New beta-function and the QCD coupling at the Z-boson pole mass

Kieran Holland
University of the Pacific
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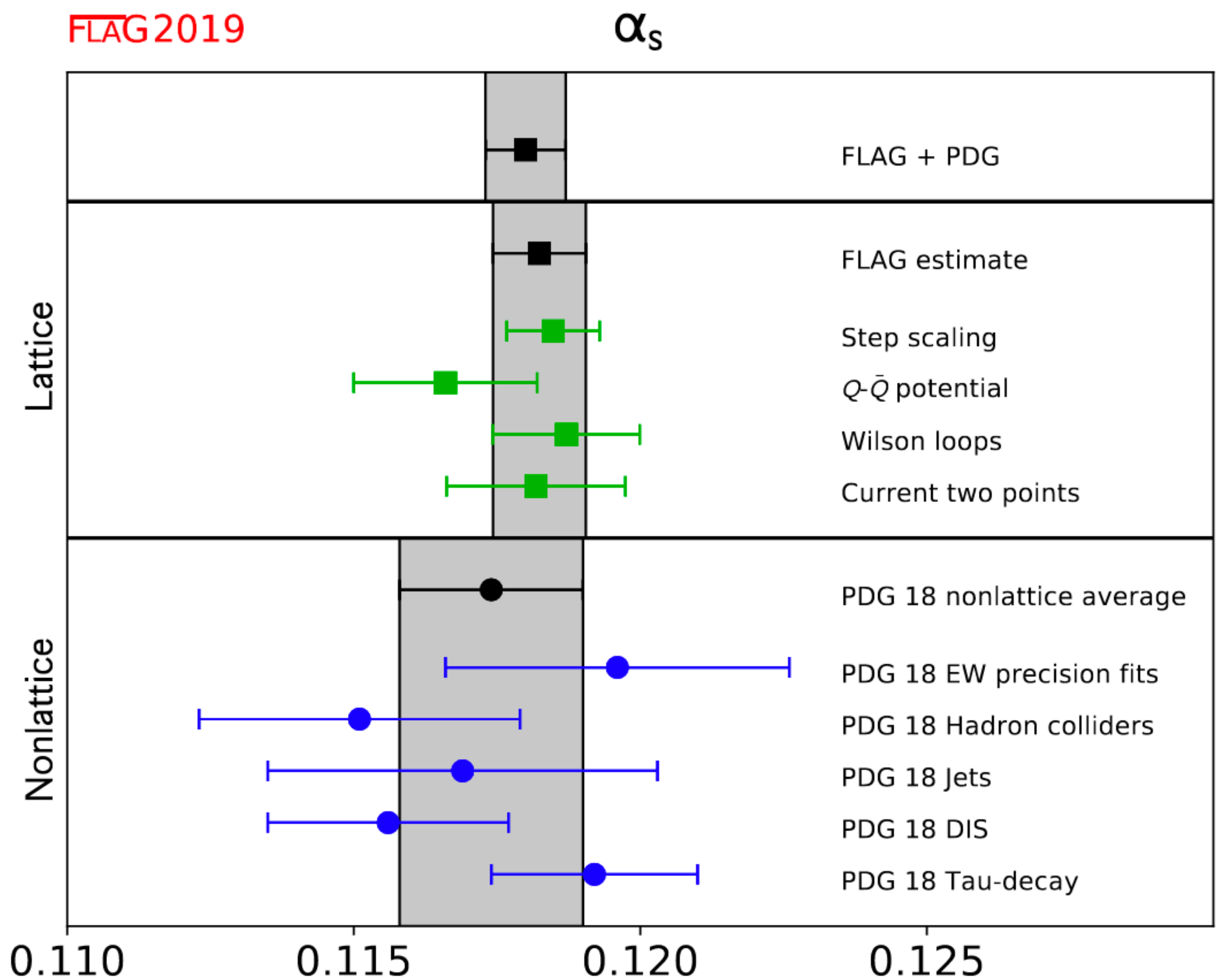
on behalf of the Lattice Higgs Collaboration (LatHC):

with Julius Kuti, Zoltan Fodor, and Chik-Him Wong

Overview

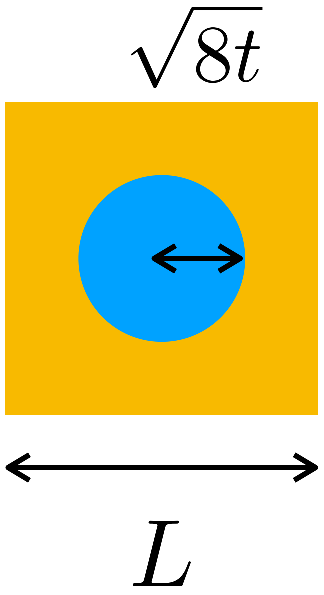


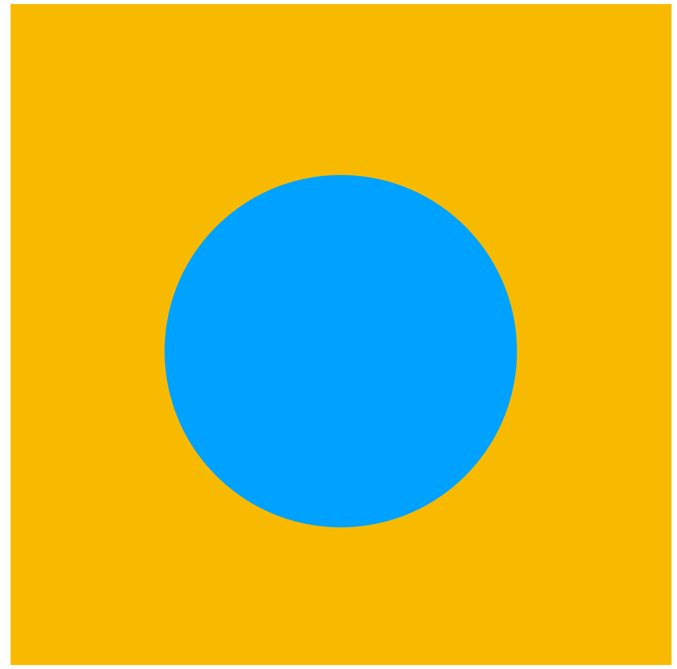
QCD coupling evaluated at the Electroweak scale is a crucial ingredient in Standard Model phenomenology

lattice determination of QCD coupling at Electroweak scale dominated by step-scaling calculation

lattice results dominate over non-lattice determinations

is there another possible scheme that can be competitive with step-scaling, for consistency check?





gradient flow scheme, flow time t

step-scaling: RG scale set through lattice volume

$$g^2(2L) - g^2(L)$$

with fixed ratio of flow time to lattice size t/L^2

continuum limit $L/a \to \infty$

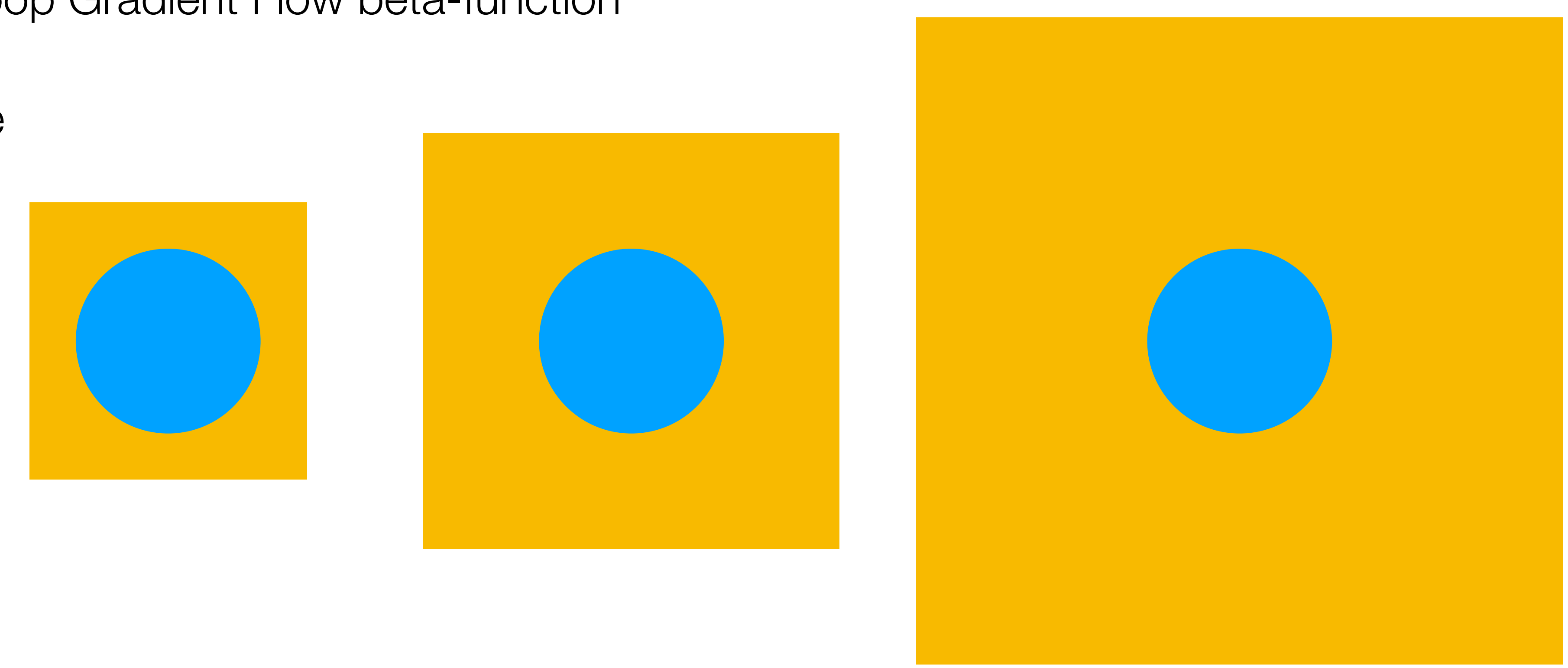
alternative: gradient flow scheme, in the infinite volume limit, at fixed t/a^2

continuum extrapolation $t/a^2 \to \infty$ is a separate stage

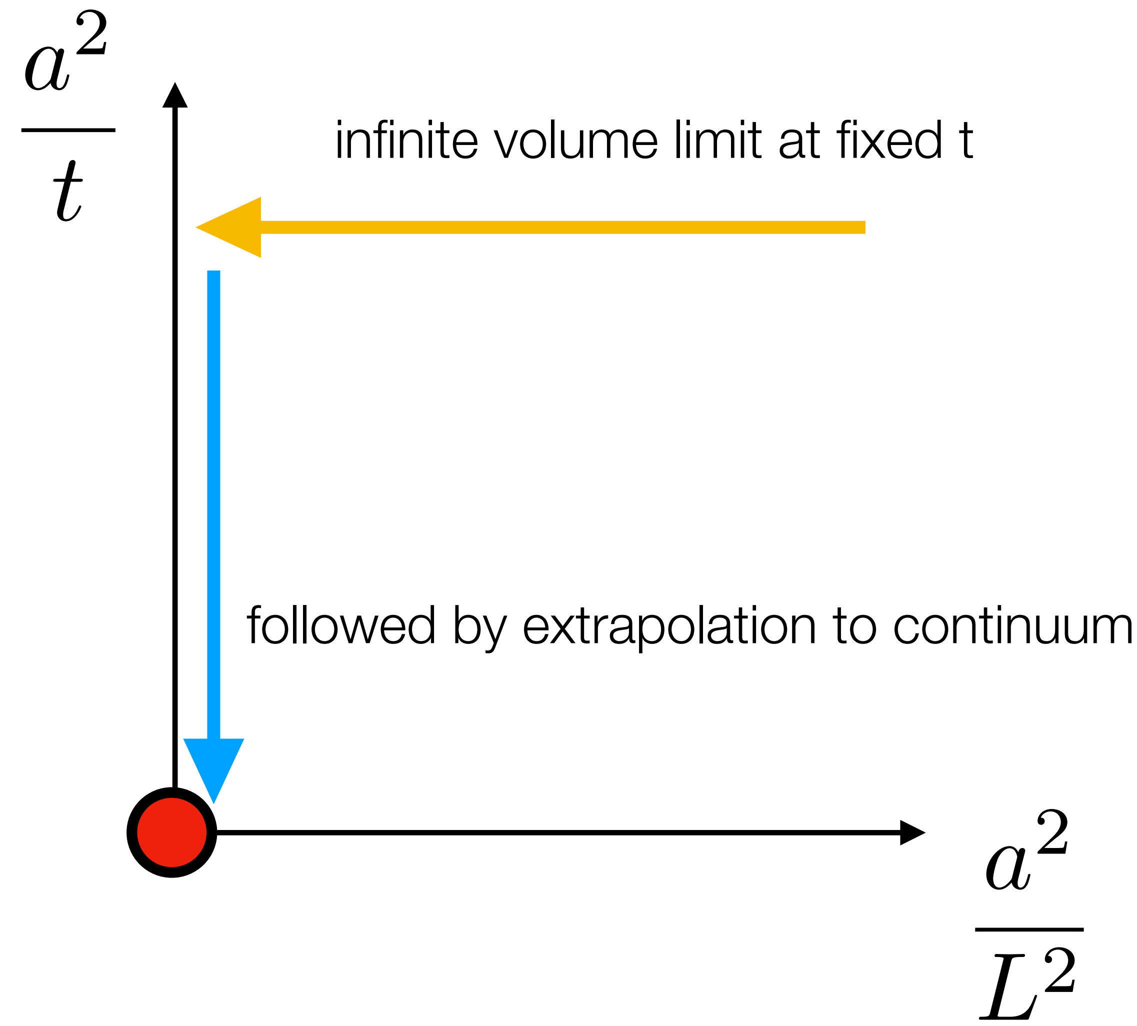
connects to Harlander et al 3-loop Gradient Flow beta-function

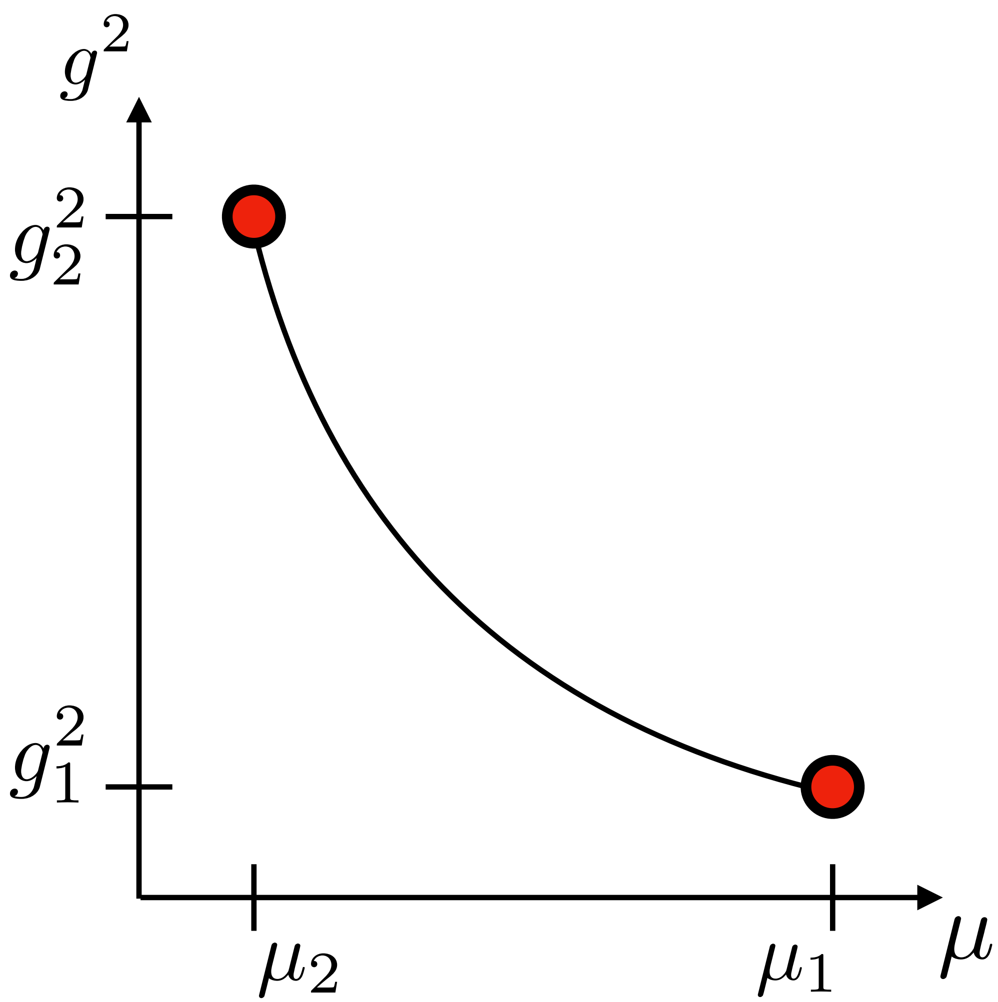
beta-function through derivative

$$t\frac{dg^2}{dt}$$



infinite volume beta-function requires 2-stage procedure towards the continuum limit





goal: connect weak to strong coupling through non-perturbative measurement of beta-function throughout

need additional information in strong coupling regime to express ratio of scale change in physical units e.g. using hadronic mass spectrum

Set-up

apply new procedure to **SU(3) gauge theory with Nf = 3 massless quarks**

use experience from beta-function studies of BSM models Nf = 10 and 12

lattice action: staggered fermions with 4 levels of stout smearing rho = 0.12
tree-level improved Symanzik gauge action

anti-periodic boundary conditions in all directions

first test: explore weak renormalized coupling regime, is it possible to make contact with perturbation theory

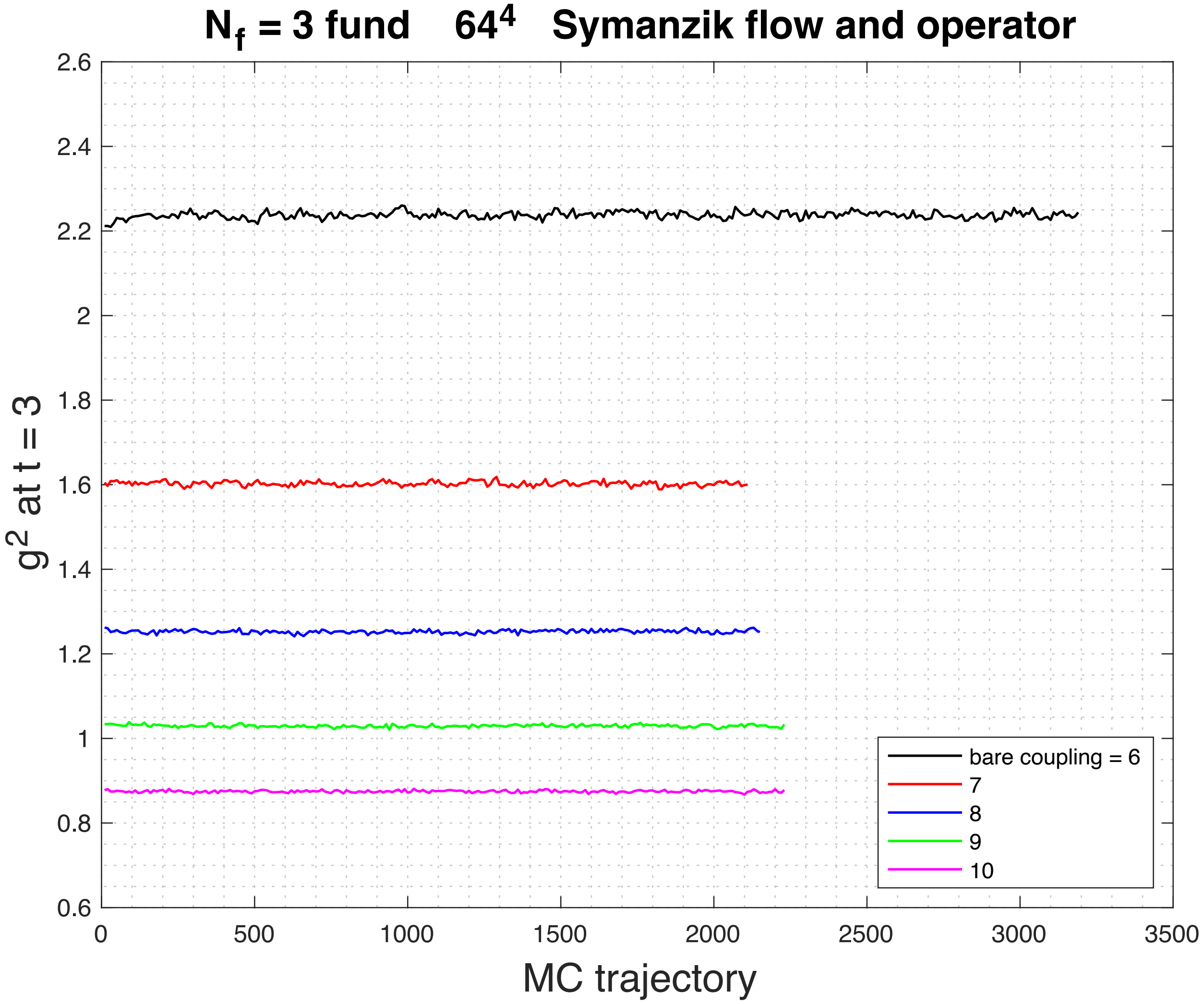
set of lattice ensembles with 5 bare couplings

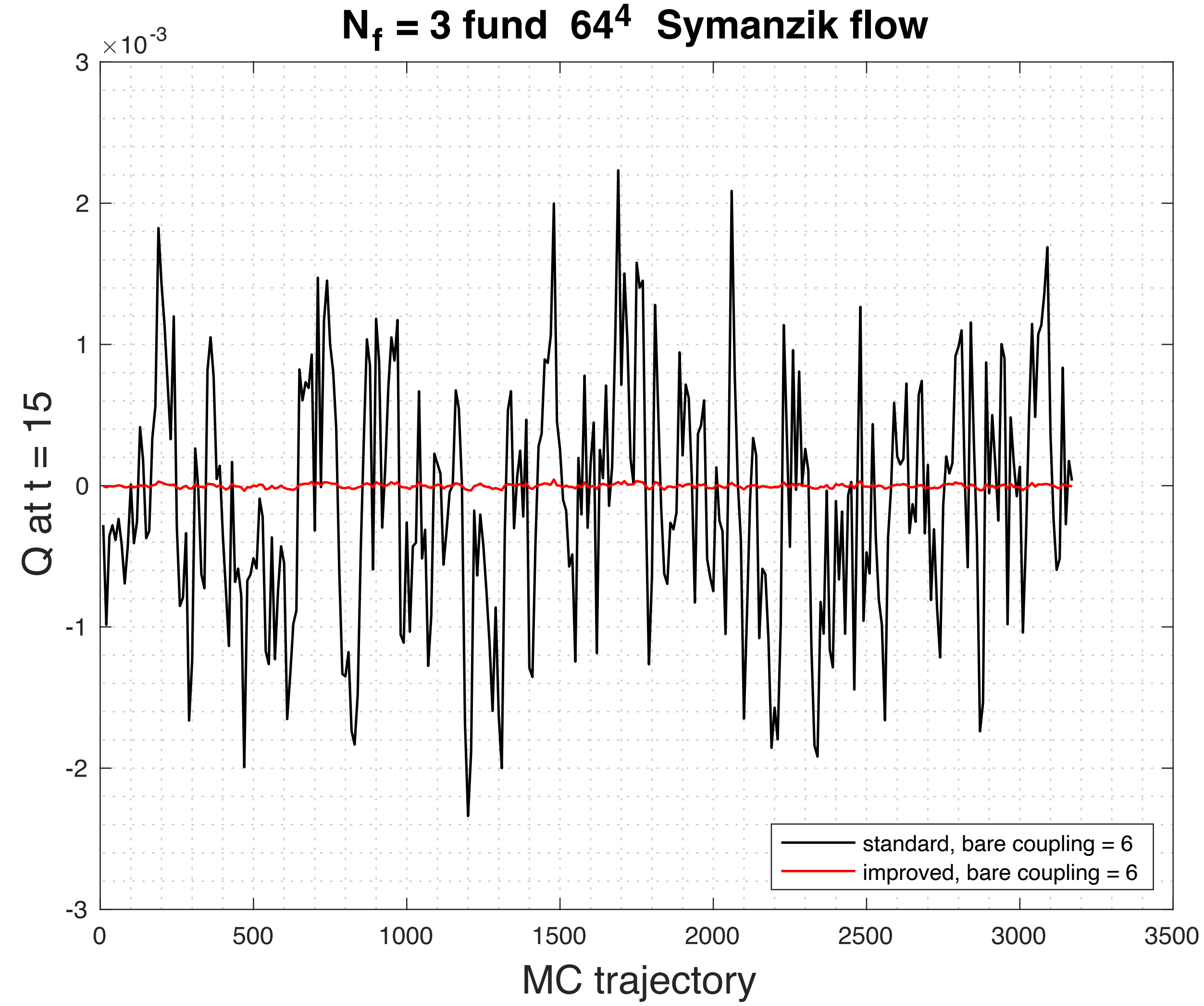
hypercubic lattice volumes L^4 with L /a = 24, 32, 40, 48, 64

25 ensembles in total

initial target: renormalized coupling in the range $g^2 \sim 1 - 2$

idea of derivative beta-function for gradient flow has previously been explored by *Fodor et al arXiv:1711.04833*, and *Hasenfratz et al arXiv:1910.06408*

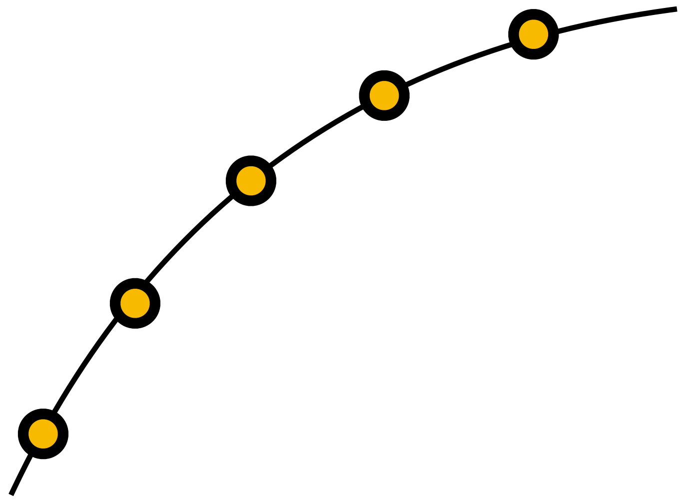




Monte Carlo ensembles range from over 20,000 trajectories on smallest volume (L = 24) to 2,000 trajectories on largest volume (L = 64)

all ensembles are in the topological charge sector Q = 0, no tunneling to other sectors at weak coupling the gradient flow renormalized coupling is measured at equally spaced intervals in flow time

$$g^2(\epsilon), g^2(2\epsilon), ..., g^2(n\epsilon), ...$$



use 5-point stencil to approximate derivative $t\frac{dg^2}{dt}$

to order $\mathcal{O}(\epsilon^4)$ at target t value

with $\epsilon = 0.05$ this is sufficiently accurate

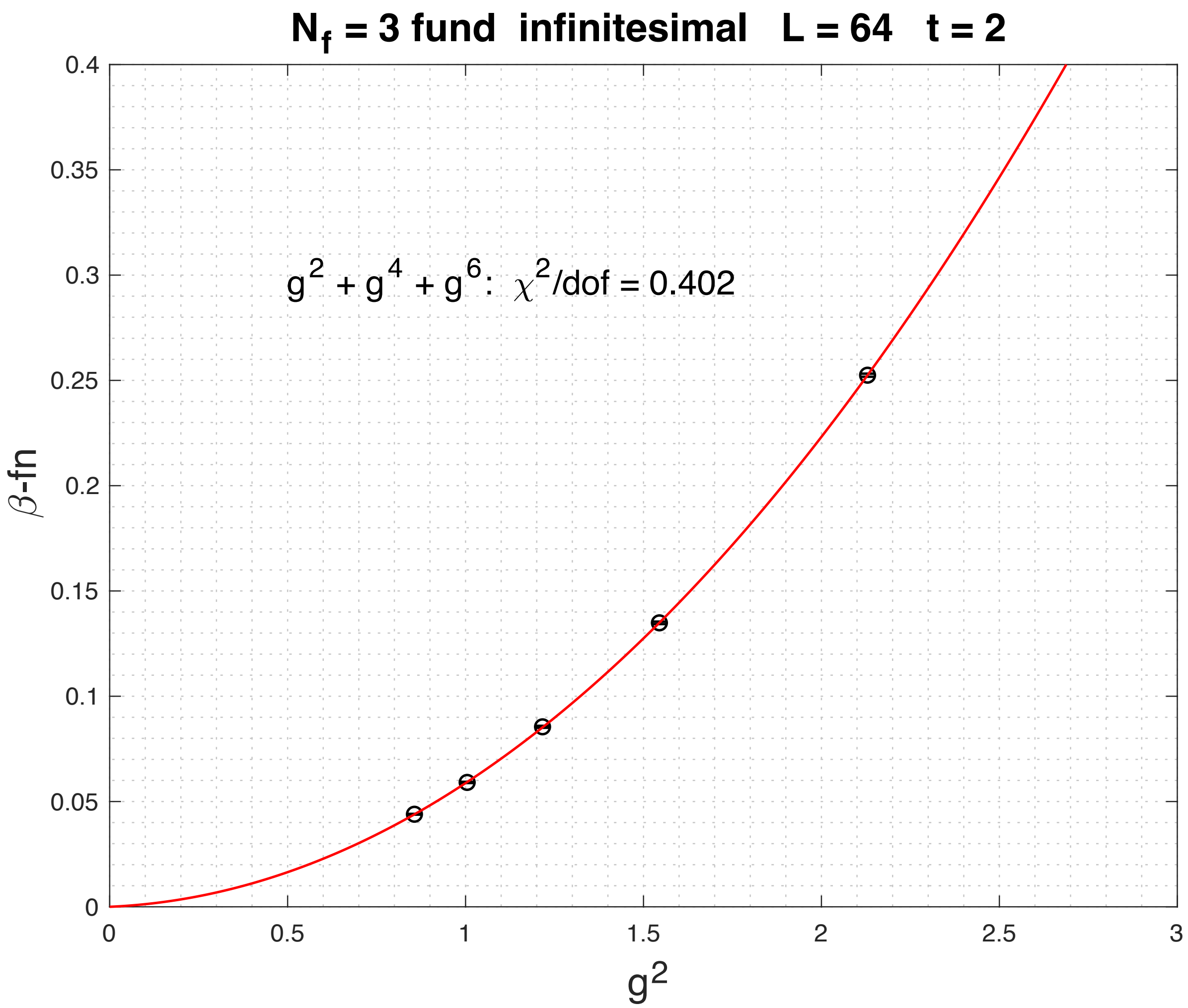
(also compared with 3-point and 7-point stencils)

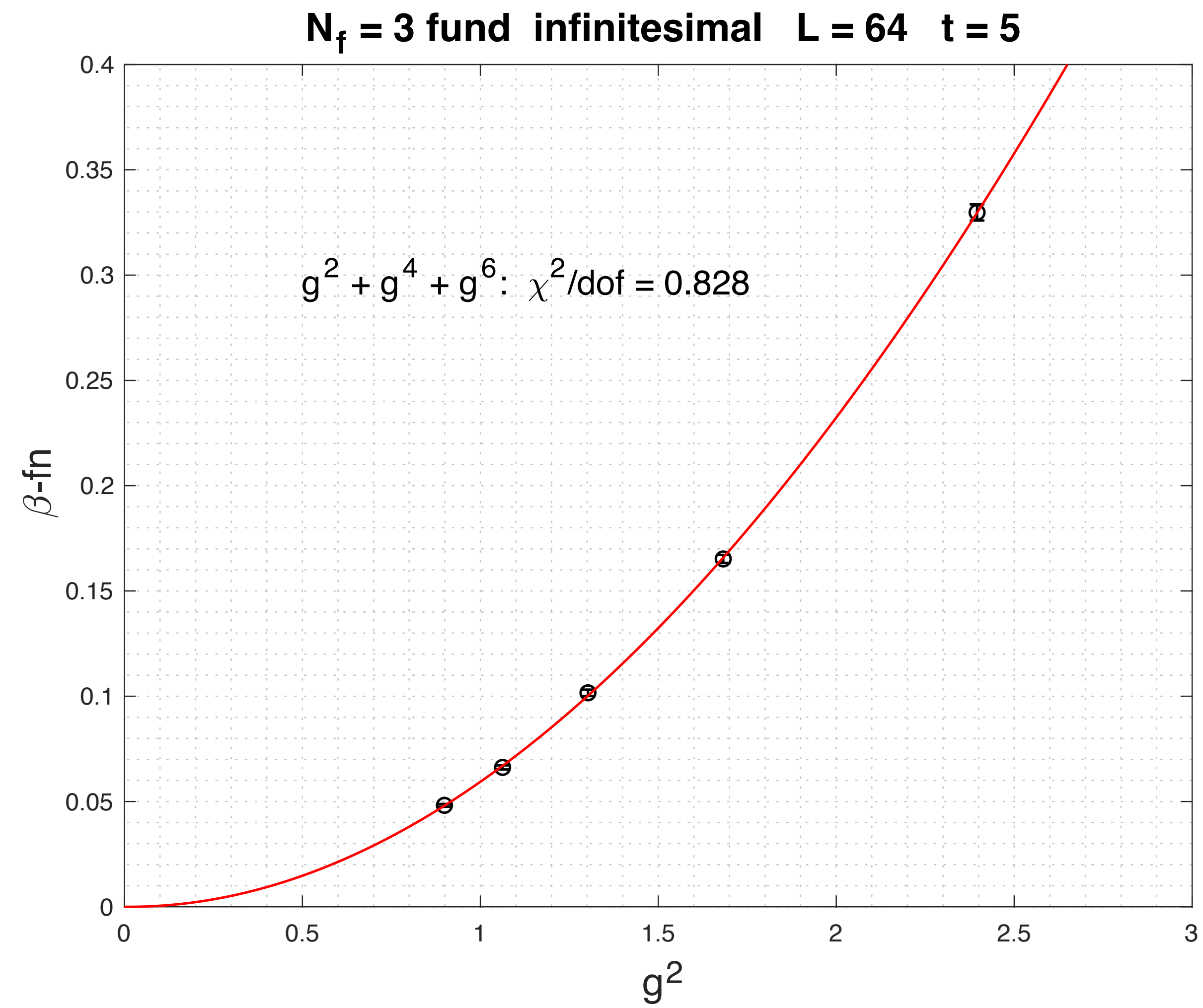
have both Symanzik (S) and Wilson (W) discretizations of the gradient flow

have both Symanzik (S) and Clover (C) discretizations of the observable $\langle t^2 E \rangle$

sometimes use shorthand notation e.g. SSC or SSS

(gradient flow—MC action—observable, with MC action always Symanzik)





5 ensembles for each lattice volume give a range of renormalized couplings and derivative at each chosen t value

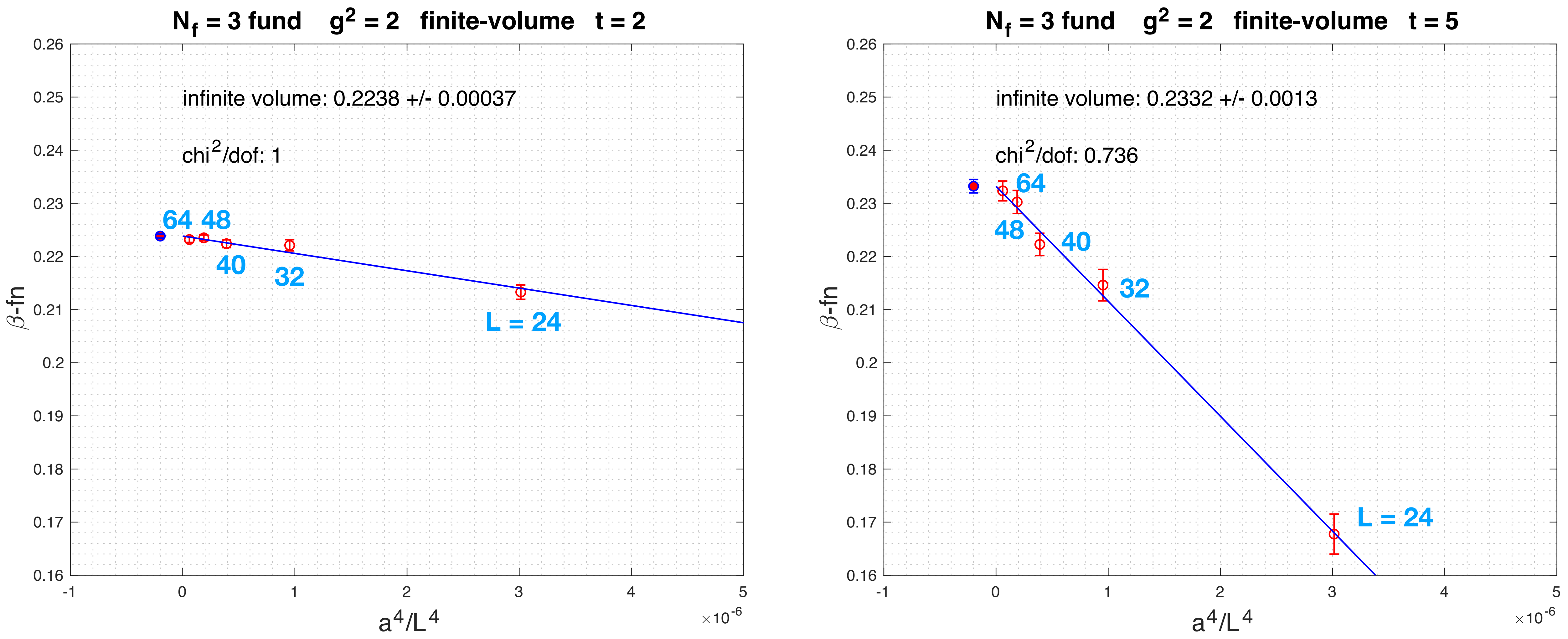
$$\beta\text{−fn} = t\,\frac{dg^2}{dt}$$

stage 1: pick a target value for the renormalized coupling e.g. $g^2 = 2$

on each volume and for each t value, interpolate beta-function in g^2 to target point

this replaces the tuning step, which is often used in step-scaling studies

figures above are for Symanzik flow and Symanzik observable



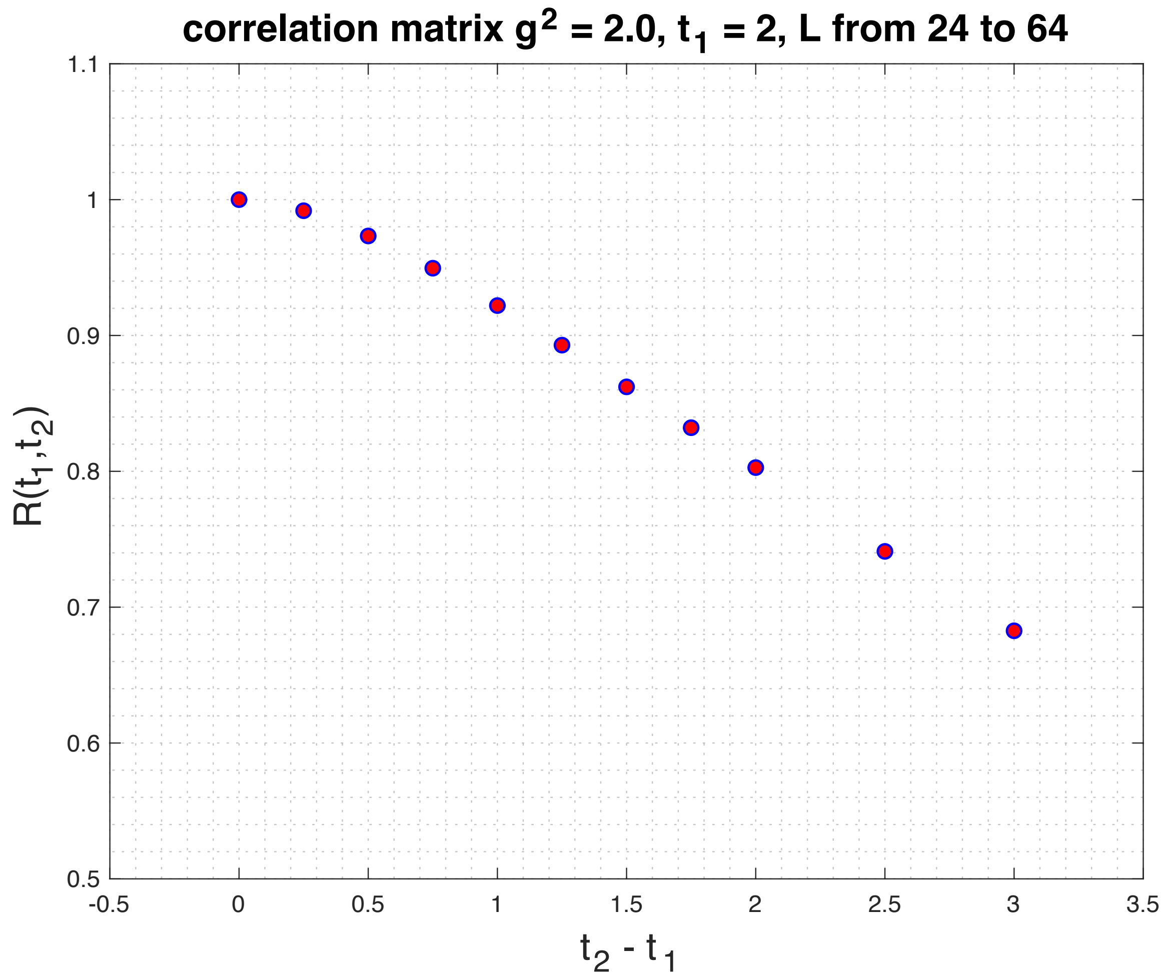
stage 2: extrapolation to infinite volume at fixed t and at fixed renormalized coupling

volume-dependence is completely consistent with $1/L^4$

volume variation naturally larger at larger fixed t value

L = 64 volume practically at infinite-volume limit, even at larger t values

choice of t range: avoid too small t (cutoff effects) and too large t (volume dependence)



infinite-volume results are strongly correlated across t values, being measured on same ensembles

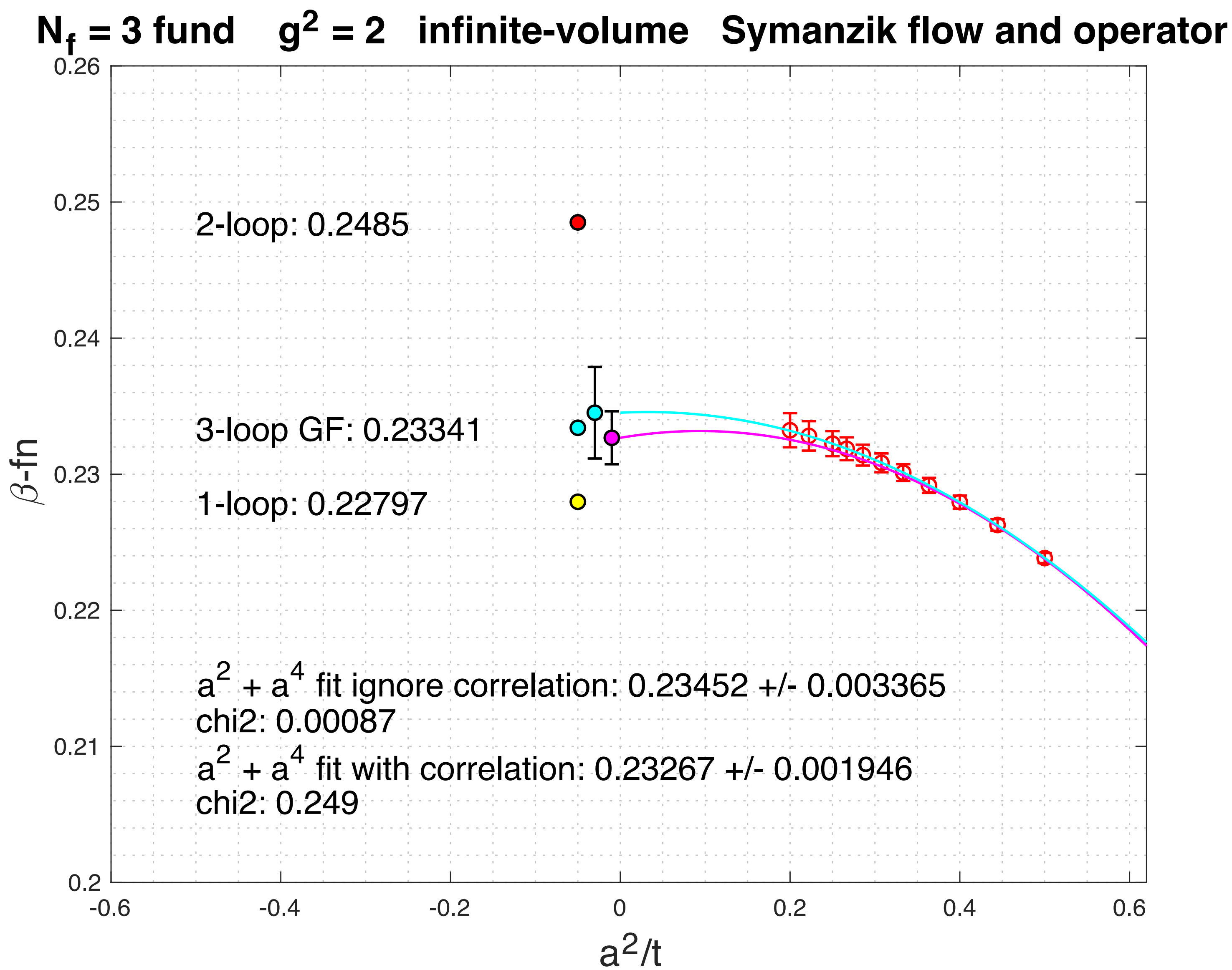
continuum extrapolation includes this correlation, to produce likely chi-squared and efficient error estimation

$$\Sigma = S \cdot R \cdot S$$

$$\chi^2(b) = (Y - X \cdot b)^t \, \Sigma^{-1} \, (Y - X \cdot b)$$

$$\hat{b} = (X^t \, \Sigma^{-1} \, X)^{-1} \cdot X^t \, \Sigma^{-1} \, Y$$

Y: data X: fitting function b: fit parameters



stage 3: continuum extrapolation of infinite-volume results at fixed renormalized coupling

including the correlation in chi-squared gives a more efficient estimate of the continuum error and a more natural chi-squared value

range of t-values chosen such that large cutoff effects at too small t excluded, and volume effects at larger t under control

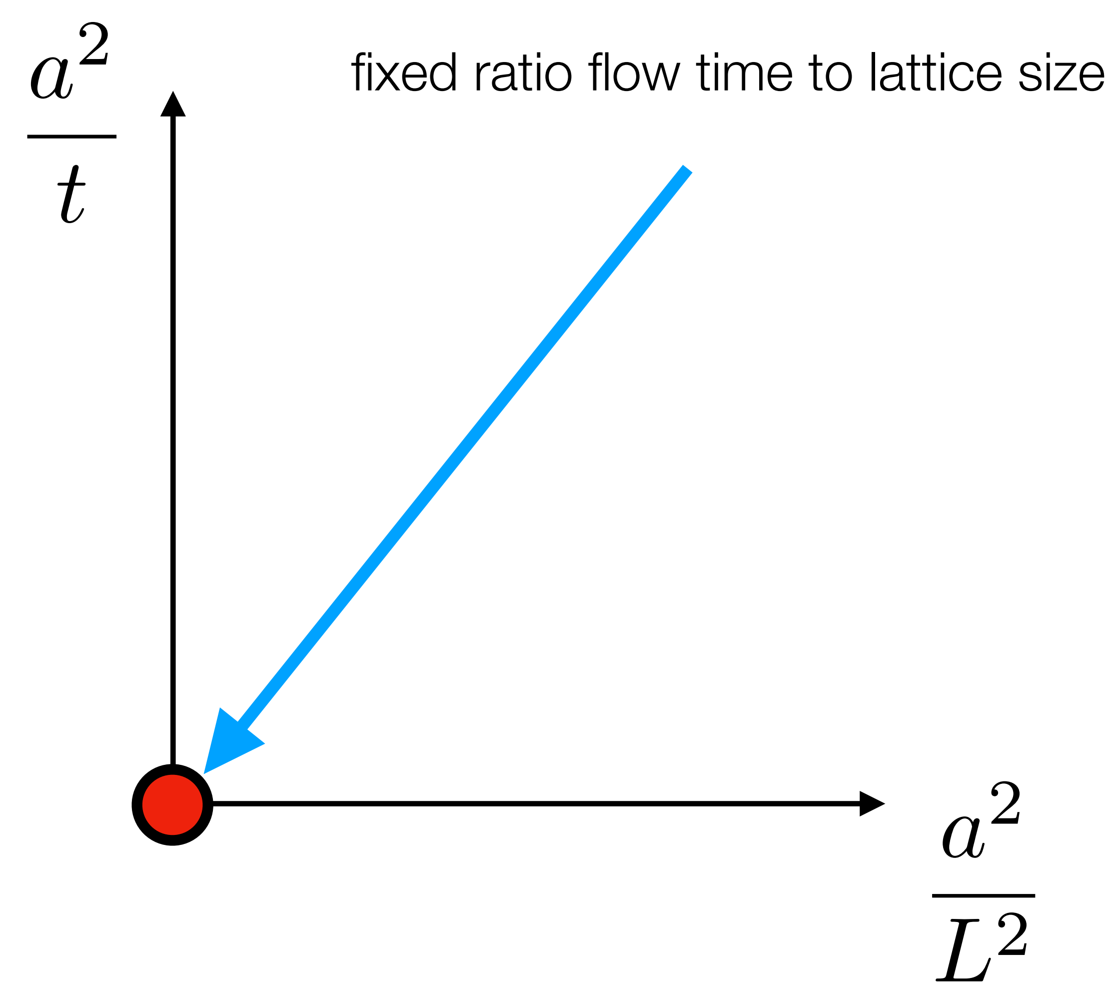
alternate procedure:

combine volume and continuum extrapolations into one step

slope in plane corresponds to choice of fixed ratio of flow time to lattice size

$$c^2 = \frac{8t}{L^2}$$

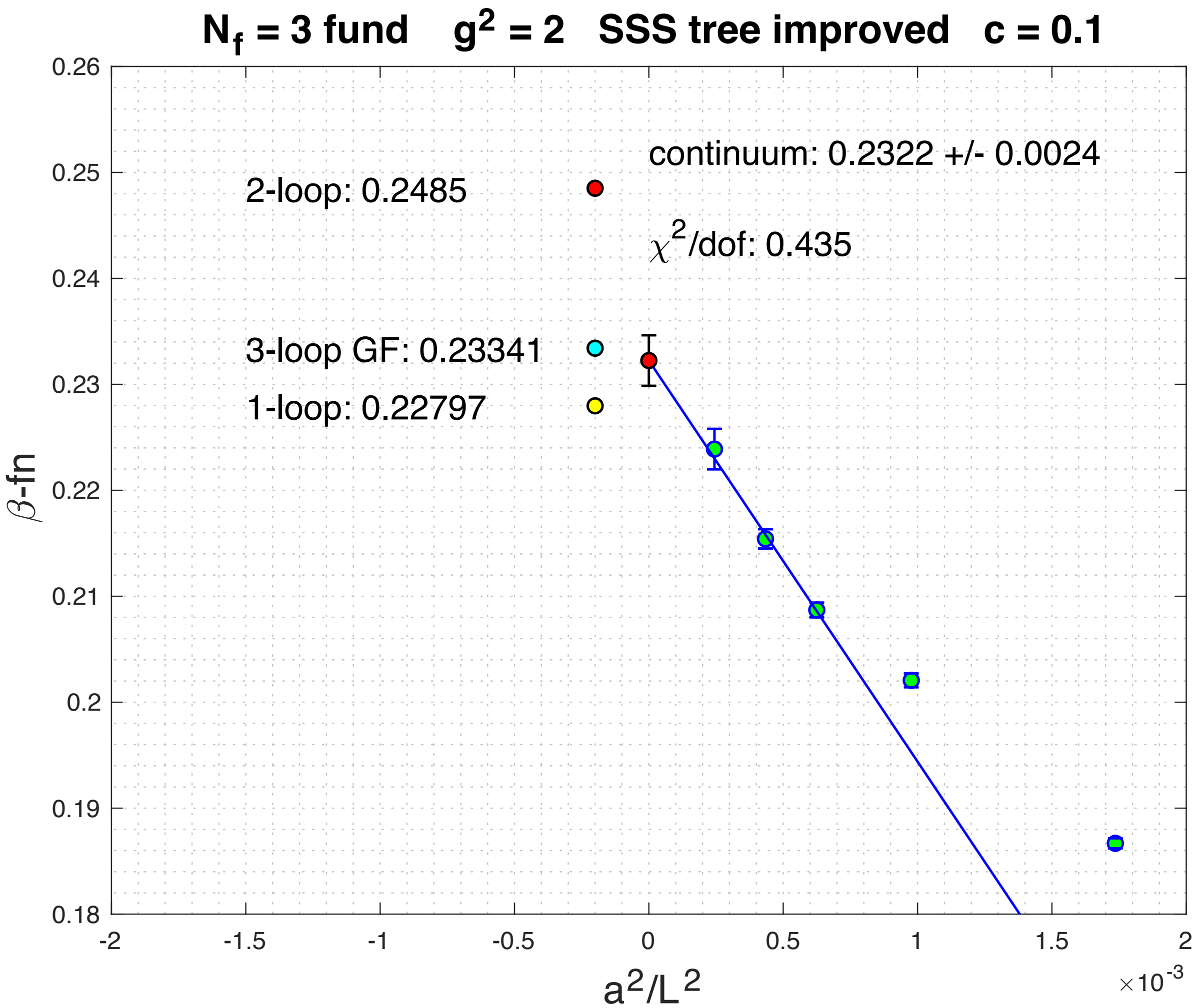
at sufficiently small values of c, the remnant finite-volume dependence should be very small

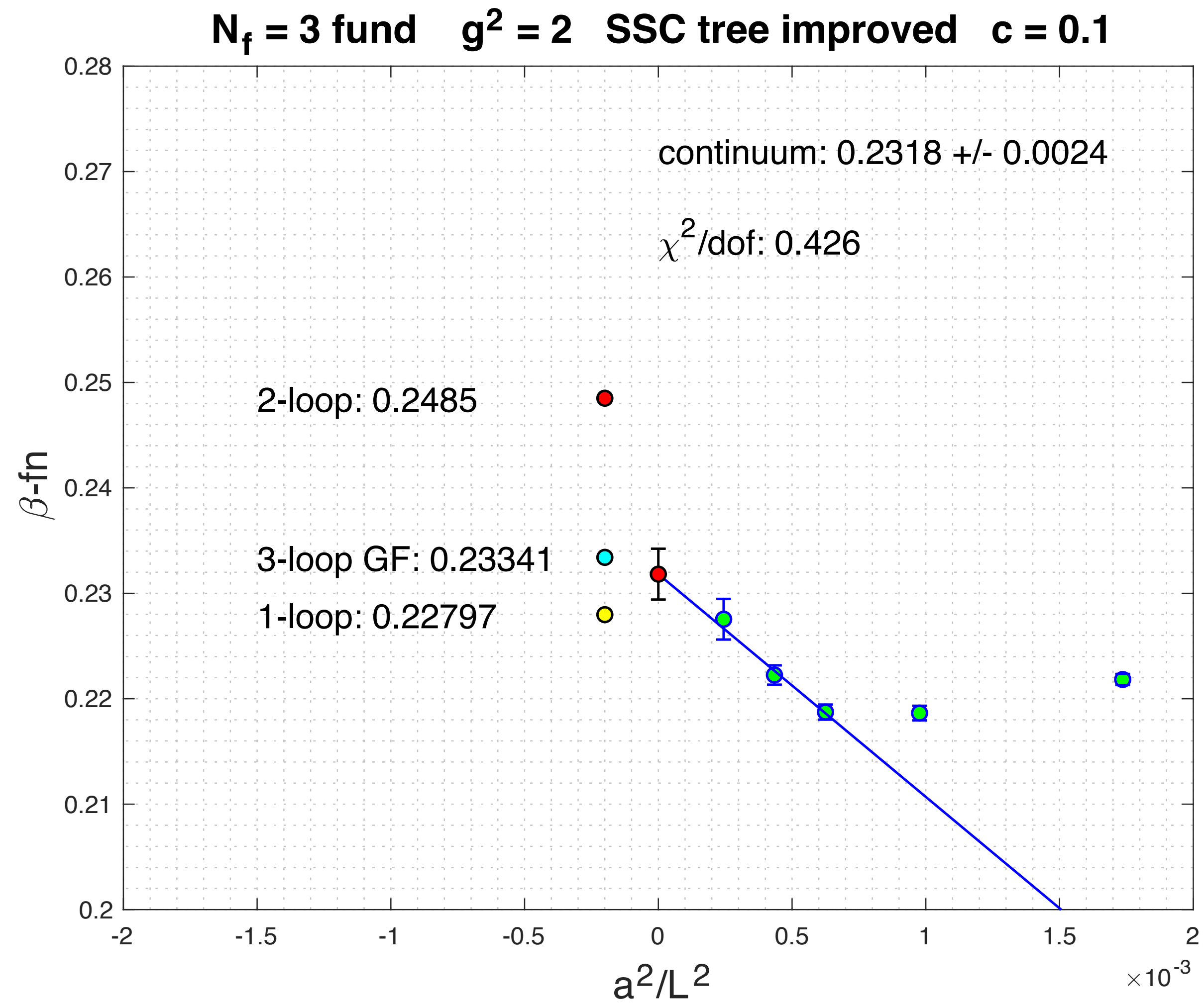


will show example at c = 0.1, which corresponds on largest volume L = 64 to t = 5.12

— matches with the t-range in the previous method

use the same set of lattice ensembles





choice c = 0.1

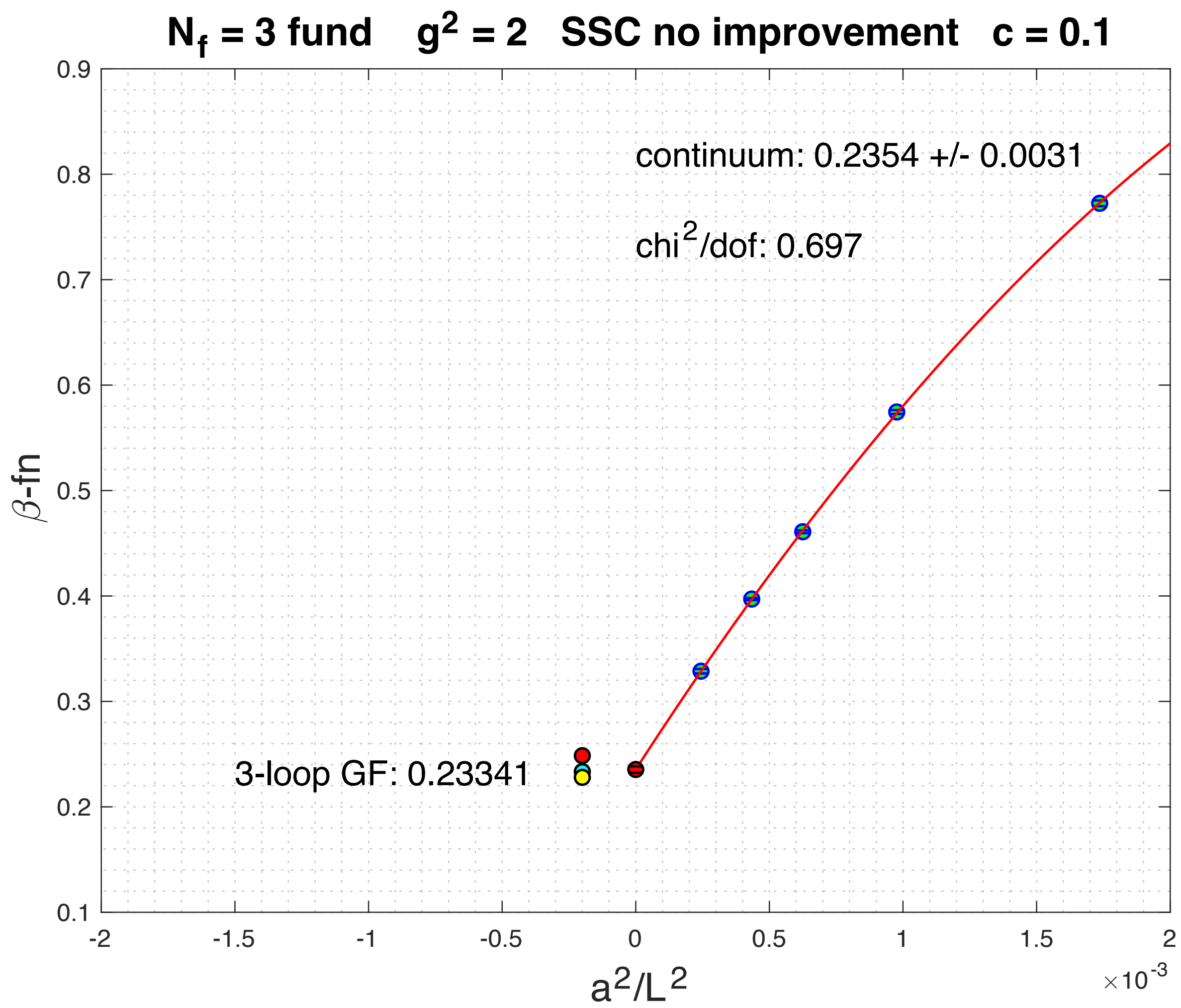
consistent results with different discretization of observable: Symanzik (S) and Clover (C)

tree-level improvement of the renormalized coupling through finite-lattice sums

Fodor et al arXiv:1406.0827

$$\langle t^2 E(t)\rangle = g^2 \frac{3(N^2-1)}{128\pi^2} C(a^2/t, \sqrt{8t}/L) \tag{6.2}$$

$$C(a^2/t, \sqrt{8t}/L) = \frac{128\pi^2 t^2}{3L^4} + \frac{64\pi^2 t^2}{3L^4} \sum_{n_\mu=0,\, n^2\neq 0}^{L/a-1} \mathrm{Tr}\, \left(e^{-t(\mathcal{S}^f+\mathcal{G})} (\mathcal{S}^g+\mathcal{G})^{-1} e^{-t(\mathcal{S}^f+\mathcal{G})} \mathcal{S}^e \right) \ ,$$



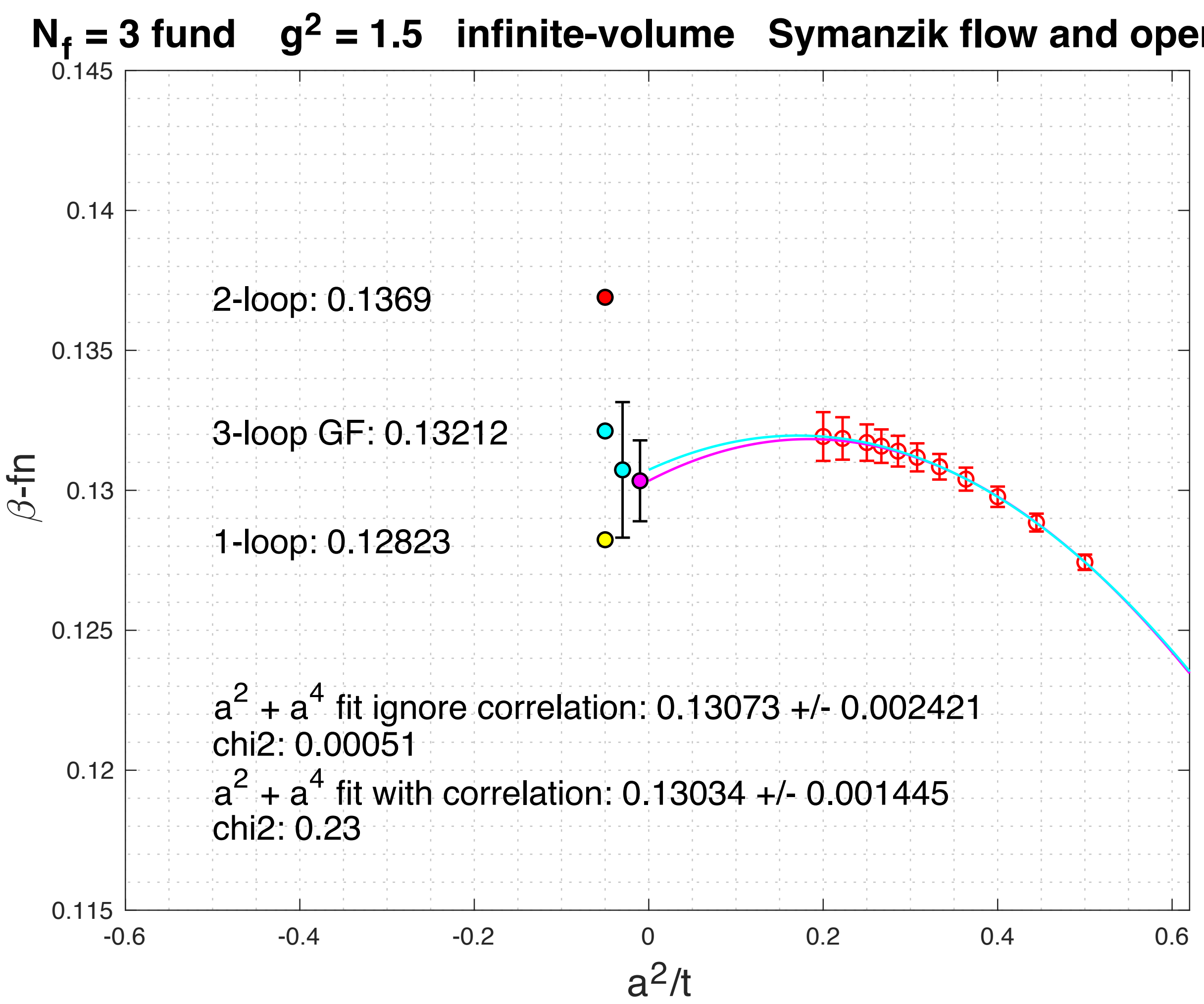
tree-level improvement is a large effect for fixed c method and the Clover observable

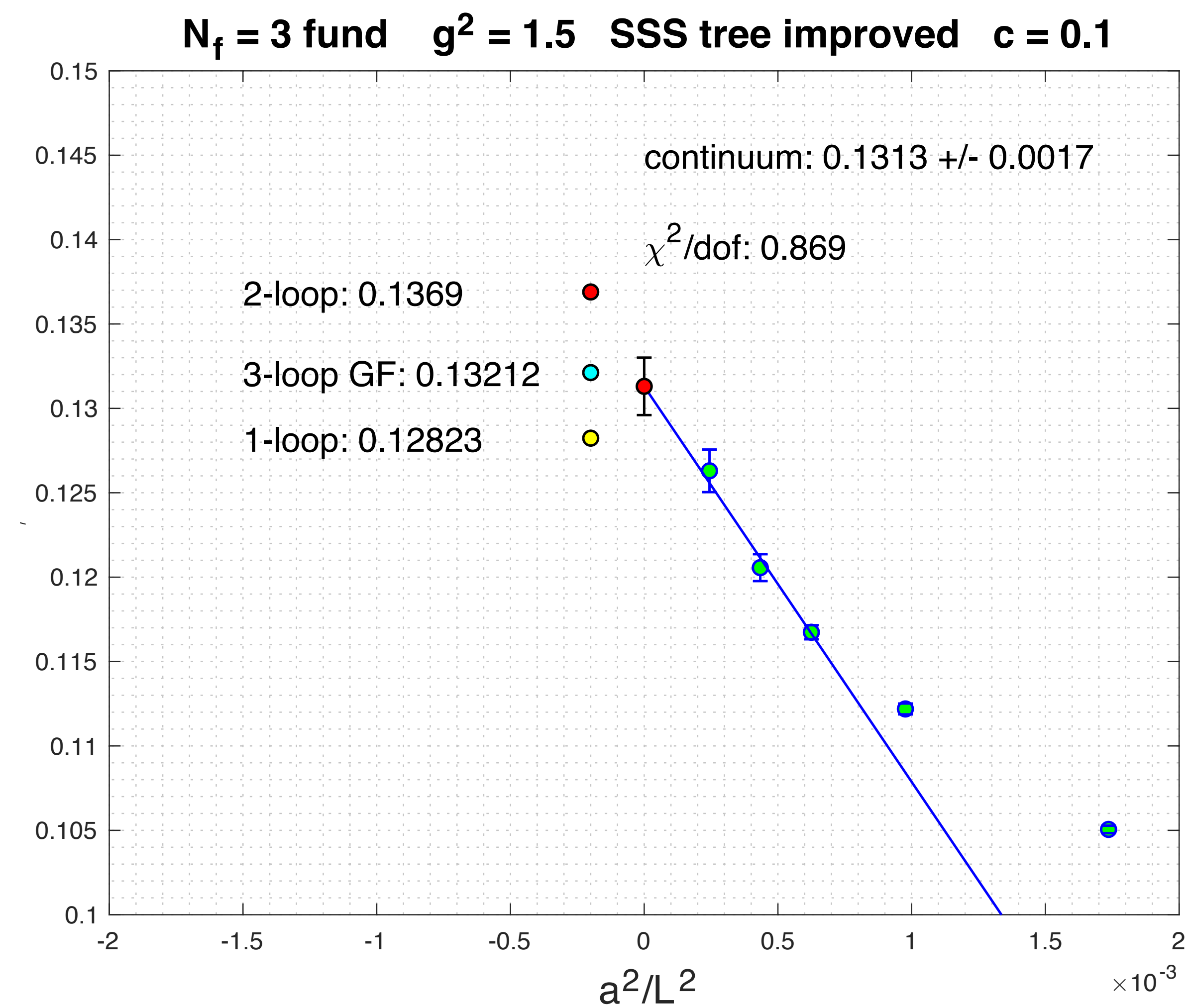
without improvement, results approach continuum result but with much larger cutoff effects

tree-level correction for Symanzik observable is much less visible

c = 0.1: L = 24, t = 0.72 L = 40, t = 2.0 L = 64, t = 5.12

no surprise that smaller L has large cutoff effect, given the corresponding small t values





previous analysis was at $g^2 = 2$

similar good agreement of infinite-volume determination with perturbation theory at weaker renormalized coupling $g^2 = 1.5$

continue to see consistency between infinite-volume method and fixed ratio approach at c = 0.1 and same coupling

for orientation: the value $g^2 = 1.5$ corresponds to $\alpha_s = 0.12$ coupling at Electroweak scale

Outlook

determination of QCD coupling at Electroweak scale $\alpha_s(M_z)$ requires:

(1) making contact with perturbation theory at weak coupling to high accuracy

and

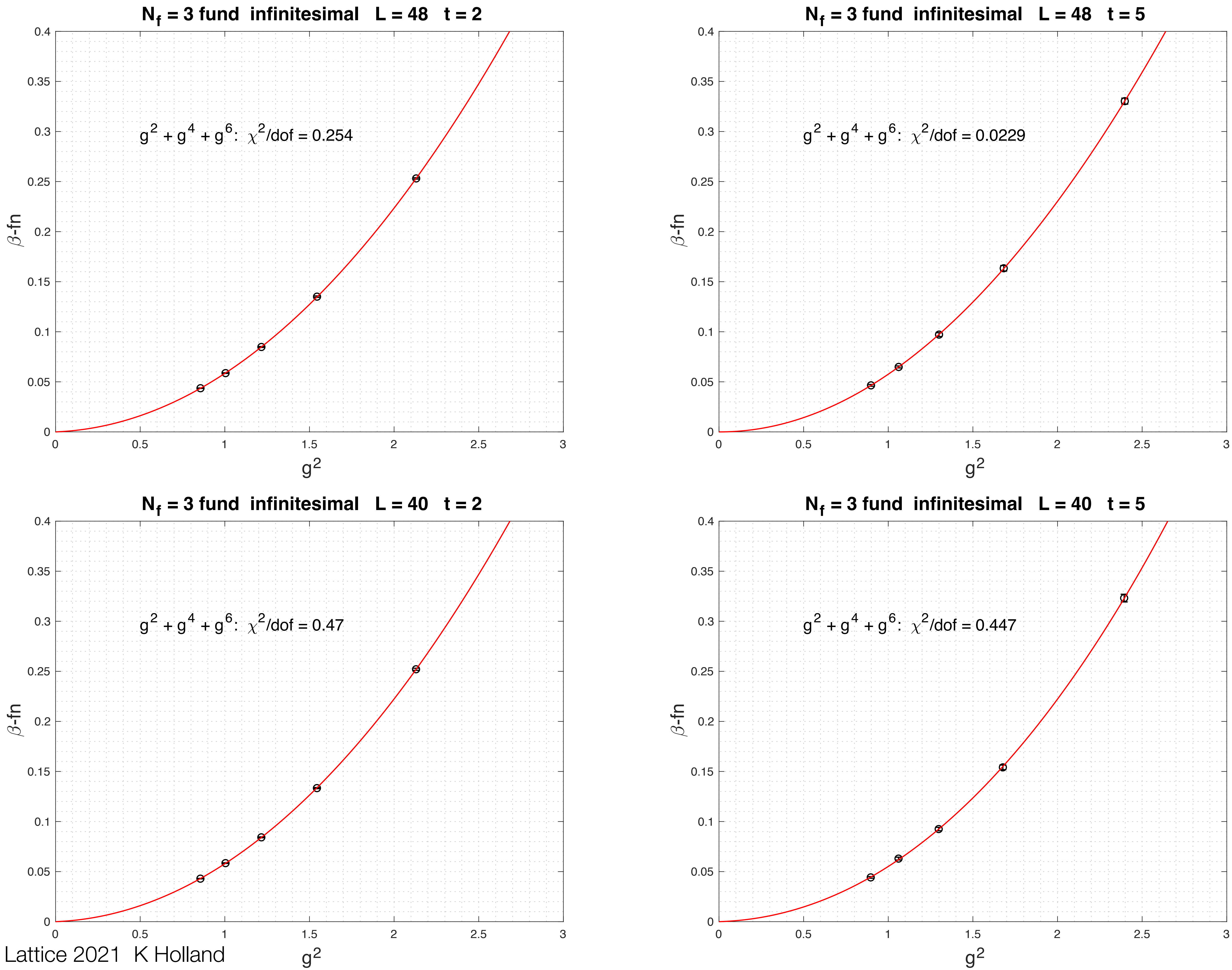
(2) connecting to a physical scale at strong coupling

test study of part (1) for **SU(3) with Nf = 3 massless quarks** looks promising, reaching ~ 1% error in the beta-function in the continuum limit for $g^2 \sim 2$

continuing to part (2) for this approach looks worthwhile, and could be competitive with step-scaling determination thank you

Extra slides

more examples of interpolation at fixed volume and t value



tree-level improvement through finite-lattice sum at each fixed t value via

$$\langle t^2 E(t) \rangle = g^2 \frac{3(N^2-1)}{128\pi^2} C(a^2/t, \sqrt{8t}/L) \qquad (6.2)$$

arXiv:1406.0827

$$C(a^2/t, \sqrt{8t}/L) = \frac{128\pi^2 t^2}{3L^4} + \frac{64\pi^2 t^2}{3L^4} \sum_{n_\mu=0,\, n^2\neq 0}^{L/a-1} \mathrm{Tr}\left(e^{-t(\mathcal{S}^f+\mathcal{G})} (\mathcal{S}^g+\mathcal{G})^{-1} e^{-t(\mathcal{S}^f+\mathcal{G})} \mathcal{S}^e \right) ,$$

choice of lattice gauge action for gradient flow, MC simulation, and action density observable

tree-level improvement does not include fermion effects

more likely to be useful for smaller Nf value Nf = 3 than in BSM studies with Nf = 10 or 12 gradient flow beta-function to 3-loop order and connection to MS-bar scheme

$$\alpha_{\rm s} = \hat{\alpha}_\rho \left[1 - e_1(\rho)\frac{\hat{\alpha}_\rho}{4\pi} + (2e_1^2(\rho) - e_2(\rho))\left(\frac{\hat{\alpha}_\rho}{4\pi}\right)^2 + \ldots\right],$$

Harlander et al arXiv:1905.00882

$$\mu^2 \frac{\mathrm{d}}{\mathrm{d}\mu^2}\hat{\alpha}_\rho(\mu) = \hat{\alpha}_\rho(\mu)\hat{\beta}_\rho(\hat{\alpha}_\rho),$$

$$\hat{\beta}_\rho(\hat{\alpha}_\rho) = -\sum_{n=0}^{\infty}\hat{\beta}_{\rho,n}\left(\frac{\hat{\alpha}_\rho}{4\pi}\right)^n = -\frac{\hat{\alpha}_\rho}{4\pi}\beta_0 - \left(\frac{\hat{\alpha}_\rho}{4\pi}\right)^2\beta_1 - \sum_{n=2}^{\infty}\hat{\beta}_{\rho,n}\left(\frac{\hat{\alpha}_\rho}{4\pi}\right)^n,$$

with β_0 and β_1 from Eq. (62). The third coefficient is given by

$$\hat{\beta}_{\rho,2} = \beta_2 - e_1(\rho)\,\beta_1 + (e_2(\rho) - e_1^2(\rho))\,\beta_0,$$

with the $\overline{\rm MS}$ coefficient β_2 which we quote here for the SU(3) gauge group:

$$\beta_2 = \frac{2857}{2} - \frac{5033}{18}n_{\rm F} + \frac{325}{54}n_{\rm F}^2.$$

$$e_0(z) = e_{0,0}, \qquad e_1(z) = e_{1,0} + \beta_0\,L(z),$$
$$e_2(z) = e_{2,0} + (2\beta_0\,e_{1,0} + \beta_1)\,L(z) + \beta_0^2\,L^2(z),$$

$$e_{0,0} = 1, \qquad e_{1,0} = \left(\frac{52}{9} + \frac{22}{3}\ln 2 - 3\ln 3\right)C_{\rm A} - \frac{8}{9}T_{\rm F}$$
$$e_{2,0} = 27.9786\,C_{\rm A}^2 - (31.5652\ldots)\,T_{\rm F}C_{\rm A} + \left(16\zeta(3) - \frac{43}{3}\right)T_{\rm F}C_{\rm F} + \left(\frac{8\pi^2}{27} - \frac{80}{81}\right)T_{\rm F}^2,$$