Nucleon isovector form factors from physical-mass 2+1-flavor dynamical domain-wall QCD

Shigemi Ohta*† for LHP+RBC+UKQCD

Lattice 2021, 5:15-5:30am ET, July 27, 2021

Domain-wall fermions (DWF) lattice Quantum Chromodynamics (QCD):

- preserves both chiral and flavor symmetries,
- started by RIKEN-BNL-Columbia Collaboration 23 years ago, using purpose-built parallel supercomputers.

Joint RBC+UKQCD Collaborations have been generating 2+1-flavor dynamical DWF ensembles:

- for more than a decade, and at physical mass for several years,
- with a range of momentum cuts off, 1-3 GeV, and volumes $m_\pi L \sim 4$.

We have been calculating pion, kaon, $(g-2)_\mu$, and nucleon electroweak matrix elements.

An update.

*Institute of Particle and Nuclear Studies, High-Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan
†RIKEN BNL Research Center, Brookhaven National Laboratory, Upton, NY 11973, USA

RBC/UKQCD $N_f = 2 + 1$-flavor dynamical DWF ensembles[1] with $a^{-1} = 1.730(4)$ and $2.359(7)$ GeV :

- $L \sim 5.5$ fm, with pion mass of 139.2(4) and 139.2(5) MeV respectively,
- $m_\pi L \sim 4$, small volume corrections.

Chiral and continuum limit with good flavor and chiral symmetries (and so renormalizations wherever needed):

- meson decay constants: $f_\pi = 130.2(9)$ MeV, $f_K = 155.5(8)$ MeV, $f_K/f_\pi = 1.195(5)$;
- quark mass: $m_{\rm s}^{\overline{\rm MS}(3{\rm GeV})} = 81.6(1.2)$ MeV, $m_{\rm ud}^{\overline{\rm MS}(3{\rm GeV})} = 3.00(5)$ MeV, $m_s/m_{ud} = 27.34(21)$;
- chiral condensate $\Sigma^{1/3}(\overline{\rm MS}, 3{\rm GeV}) = 0.285(2)_{\rm stat.}(1)_{\rm pert.}$ GeV;
- kaon mixing parameter: $B_K^{\overline{\rm RGI}} = 0.750(15)$, $B_K^{\overline{\rm MS}(3{\rm GeV})} = 0.530(11)$,
- K_{l3}[2] $f_+(0) = 0.9685(34)_{\rm stat}(14)_{\rm FV}$, $|V_{us}| = 0.2233(5)_{\rm exp}(9)_{\rm lat}$;
- $SU(2)$ low-energy constants[3] $B^{\overline{\rm MS}}$, f, $\Sigma^{1/3,\overline{\rm MS}}$, f_π/f=1.064(2)(5), $l_{1,2,3,4,7}$;
- SU(3)-breaking ratios for D- and B-mesons[4], $|V_{cd}/V_{cs}|$, $|V_{td}/V_{ts}|$;
- BSM kaon mixing[5] are also being calculated, testing the SM, or constraining the BSM.

Contribute to determining SM parameters from meson calculations and constraining the BSM[6].

[1] T. Blum et al., RBC and UKQCD Collaborations, Phys.Rev. D93 (2016) 074505, arXiv:1411.7017 [hep-lat].
[2] D. Murphy et al., RBC and UKQCD Collaborations, PoS LATTICE2014 (2015) 369
[3] P.A. Boyle et al., RBC and UKQCD Collaborations, Phys.Rev. D93 (2016) 054502, arXiv:1511.01950 [hep-lat].
[4] P.A. Boyle et al., RBC and UKQCD Collaborations, arXiv:1812.08791 [hep-lat].
[5] P.A. Boyle et al., arXiv:1812.04981 [hep-lat].
[6] S. Aoki et al., Eur.Phys.J. C77 (2017) 112 DOI: 10.1140/epjc/s10052-016-4509-7 e-Print: arXiv:1607.00299 [hep-lat].

In contrast, systematics in the baryon sector is not well understood yet:

- Proton mean squared charge radius, though Lamb shift discrepancy might have been resolved[7],
- Nucleon axial charge, g_A,
- Nucleon electroweak form factors, $F_V(q^2)$, $F_T(q^2)$, $F_A(q^2)$, $F_P(q^2)$,
- Nucleon structure functions and parton distribution functions,
- Proton spin puzzle,

despite potentials for new physics:

- dark matter via g_T and g_S,
- neutron electric dipole moment,
- proton decay,
- $n\bar{n}$ mixing, ...

[7] N. Bezginov, T. Valdez, M. Horbatsch, A. Marsman, A.C. Vutha, E.A. Hessels, Science 06 Sep 2019: Vol. 365, Issue 6457, pp. 1007-1012 DOI: 10.1126/science.aau7807.

Nucleon form factors, measured in elastic scatterings or β decay or muon capture:

$$\langle p|V_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_\mu F_1(q^2) - i\sigma_{\mu\lambda} q_\lambda \frac{F_2(q^2)}{2m_N}\right] u_n e^{iq\cdot x},$$

$$\langle p|A_\mu^+(x)|n\rangle = \bar{u}_p \left[\gamma_\mu \gamma_5 F_A(q^2) + \gamma_5 q_\mu \frac{F_P(q^2)}{2m_N}\right] u_n e^{iq\cdot x},$$

$$F_V = F_1, F_T = F_2; G_E = F_1 - \frac{q^2}{4m_N^2} F_2, G_M = F_1 + F_2.$$

Related to

- mean-squared charge radii, $F_1 = F_1(0) - \frac{1}{6}\langle r_E^2\rangle Q^2 + ...$
- anomalous magnetic moment, $F_2(0)$,
- $g_A = F_A(0) = 1.2752(13) g_V$ ($g_V = F_1(0) = G_{\text{Fermi}} \cos\theta_{\text{Cabibbo}}$).

$\langle r_E^2\rangle$ and g_A, in particular, are being revised:

- $\sqrt{\langle r_E^2\rangle} = 0.875(6)$ fm from electron scattering, 0.8409(4) and 0.833(10) from μ and e Lamb shift;
- $g_A/g_V = 1.264(2)$ pre 2002 (“cold neutron,”) 1.2755(11) post, (“ultra cold neutron.”)

The latter, with Goldberger-Treiman relation, $m_N g_A \propto f_\pi g_{\pi NN}$, determines much of nuclear physics, such as primordial and neutron-star nucleosyntheses.

Ratio of two- and three-point correlators, $\dfrac{C^{\Gamma,O}_{3\text{pt}}(t_{\text{src}}, t, t_{\text{snk}})}{C_{2\text{pt}}(t_{\text{src}}, t_{\text{snk}})}$ with

$$C^{(2)}(t_{\text{src}}, t_{\text{snk}}) = \sum_{\alpha,\beta} \left(\frac{1+\gamma_t}{2}\right)_{\alpha\beta} \langle N_\beta(t_{\text{snk}})\bar{N}_\alpha(t_{\text{src}})\rangle,$$

$$C^{(3)\Gamma,O}(t_{\text{src}}, t, t_{\text{snk}}) = \sum_{\alpha,\beta} \Gamma_{\alpha\beta}\langle N_\beta(t_{\text{sink}})O(t)\bar{N}_\alpha(0)\rangle,$$

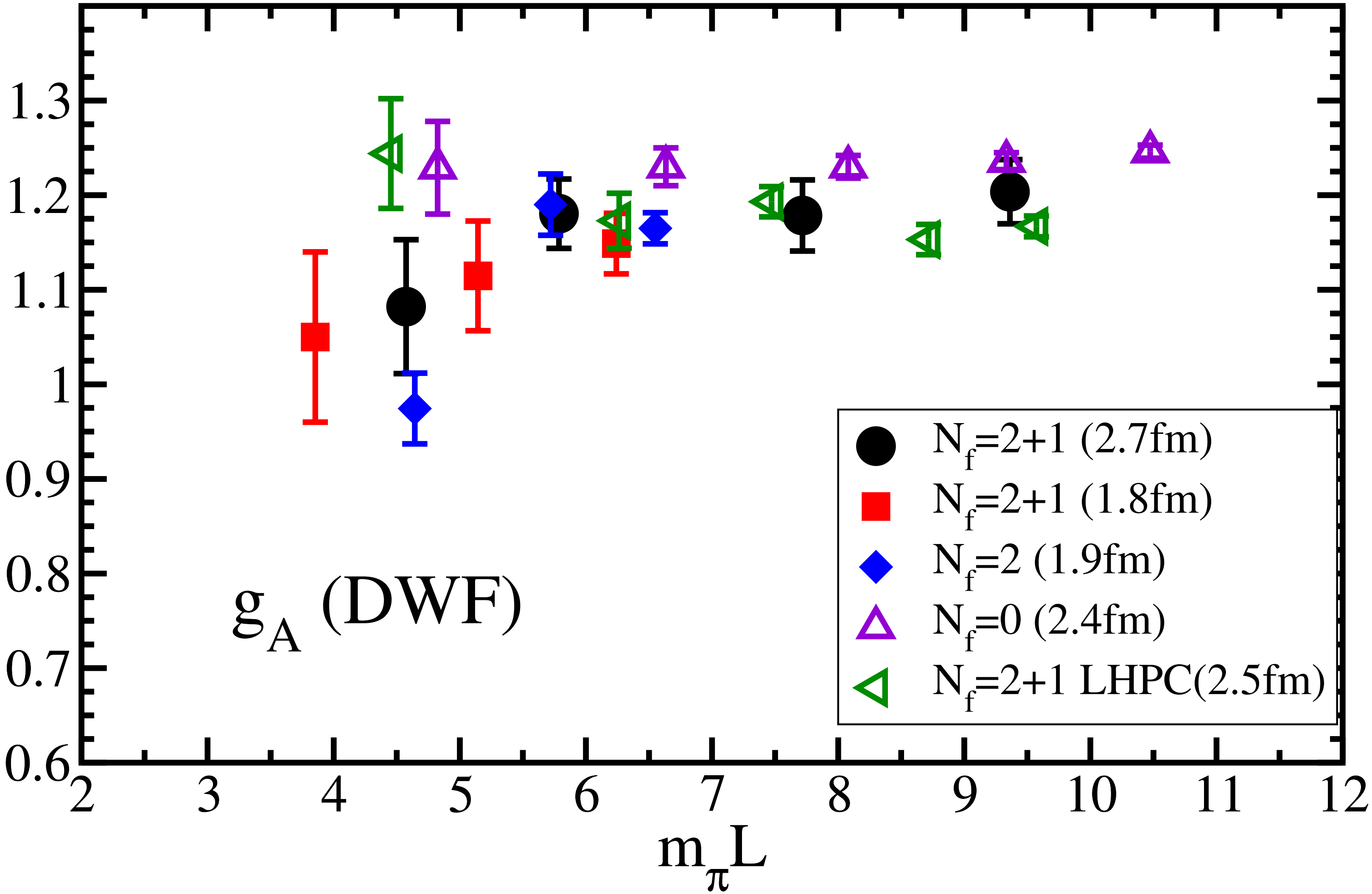
with appropriate nucleon operator, eg, $N = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$, gives a plateau in t for a lattice bare value $\langle O\rangle$ for the relevant observable, with appropriate spin ($\Gamma = (1+\gamma_t)/2$ or $(1+\gamma_t)i\gamma_5\gamma_k/2)$ or momentum-transfer (if any) projections.

More specifically for the form factors, ratios such as

$$\frac{C^{(3)\Gamma,O}_{\text{GG}}(t_{\text{src}}, t, t_{\text{snk}}, \vec{p}_{\text{src}}, \vec{p}_{\text{snk}})}{C^{(2)}_{\text{GG}}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{src}}, \vec{p}_{\text{snk}})} \times \sqrt{\frac{C^{(2)}_{\text{LG}}(t, t_{\text{snk}}, \vec{p}_{\text{src}}))C^{(2)}_{\text{GG}}(t_{\text{src}}, t, \vec{p}_{\text{snk}}))C^{(2)}_{\text{LG}}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{snk}}))}{C^{(2)}_{\text{LG}}(t, t_{\text{snk}}, \vec{p}_{\text{snk}}))C^{(2)}_{\text{GG}}(t_{\text{src}}, t, \vec{p}_{\text{src}}))C^{(2)}_{\text{LG}}(t_{\text{src}}, t_{\text{snk}}, \vec{p}_{\text{src}}))}}$$

with point (L) or Gaussian (G) smearings, give plateaux dependent only on momentum transfer.

Some time ago (2007) Takeshi Yamazaki reported unexpectedly large deficit in lattice calculation [8]:



[8] T. Yamazaki *et al.* [RBC+UKQCD Collaboration], Phys. Rev. Lett. **100**, 171602 (2008).

Long story short, by 2017: deficit in nucleon g_A/g_V calculated in lattice QCD with small volumes and heavy mass.

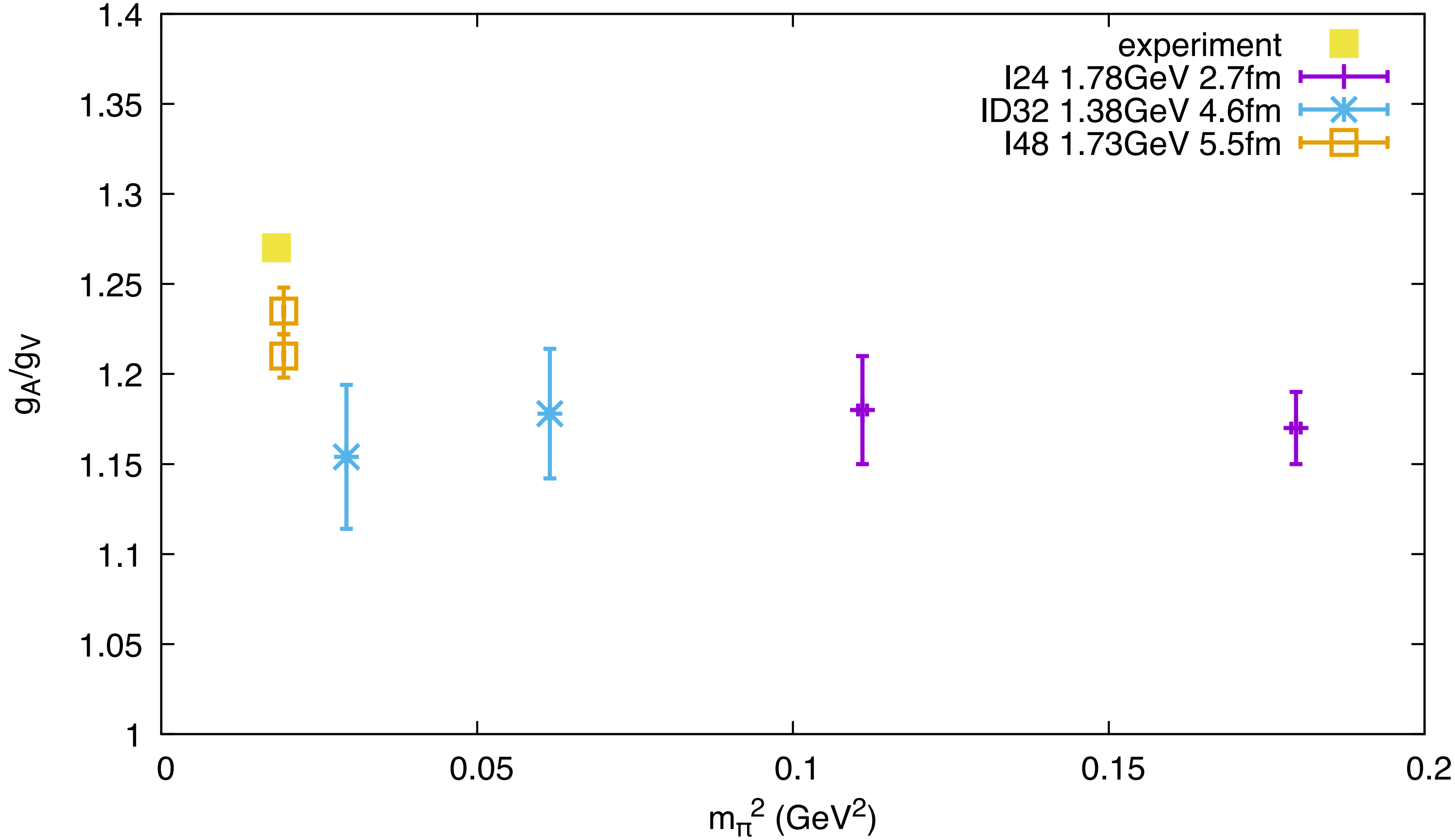
Yet a validation of lattice QCD: As of Lattice 2017, with similar quark mass and lattice cuts off,

- Calculations with overlap-fermion valence quarks on RBC+UKQCD DWF ensembles: ~ 1.2[9],
- Wilson-fermion unitary calculations now agree too once $O(a)$ systematics is removed:
  - PACS, 1.16(8)[10],
  - QCDSF$\sim$ 1.1[11],
- and even a Wilson valence on HISQ, PNDME[12], ~ 1.2,
- except the then latest DWF valence[13] on HISQ staggered ensembles after an extrapolation.

g_A from different actions "blindedly" agree with deficits once $O(a)$ systematics is removed,

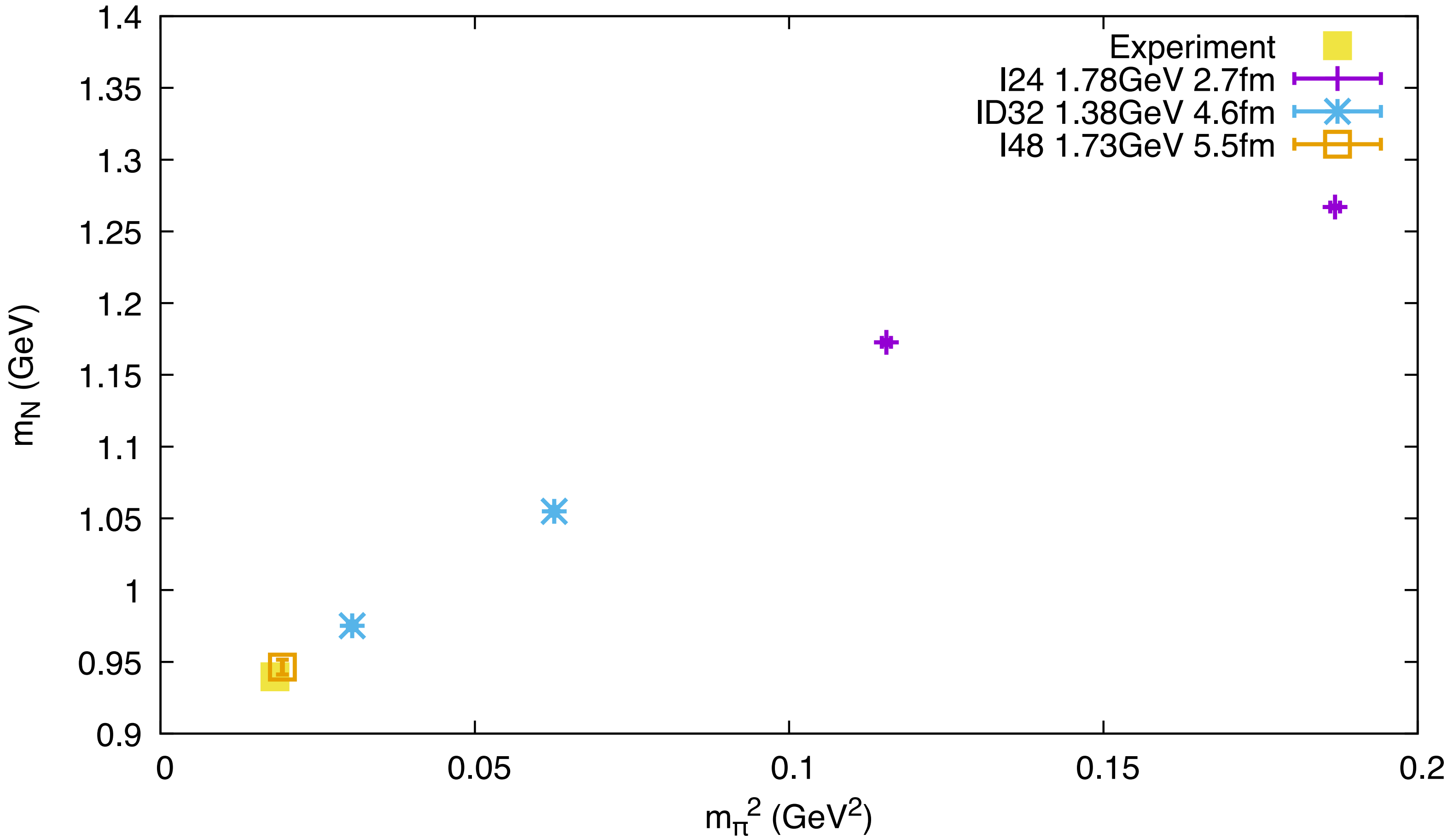
[9] J. Liang, Y. B. Yang, K. F. Liu, A. Alexandru, T. Draper and R. S. Sufian, arXiv:1612.04388 [hep-lat].
[10] A parallel talk by Tsukamoto at Lattice 2017, Granada; K. I. Ishikawa *et al.* [PACS Collaboration], Phys. Rev. D **98**, no. 7, 074510 (2018) doi:10.1103/PhysRevD.98.074510 [arXiv:1807.03974 [hep-lat]].
[11] J. Dragos *et al.*, Phys. Rev. D **94**, no. 7, 074505 (2016) doi:10.1103/PhysRevD.94.074505 [arXiv:1606.03195 [hep-lat]].
[12] T. Bhattacharya, V. Cirigliano, S. Cohen, R. Gupta, H. W. Lin and B. Yoon, Phys. Rev. D **94**, 054508 (2016) [arXiv:1606.07049].
[13] E. Berkowitz *et al.*, arXiv:1704.01114 [hep-lat]; C. C. Chang *et al.*, Nature **558**, no. 7708, 91 (2018) [arXiv:1805.12130 [hep-lat]].

More recently on RBC+UKQCD 2+1-flavor DWF, Iwasaki gauge, at 1.730(4) GeV and 5.5fm:

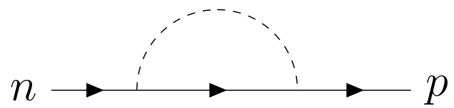


g_A/g_V calculated by RBC+LHP trends to the experiment.
Almost all other groups saw similar trends.

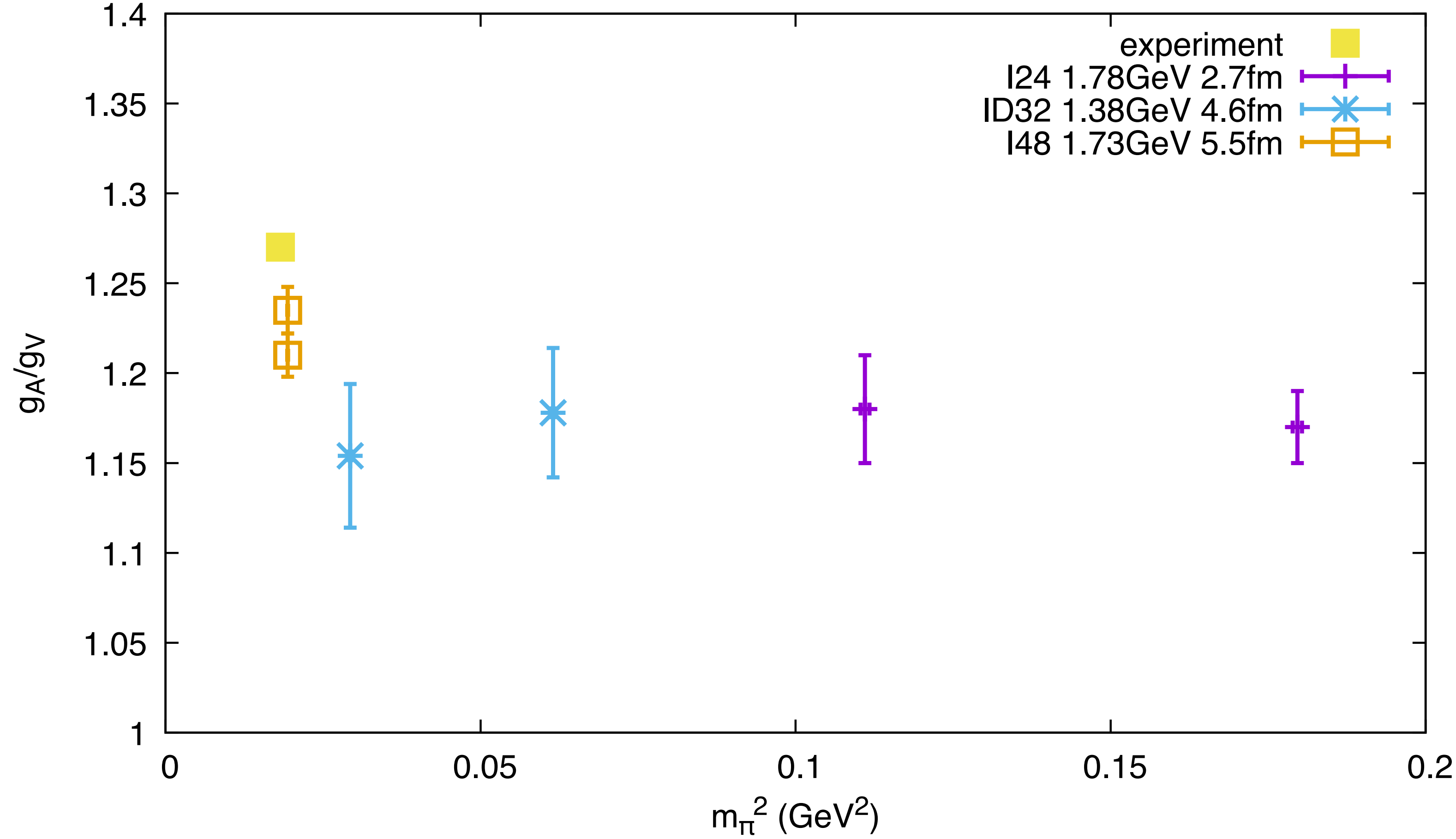
Along with the nucleon mass with non-linear dependence on quark mass:



much expected chiral log, $m_\pi^2 \log m_\pi^2 \sim m_q \log m_q$?



And we still see some deficit:



So we like to check the systematic errors.

Previous RBC and RBC+UKQCD calculations addressed two important sources of systematics:

- Time separation between nucleon source and sink,
- Spatial volume.

And though not explicitly addressed yet, a better understanding of quark mass dependence is necessary.

No source or sink is purely ground state:

$$e^{-E_0 t}|0\rangle + A_1 e^{-E_1 t}|1\rangle + ...,$$

resulting in dependence on source-sink separation, $t_{\rm sep} = t_{\rm sink} - t_{\rm source}$,

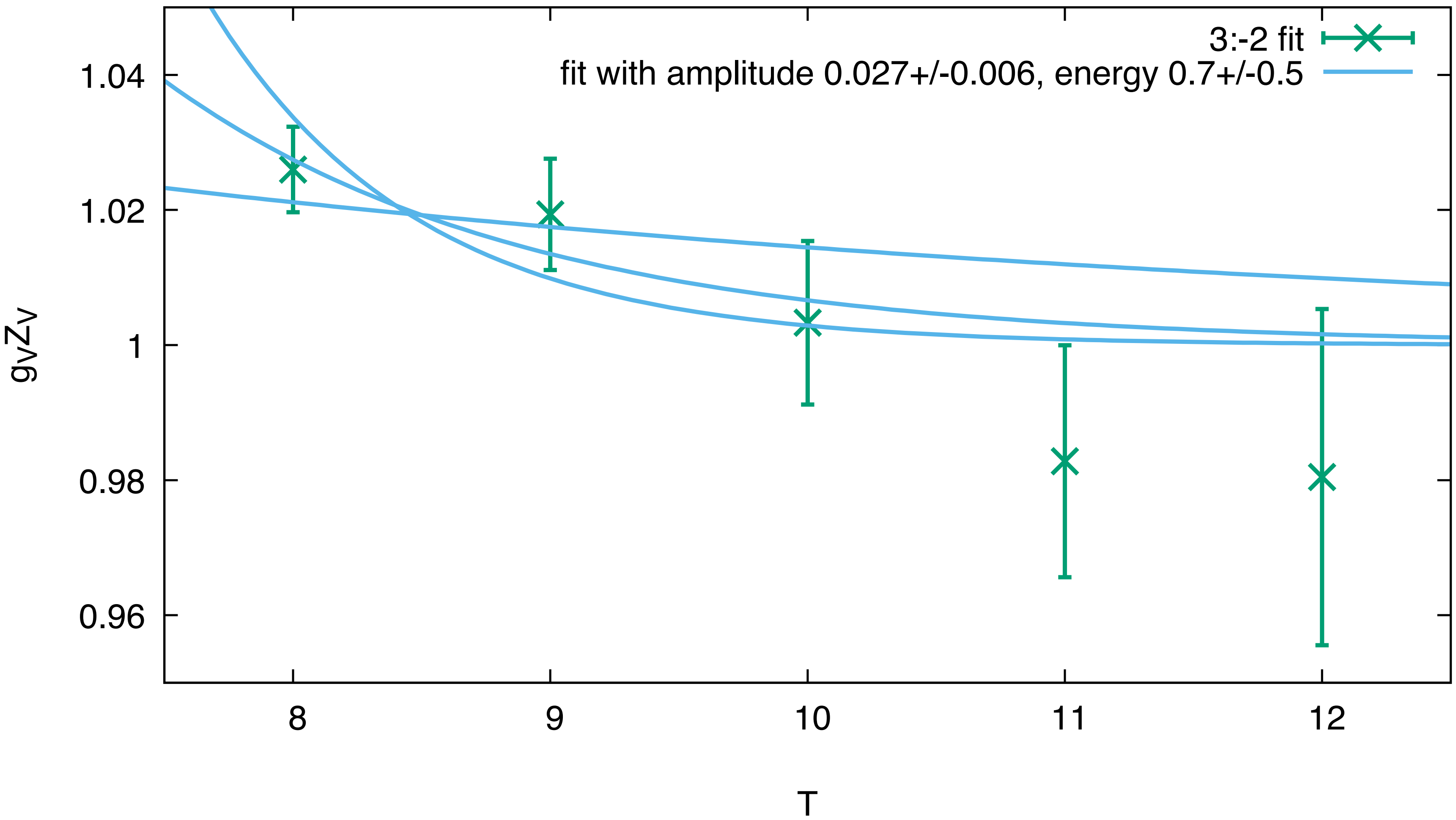
$$\langle 0|O|0\rangle + A_1 e^{-(E_1-E_0)t_{\rm sep}}\langle 1|O|0\rangle + ...$$

Any conserved charge, $O = Q$, $[H, Q] = 0$, is insensitive because $\langle 1|Q|0\rangle = 0$.

- g_V is clean,
- g_A does not suffer so much, indeed we never detected this systematics,
- structure function moments are not protected, so we saw the problem.

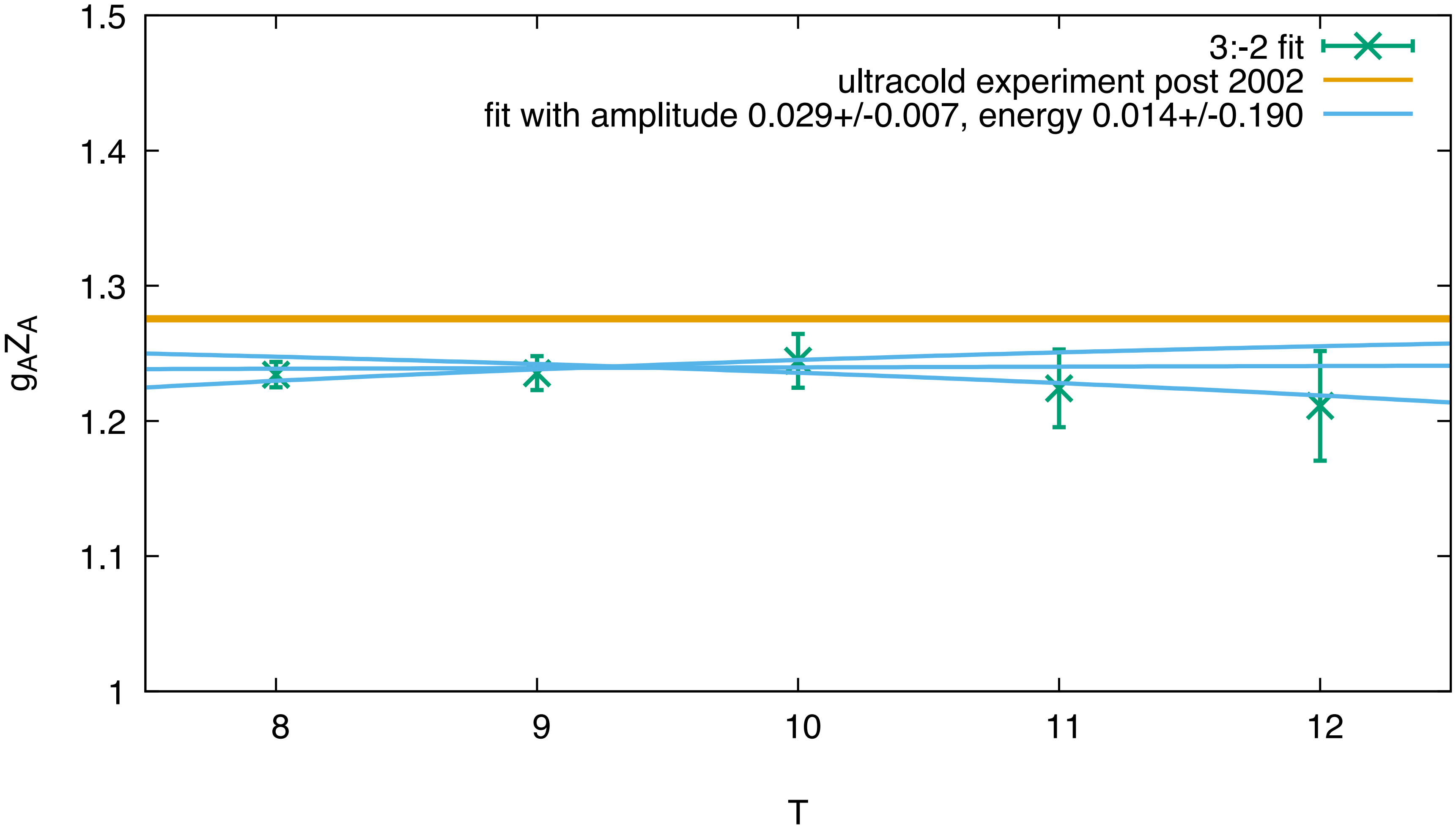
We can optimize the source so that A_1 is small, and we take sufficiently large $t_{\rm sep}$: Indeed with AMA we established there is no excited-state contamination present in any of our 170-MeV calculations.

Isovector vector charge, g_V, at $T = 8$ and 9, deviates from unity: possibly $O(a^2)$ mixing with excited states,



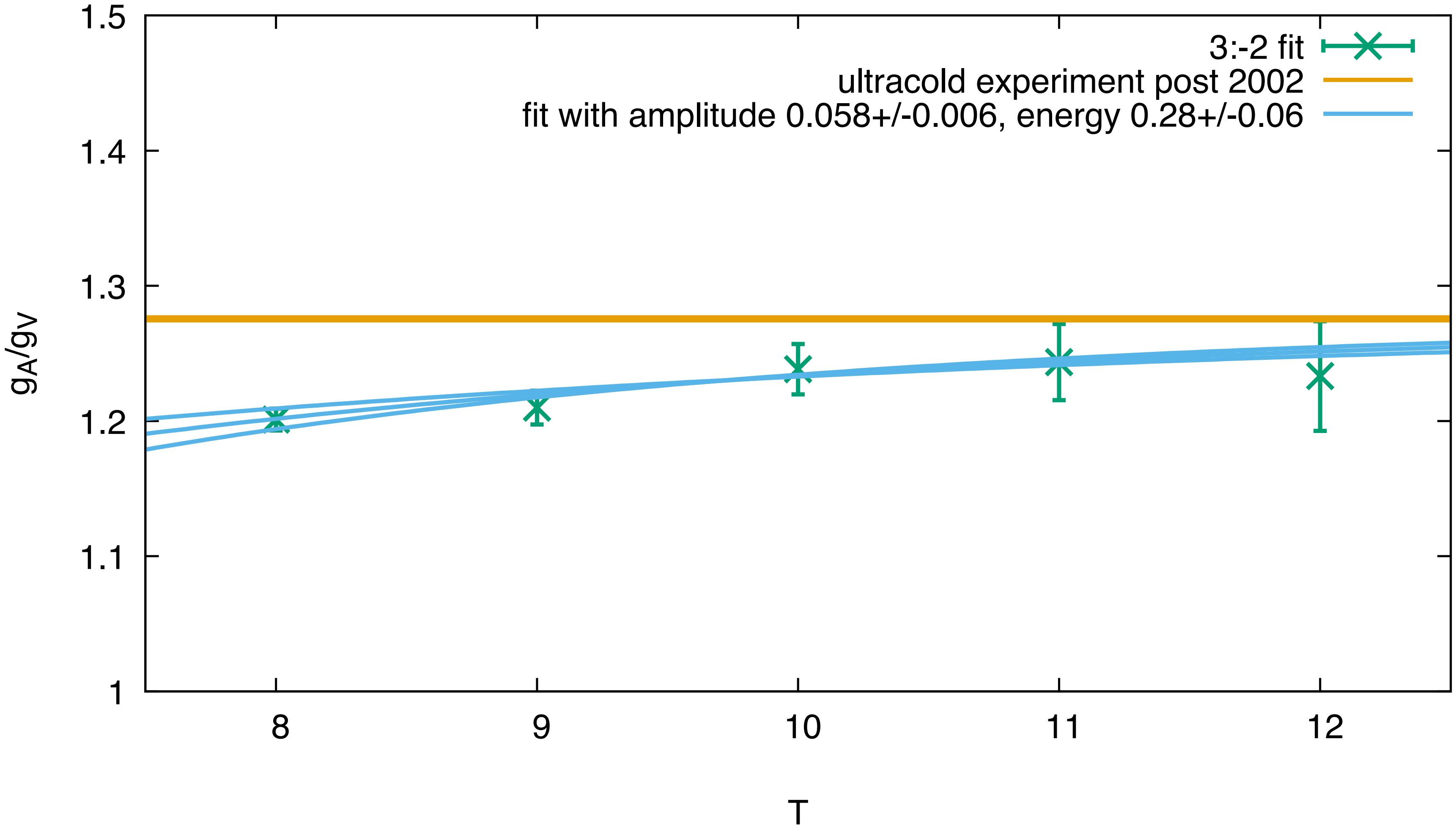
single-excitation fit is not so precise: we need shorter $T = 7$ and 6 calculations for further investigation.

Isovector axialvector charge, g_A, renormalized with $Z_A^{\rm meson}$, undershoots the experiment by a few percent.



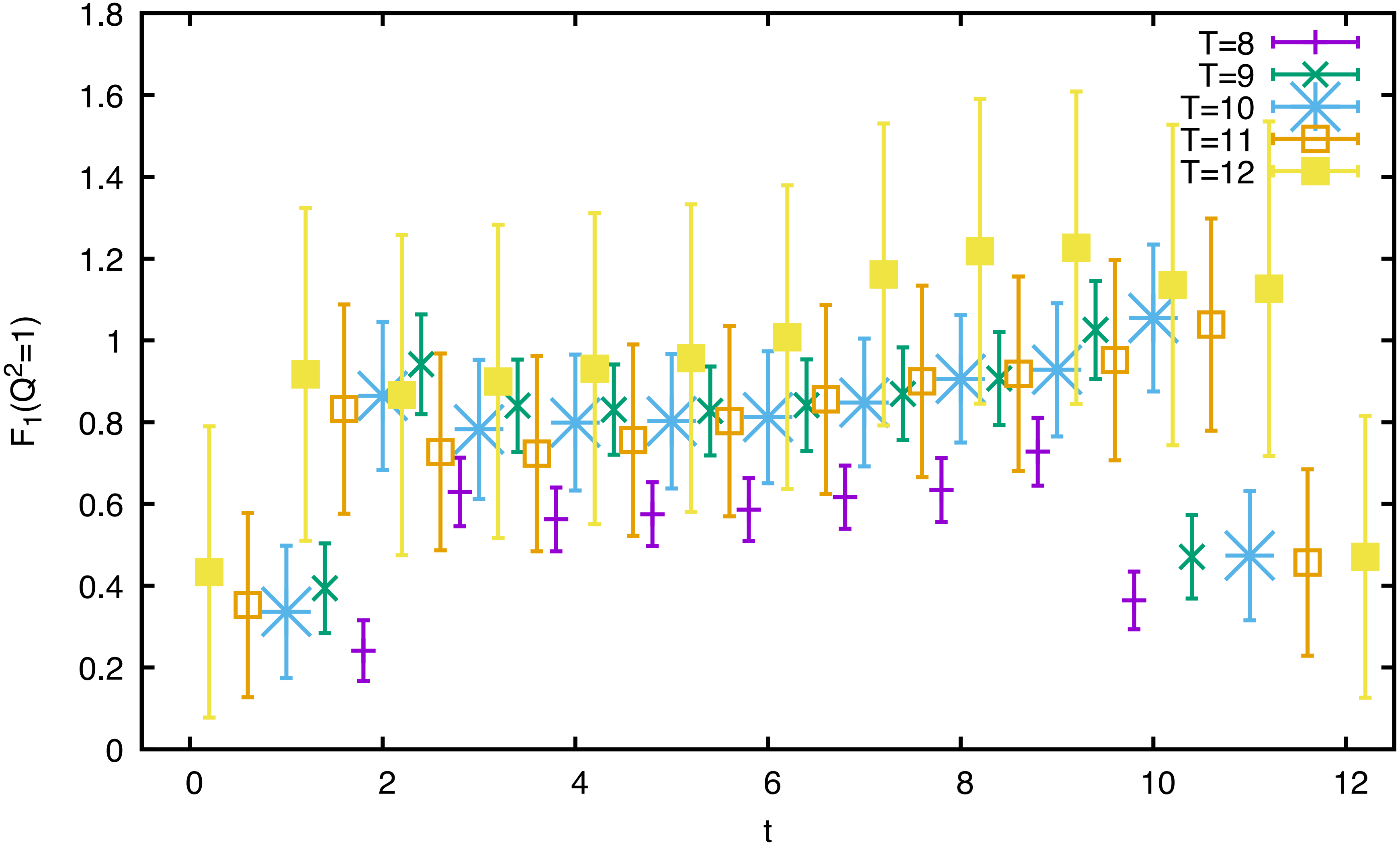
Excitation consistent with 0: this deficit appears independent of excited state contamination.

Isovector axialvector to vector charge ratio, g_A/g_V, undershoots the experiment by several percent.



On top of the g_V $T-$dependence, a better precision.

Can we quantify such possible excited state in momentum-dependent form factors?



$F_1(Q^2 = 1)$ from $T = 8$ may deviate from the others.
$T = 12$ is very noisy. Larger Q^2 are noisier.

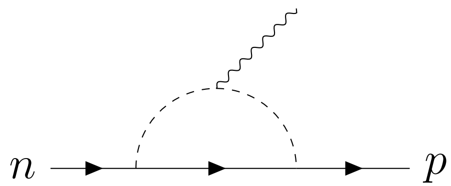
RBC+UKQCD DWF ensembles at physical mass, plans/wishes as of 2021:

Immediate plan: we are finishing up on I48 form factors.

Short term: we are improving the statistics so we know better about excited-state and other systematics.

Mid term: isospin breaking,

- both u-d mass difference, $\delta_{u-d} \log \delta_{u-d}$,



- and EM.

Longer term: finer lattice spacing,

- $a^{-1} = 2.359(7)$ GeV, and
- then ≥ 3 GeV to unquench charm, ...