t/2} e^{+E_H (T-t)/2}$$

$$\xrightarrow{1 \ll t \ll T} f_J$$

- The ratio asymptotically approaches the form factors
- Our analysis of 2pt and 3pt functions fits the full spectral decomposition
- The ratio gives a valuable visual check on fit results

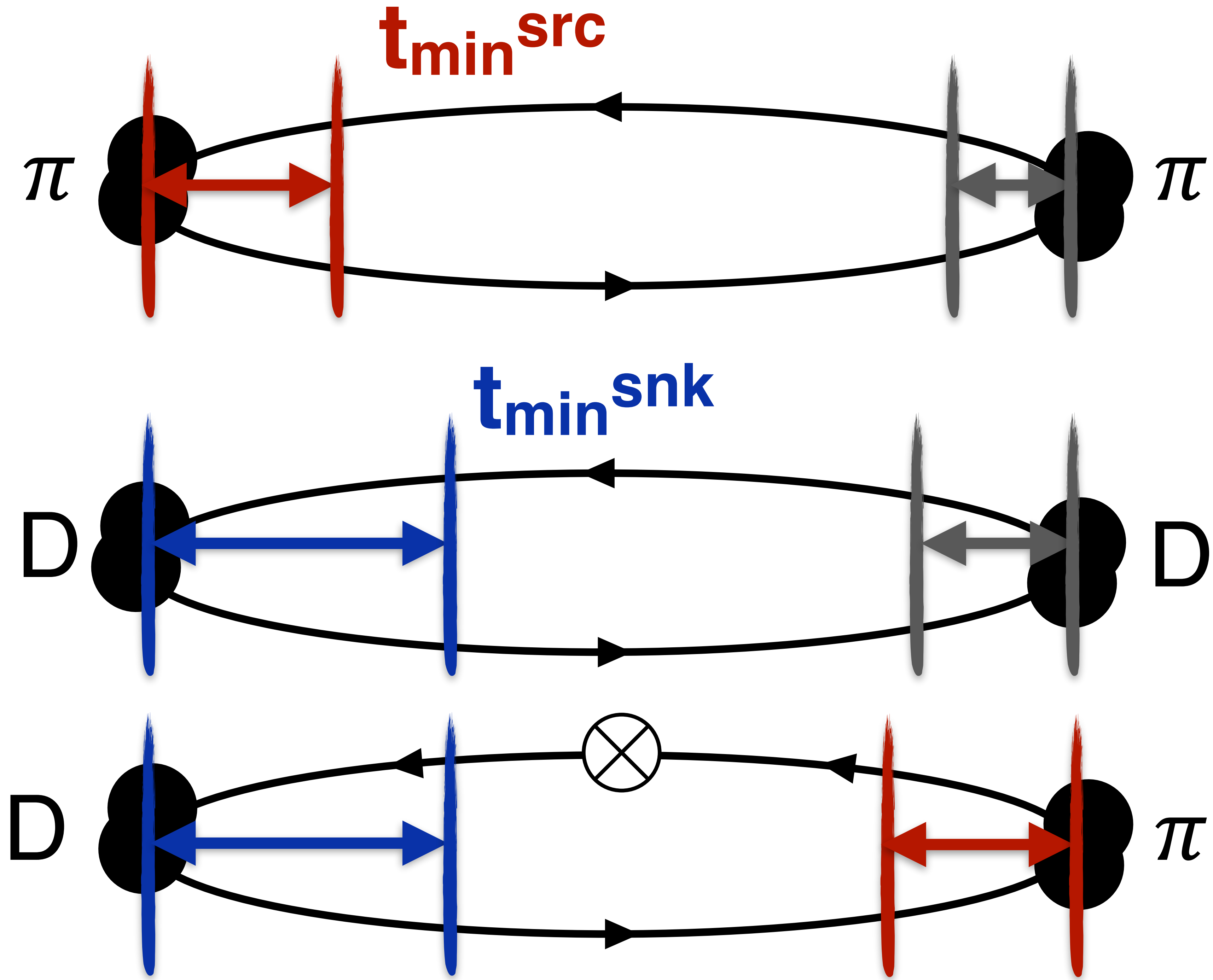
2pt + 3pt

- π: (n+n$_o$) states
- D: (m+m$_o$) states
- Fix distances

  t_{min}^{src} and t_{min}^{snk}

  for all correlators

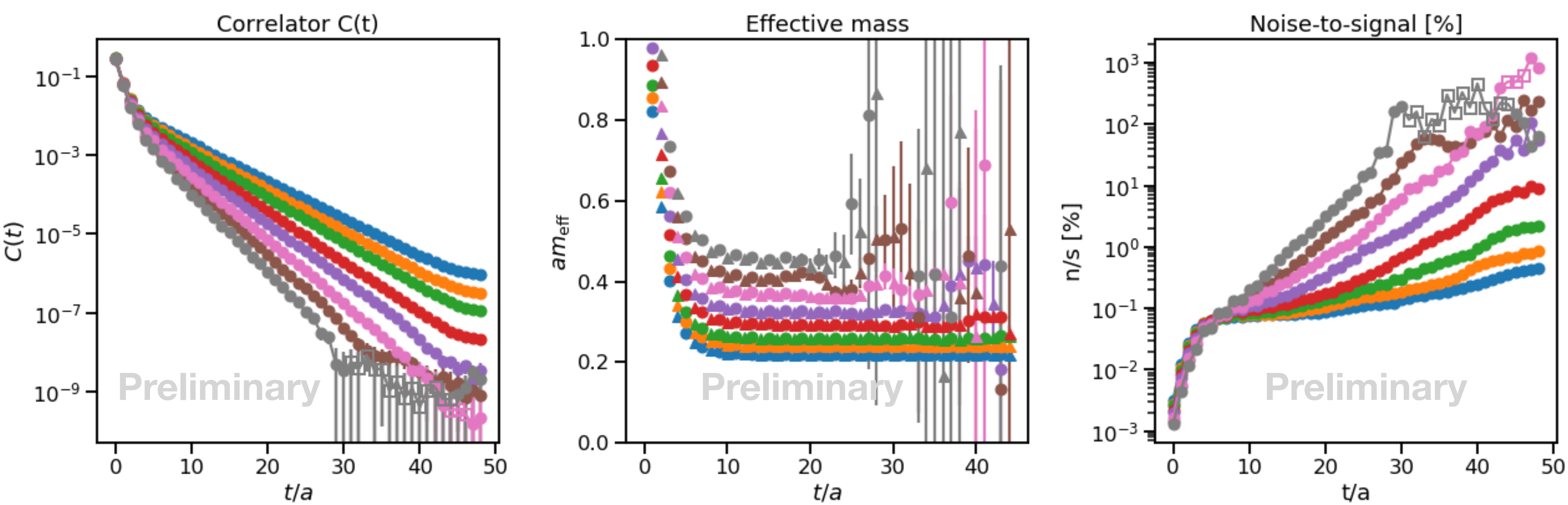
  in physical units



Preliminary results

2pt functions

- Consider kaon 2pt functions at a=0.09 fm
- Fit each 2pt function
- Find appropriate values for t_{min} and number of states



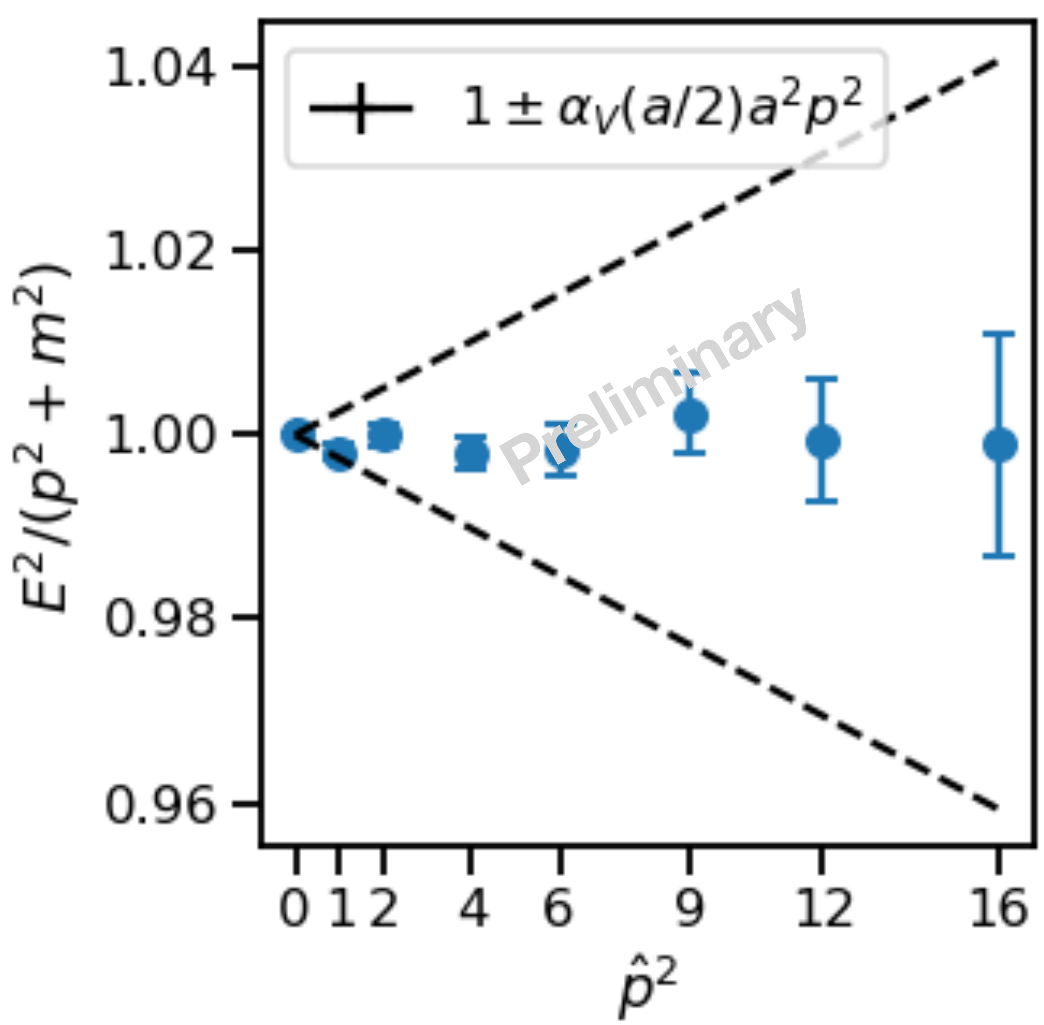
2pt functions

- Consider kaon 2pt functions at a=0.09 fm
- Fit each 2pt function
- Find appropriate values for t_{min} and number of states
- Check relativistic dispersion relation

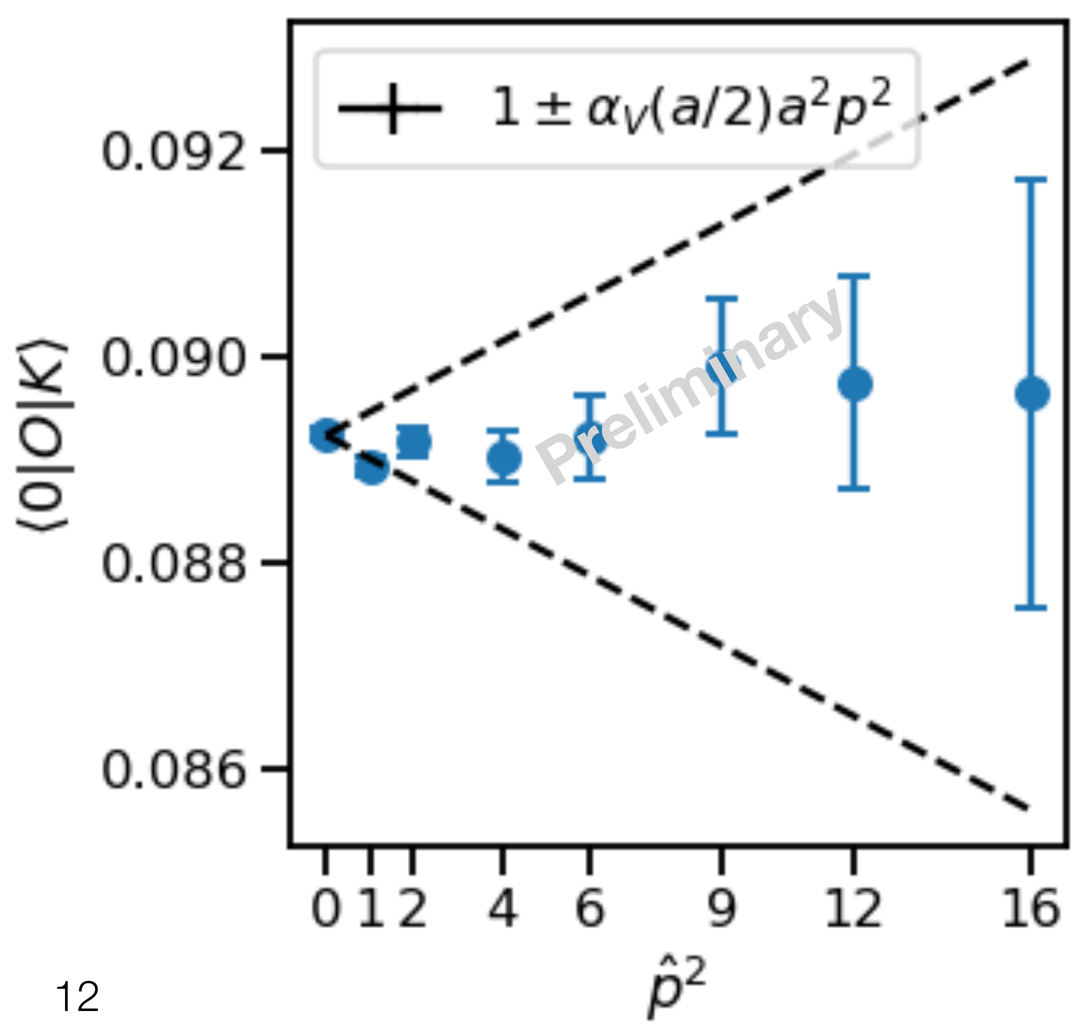
2pt functions

- Consider kaon 2pt functions at a=0.09 fm
- Check relativistic dispersion relation

✔ $E^2 = p^2 + m^2$



✔ ⟨0|O|K⟩=const

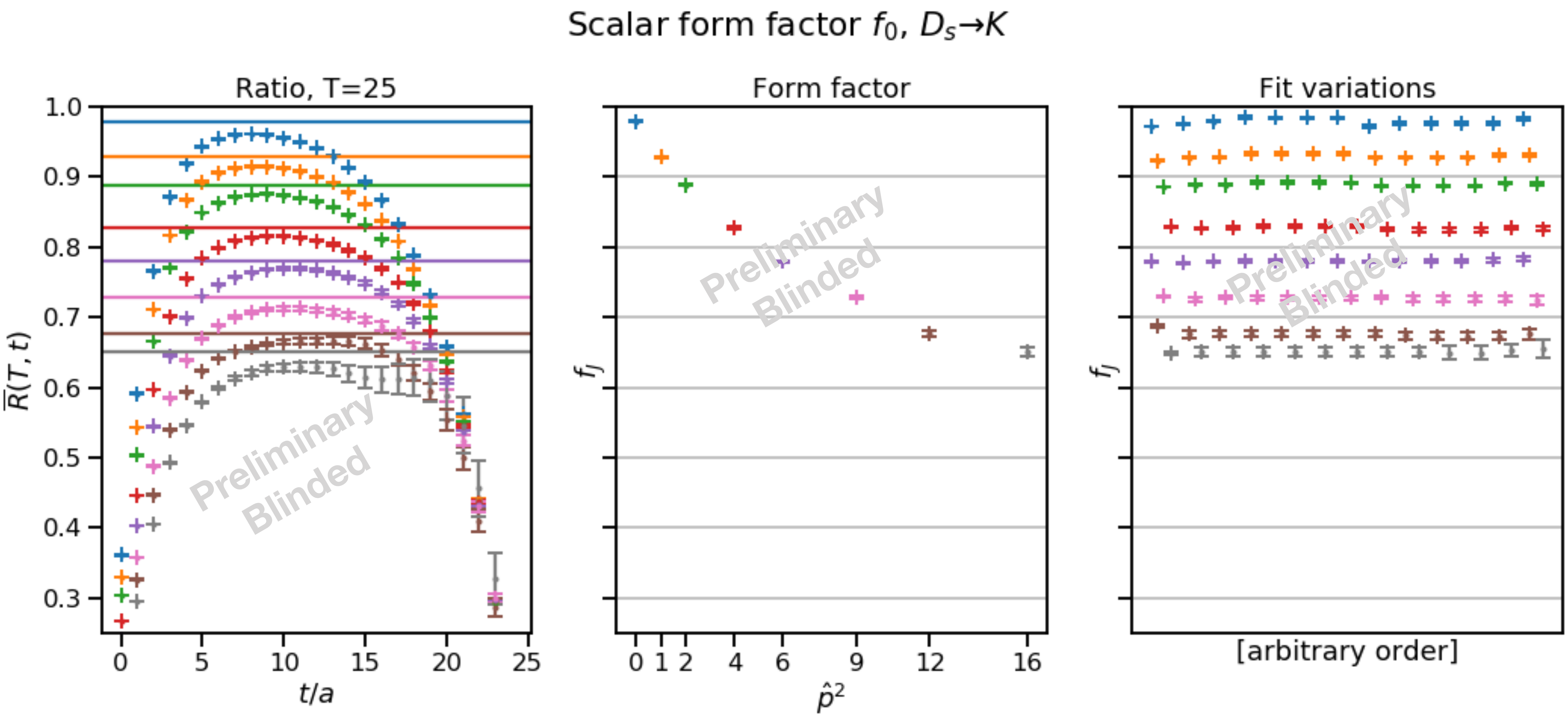


2pt functions

- Consider kaon 2pt functions at a=0.09 fm
- Check relativistic dispersion relation
- Use knowledge from 2pt functions as inputs for 3pt fits
  - Fix t_{min} in physical units
  - Fix number of states $(n+n_o)$ and $(m+m_o)$
  - Impose knowledge $E^2/(\mathbf{p}^2+m^2) = 1 + \mathcal{O}(\alpha_s(ap)^2)$
  - Verify results visually for sanity

Form factor fits

- $D_s \to K$, physical masses, a=0.09 fm
- Joint fit with two 2pt functions and five 3pt functions



Form factor fits

- $D_s \rightarrow K$, physical masses, a=0.09 fm
- Joint fit with two 2pt functions and five 3pt functions
- Results are qualitatively very similar for
  - Different currents (V^i, V^4, and T^{ij})
  - Different masses (i.e., $D \rightarrow K$ and $D \rightarrow \pi$)
  - Different lattice spacings
- Note: 3pt results are fully blinded

Renormalization

- PCVC: $\nabla_\mu V^\mu = (m_1 - m_2) S$
- This Ward Identity guarantees absolute normalization for the scalar form factor: $Z_S \equiv 1$.
- Z-factors are still needed for the local, one-link vector currents.
- PCVC defines the Z-factors non-perturbatively

$$(M_D - M_\pi)\langle D|V^4_{\text{lat}}|\pi\rangle Z_{V^4} + \mathbf{q}\cdot\langle D|\mathbf{V}_{\text{lat}}|\pi\rangle Z_{V^i} = (m_1 - m_2)\langle D|S_{\text{lat}}|\pi\rangle$$

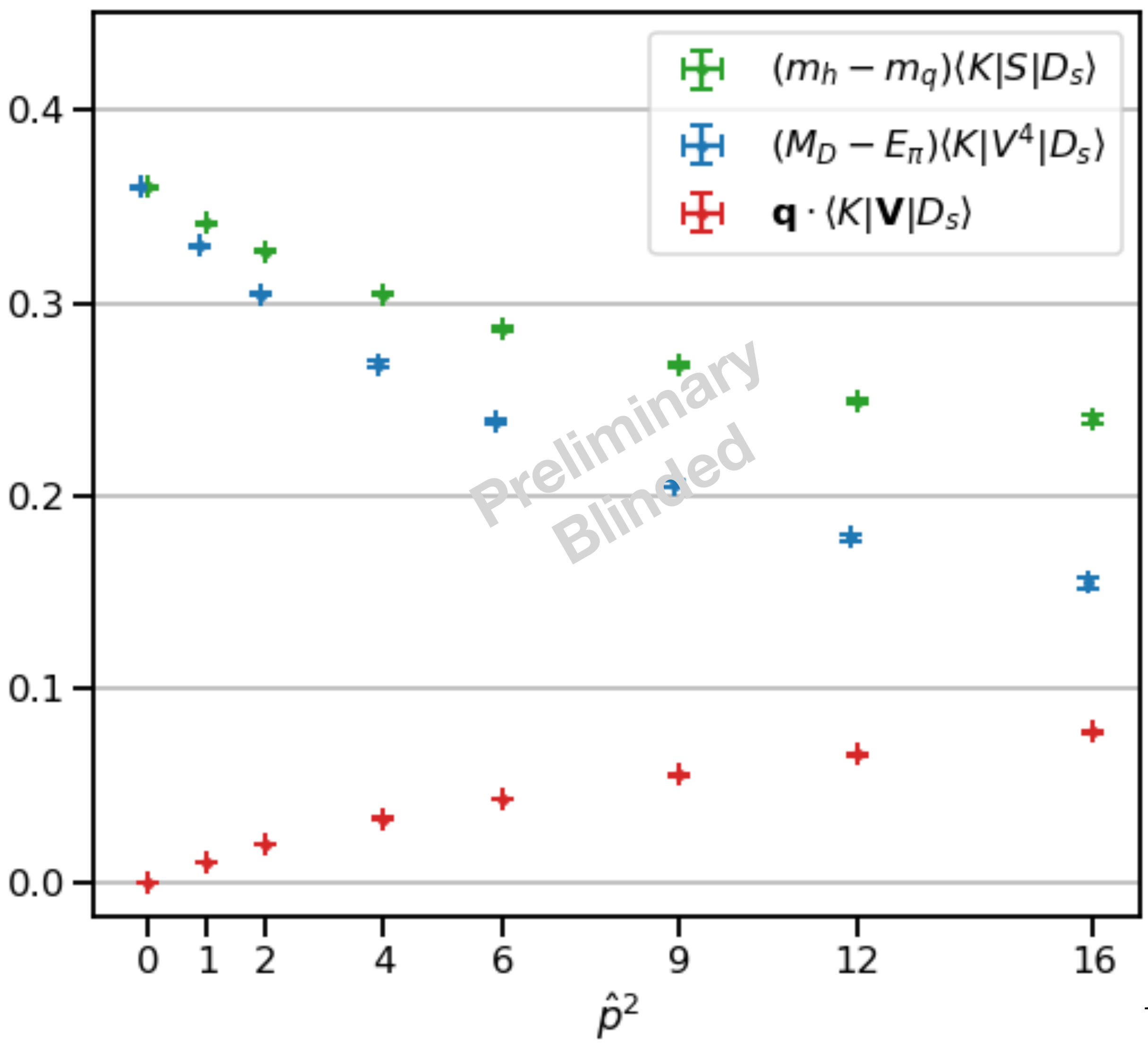
Renormalization

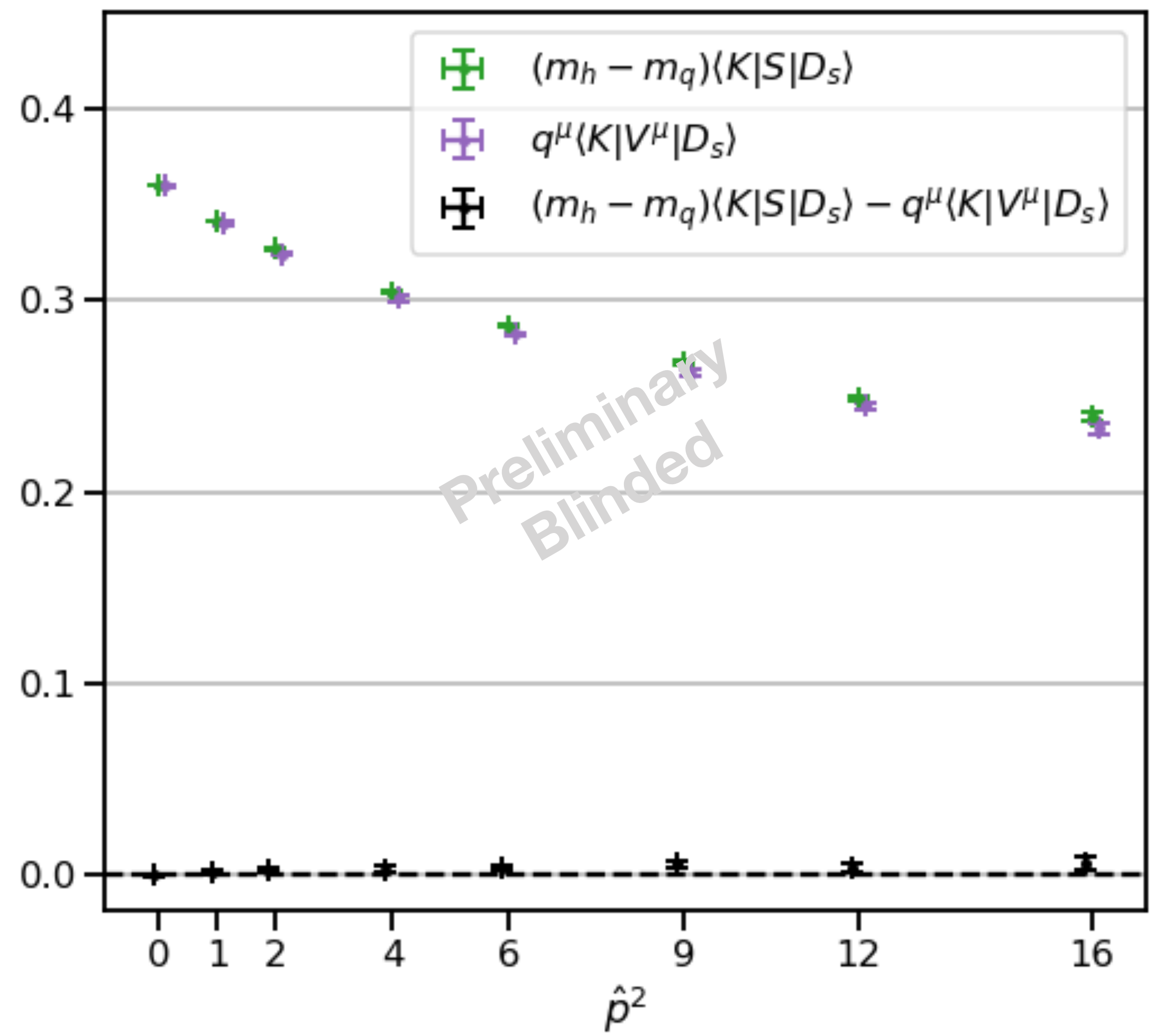
- PCVC: $\nabla_\mu V^\mu = (m_1 - m_2)S$
- This Ward Identity guarantees absolute normalization for the scalar form factor: $Z_S \equiv 1$.
- Z-factors are still needed for the local, one-link vector currents.
- PCVC defines the Z-factors non-perturbatively
- Expect: Z_{V_i}, $Z_{V_4} \rightarrow 1$ as $a \rightarrow 0$

Renormalization

- $(m_h - m_q)\langle K|S|D_s\rangle = (M_{Ds} - E_K)\langle K|V^4|D_s\rangle + \mathbf{q}\cdot\langle K|\mathbf{V}|D_s\rangle$

$$= q^{\mu}\langle K|V^{\mu}|D_s\rangle$$





Renormalization

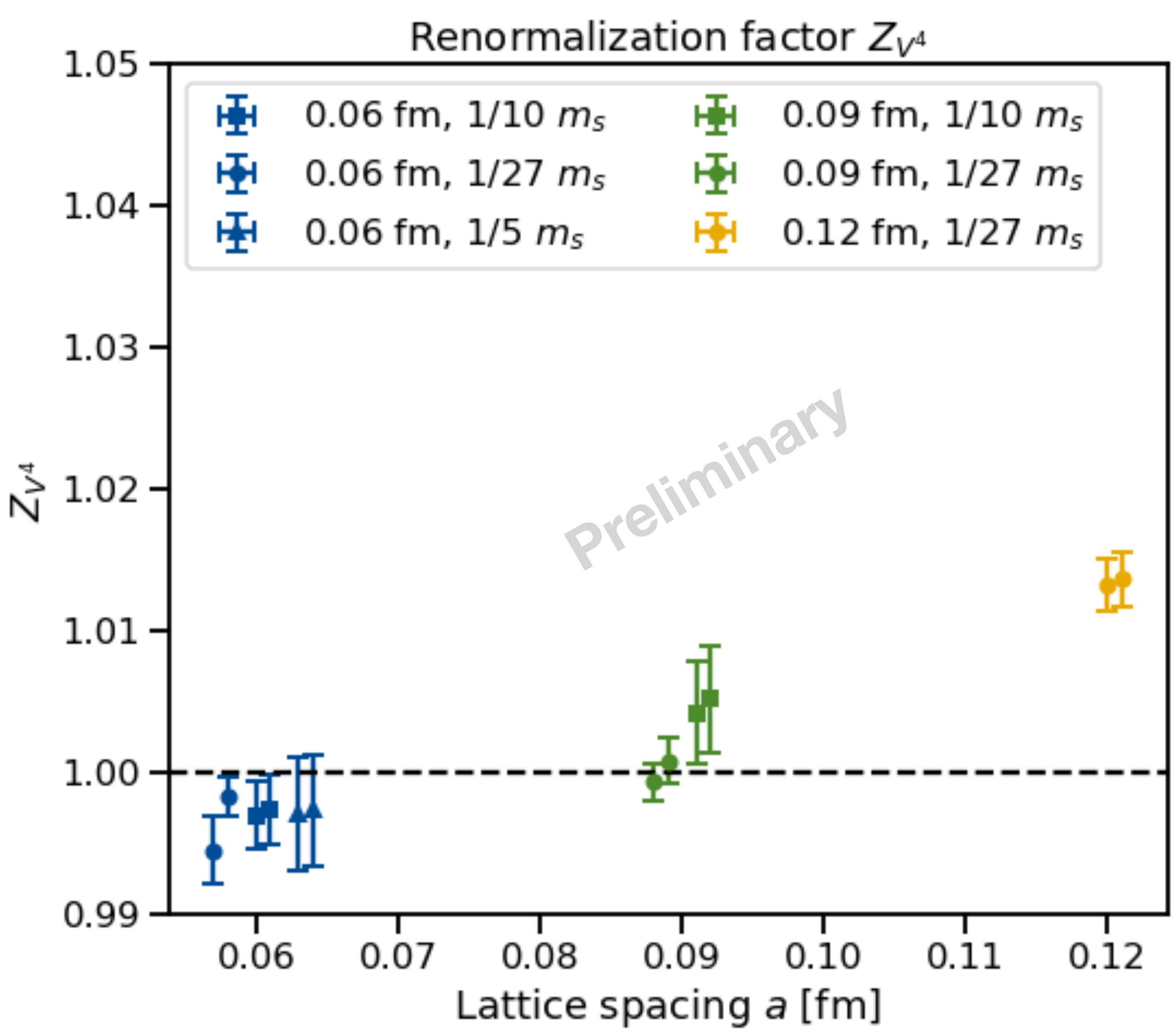
- $(m_h\text{-}m_q)\langle K|S|D_s\rangle = (M_{D_s} - E_K)\langle K|V^4|D_s\rangle + \mathbf{q}\cdot\langle K|\mathbf{V}|D_s\rangle$

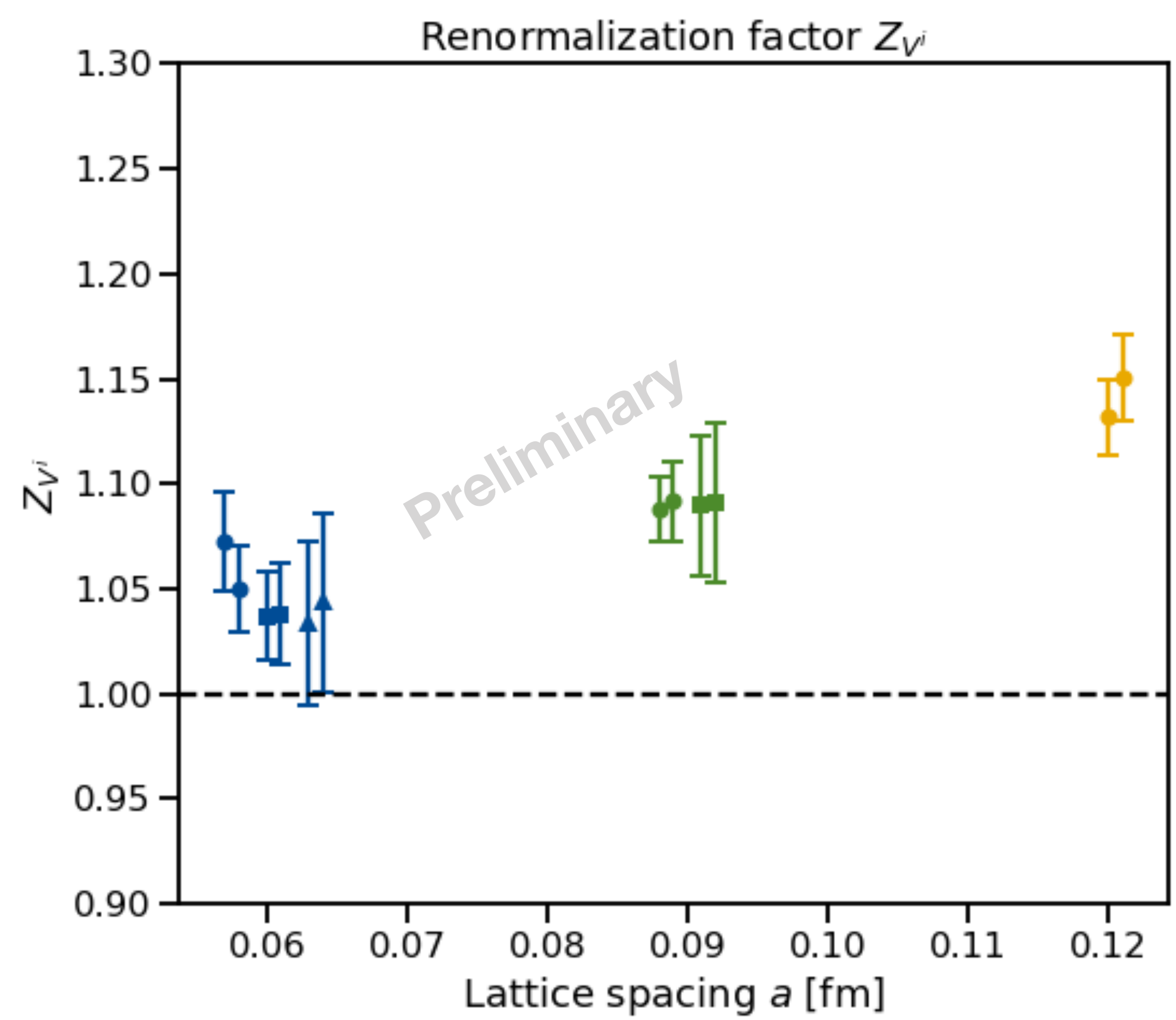
  $$= q^\mu\langle K|V^\mu|D_s\rangle$$

- At least visually, the bare matrix elements already satisfy the Ward Identity well (even for rather large momenta)
- Many options exist to extract values for Z_{V4} and Z_{Vi}
- One possible solution: extract them from a fit

Renormalization

- Fit PCVC relation with Z_{V4}, Z_{Vi} as parameters:
- $(m_h - m_q)\langle K|S|D_s\rangle = Z_{V4}\,(M_{Ds} - E_K)\langle K|V^4|D_s\rangle + Z_{Vi}\,\mathbf{q}\cdot\langle K|\mathbf{V}|D_s\rangle$





Renormalized form factors

- Armed with Z-factors, we can construct form factors relevant for phenomenology. For example:

$$f_+(q^2) = \left[f_{||}(q^2) + (M_H - E_L)f_\perp(q^2)\right]$$

$$f_0(q^2) = \frac{\sqrt{2M_H}}{M_H^2 - M_L^2}\left[(M_H - E_L)f_{||}(q^2) - (E_L^2 - M_H^2)f_\perp(q^2)\right]$$

- Equivalent forms offer consistency checks

$$f_0(q^2) = \left(\frac{m_h - m_\ell}{M_H^2 - M_L^2}\right)\langle L|S|H\rangle \qquad\qquad f_+(q^2) \propto \text{“}f_0 + f_\perp\text{”}$$

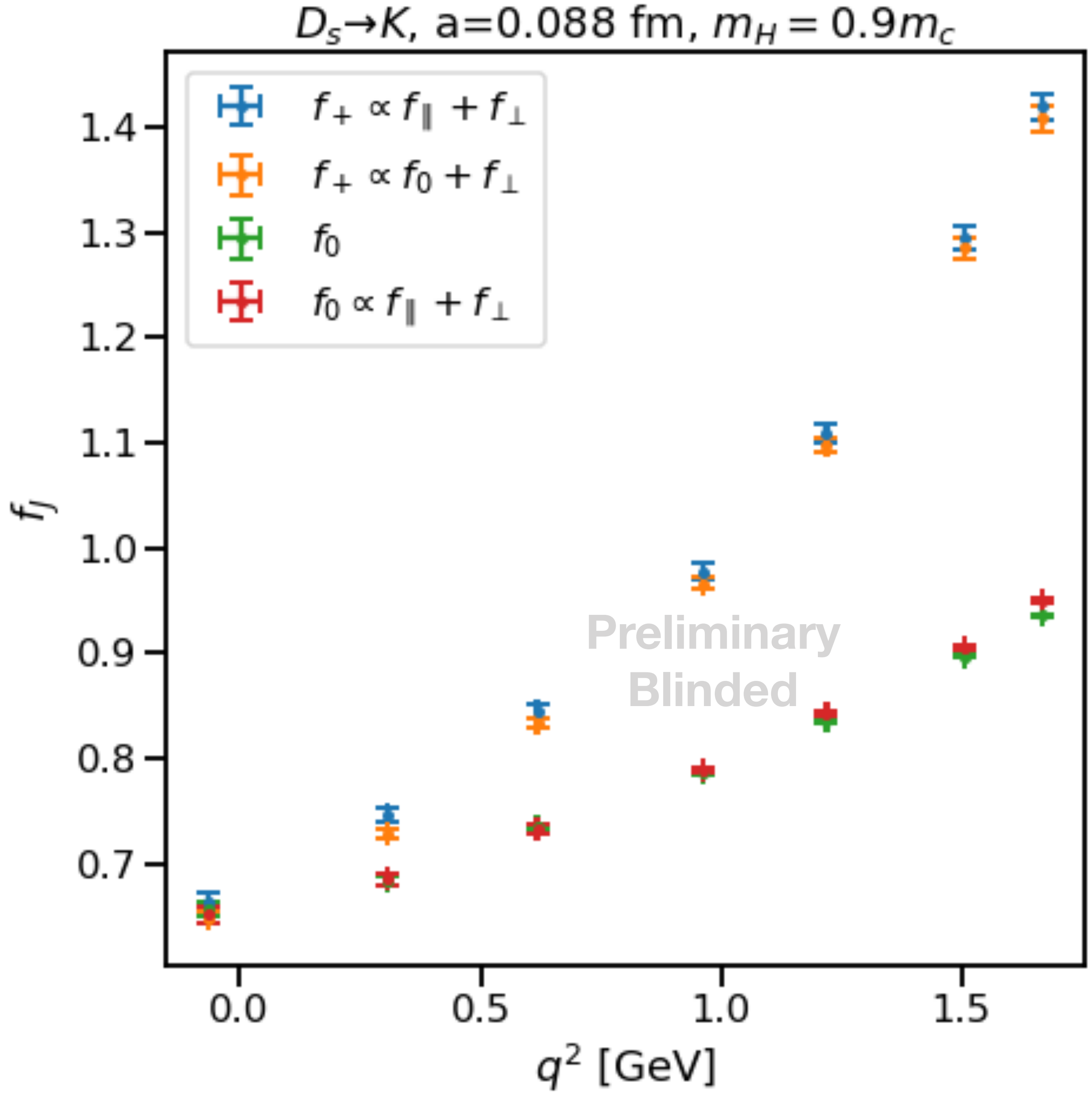
Renormalized form factors

Standard f_+ via "$f_\parallel + f_\perp$"

Alternate f_+ via "$f_0 + f_\perp$"

f_0 via scalar matrix element

Standard f_0 via "$f_\parallel + f_\perp$"



Renormalized form factors

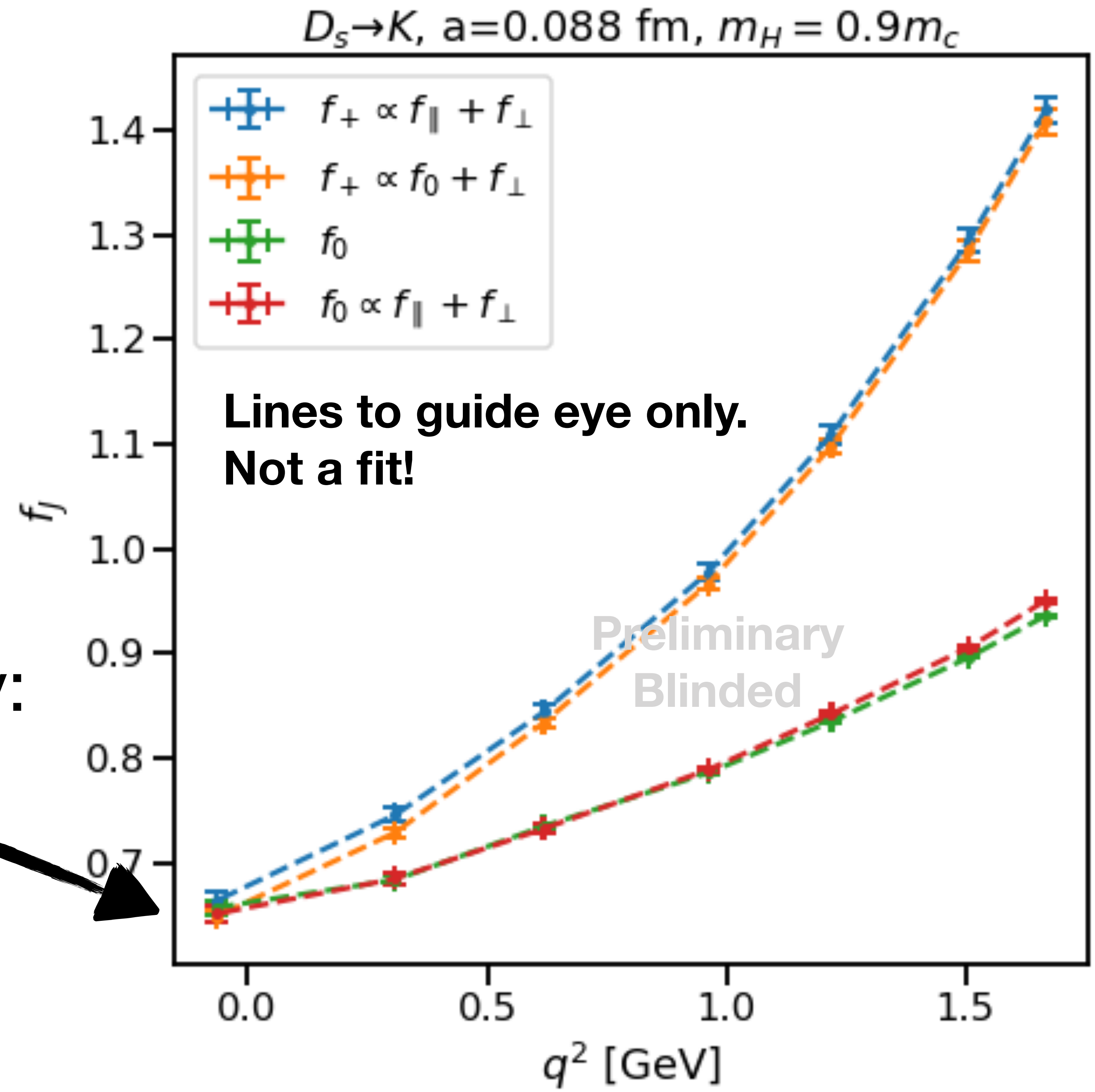
Standard f_+ via "$f_\parallel + f_\perp$"

Alternate f_+ via "$f_0 + f_\perp$"

f_0 via scalar matrix element

Standard f_0 via "$f_\parallel + f_\perp$"

Check kinematic identity:
$f_+(q^2) = f_0(q^2)$ at $q^2=0$ ✔



Summary and next steps

- We have presented preliminary results for $D_{(s)} \rightarrow \pi/K$
  - Matrix elements for S, V4, and **V** currents and range of momenta spanning full kinematic range
  - Non-perturbative renormalization via PCVC
  - Fully blinded results pass a variety of sanity checks
- In progress: chiral interpolation and continuum extrapolation

Backup slides

Three-point correlators

- We compute 40 current-momentum combinations for each mass triplet (m_{mother}, $m_{daughter}$, $m_{spectator}$)

		S	V^4	V^i			T^{ij}		
momentum	$\hat{p}^2$	f_0	$f_\parallel$	$f_\perp(1)$	$f_\perp(2)$	$f_\perp(3)$	$f_T(14)$	$f_T(24)$	$f_T(14)$
p000	0	✓	✓	✗	✗	✗	✗	✗	✗
p100	1	✓	✓	✓	✗	✗	✓	✗	✗
p110	2	✓	✓	✓	✓	✗	✓	✓	✗
p200	4	✓	✓	✓	✗	✗	✓	✗	✗
p211	6	✓	✓	✓	✓	✓	✓	✓	✓
p300	9	✓	✓	✓	✗	✗	✓	✗	✗
p222	12	✓	✓	✓	✓	✓	✓	✓	✓
p400	16	✓	✓	✓	✗	✗	✓	✗	✗

Tab. 4: Details about the correlation functions appearing the allHISQ campaign.